Effects of Ice Nucleation and Crystal Habits on the Dynamics of Arctic Mixed Phase Clouds

Muge Komurcu and Jerry Y. Harrington

I. INTRODUCTION

Arctic Mixed-phase clouds are frequently observed during the colder seasons (transition seasons and winter), and they are important for a number of reasons including their effects on Arctic climate. Significant amounts of supercooled liquid water are present in Arctic clouds, which affect the radiation and surface energy budgets considerably leading to a possible influence on the extent of the Arctic sea ice. Yet, numerical models poorly simulate Arctic clouds with substantial deviations from observations (especially in predicted liquid and ice water paths), and with results that are highly inconsistent among different models (Curry et al. 1996, Klein et al. 2009). Reasons for the deviation of the simulated ice and liquid water paths from observations are possibly due to a number of factors including problems with ice nucleation (e.g. Prenni et al., 2007, Fridlind et al., 2007), and uncertainties in ice crystal habits (vapor growth and sedimentation) used in models (Avramov and Harrington, 2009).

The relative lack of observations in the Arctic, and laboratory measurements of physical processes, increases the uncertainties related to ice nucleation and crystal habits; as a result our understanding of physical and dynamical processes taking place is quite limited. For instance, Prenni et al. (2007) propose that the deposition condensation nucleation parameterization of Meyers et al. (1992) overpredicts the amount of ice in clouds, which leads to an increased Bergeron process, and depletes the cloud through rapid precipitation. Furthermore, observations reveal that at times ice crystal number concentrations are much greater than the number concentrations of ice forming nuclei (Hobbs, 1969) though this discrepancy could be due to artificial over-estimates of cloud ice concentrations due to ice shattering on probe inlets (McFarquhar et al., 2007). Still, it appears that there is a discrepancy between measured ice concentrations and model simulated ice amounts. Therefore other nucleation mechanisms have been put forward to explain the higher ice crystal concentrations in addition to the classical heterogeneous nucleation mechanisms (i.e. depositional, condensational, immersion, and contact freezing). These mechanisms include secondary ice production processes like ice splinter production during riming, which is thought to occur between -5 and -7 ^{o}C and the fragmentation of ice crystals (Hobbs and Rangno, 1985). Some relatively controversial primary ice nucleation mechanisms have also been suggested such as Evaporation IN (Rosinki and Morgan, 1991), and Evaporation Freezing (Cotton and Field, 2002). These same theories have also been incorporated into models to allow for a better match with observed water paths. For example, by using Evaporation IN and Evaporation Freezing, Fridlind et al. (2007) improve comparisons between the simulated and observed ice concentrations.

Recent findings by Avramov and Harrington (2009) show that the choice of simplified ice crystal habit used in cloud models leads to large differences in the partitioning of phase between liquid water and ice in layered mixed-phase clouds. According to their study, large differences in simulated water paths exist among different habits (plates, columns, and spheres) assumed in the different simulations. In addition, large differences in water paths also exist within the same habit category, these differences being due to the choice of mass-dimensional and fall-speed coefficients chosen from the literature. Habits determine the fall speed, and hence in-

* *Corresponding author address:* Muge Komurcu,
Penn State University, Dept. of Meteorology,
State College, PA, email: mxk457@psu.edu.

cloud residence time of the ice crystals. Habits also influence the growth rate of ice crystals through these mass and fall-speed relations. The combined impact of simplified habits on simulated clouds is to strongly alter the predicted water paths.

In order to understand Arctic clouds and improve future climate predictions, there is a need to improve model predictions in the Arctic. In this paper, we analyze and inter-compare the impacts of different ice nucleation mechanisms and ice crystal habits on mixed-phase cloud dynamics. However, the microphysical uncertainties related to habits and nucleation are tied directly to the cloud dynamics that help maintain persistent mixed-phase clouds. For example, cloud top radiative cooling may be the driver of the circulations in cloud (Harrington et al., 1999), and the degree of cooling is dependent upon the amount of liquid water. Precipitation also induces a stabilization effect through latent heating and cooling, which may lead to a decoupling of the cloud from the surface (Stevens et al., 1998). Cutting off the moisture supply through decoupling may lead to cessation of the cloud. Another process that plays a significant role in cloud dynamics is entrainment, which brings in the warmer and drier air above cloud top; however, it is challenging to distinguish the impact of entrainment. Therefore, we analyze the relative importance of the processes that influence the dynamics of the cloud, such as radiative cooling at cloud top, cloud base stabilization, entrainment, and their connections to crystal habit and ice nucleation. In order to separate the influences of the various processes affecting the dynamics, we use sensitivity studies by fixing the radiative cooling, and the diabatic influences of ice precipitation on the cloud layers.

II. MODEL

We use the Regional Atmospheric Modeling System developed at Colorado State University (RAMS). Our initial simulations are currently being conducted in two dimensions because of computational limitations. The model is set up similarly to those in the studies in Klein et al. (2009) with 168 grid points in the horizontal direction, and 100 grid points in the vertical direction, and it is initialized with the same sounding of Klein et al (2009). The horizontal grid spacing is 60 m, and the vertical grid spacing is 30 m. Lateral boundary conditions are cyclic, the top boundary is a rigid lid with a Rayleigh damping layer, and bottom boundary is the ocean surface with constant surface fluxes. The model time step is 1 s, with simulation duration of 12 hours. Radiation is calculated every 20 s using the Fuo and Liou radiation scheme (Fu and Liou, 1992) in the infrared while solar radiation is ignored because solar influence is negligible in the Arctic in October. Subgrid scale turbulence is parameterized using the model of closure method of Deardorff (1980).

III. METHODS

We first perform simulations with a single habit choice (hexagonal plates) and use a single nucleation mechanism (deposition-condensation nucleation, contact freezing, evaporation freezing, or evaporation IN) in each simulation. The suite of simulations is then re-done with dendritic crystals. We do not include surface fluxes and large-scale forcing because we wish to isolate the impacts of nucleation and habits on the growth and evolution of mixed-phase clouds through internal dynamic processes only. Later simulations will use large scale moisture and heat sources and large-scale dynamical effects. These simulations, in which microphysical, radiative, and dynamic processes are allowed to interact, are termed our standard simulations.

To analyze the relative importance of processes that influence the dynamics, such as the radiative cooling at cloud top, and the stabilization at cloud base, we use a set of idealized sensitivity studies. Simulations are performed with fixed in-cloud radiative cooling of 30 W/m^2 or 130 W/m^2 and with fixed diabatic effects of precipitation using standard and strong cloud-base stabilization. To fix the in-cloud radiative cooling, we use the mixed-phase simulations in which all processes are allowed to interact (our standard simulations) and we calculate the radiative cooling integrated through the liquid cloud layer for every 20s starting from the 3rd hour of the simulation. During the sensitivity simulation, we weight the radiative cooling integrated through the cloud layer so that it matches the value computed from our actual mixed-phase cloud simulation. Finally, we multiply the in cloud radiative cooling at each height and location by this ratio to obtain the desired constant integrated cooling in cloud (between 30 or 130 W/m^2). In-cloud integrated cooling is shown in Figure 1, for simulations with (a) 30 W/m^2 and (b) 130 W/m^2. We apply a similar approach to the diabatic heating terms to θ_{il} in order to fix the effects of ice formation and precipitation. To do so, we compute the average change in the ice-liquid water potential temperature starting at the 3rd hour of the actual mixed-phase cloud simulations. These profiles provide an estimate of the diabatic influence of ice growth and precipitation on cloud dynamics through the cloud-base stabilization. The fixed profiles obtained from the standard mixed-phase simulations are applied in the following way. We calculate the integrated in cloud latent heating, and integrated below cloud latent cooling from our standard simulations. The obtained profile is then applied to the sensitivity simulations at each time step. Because the cloud layer changes in time as simulation evolves, at each time step, the depth of the liquid portion of the cloud layer is calculated. We then apply the latent heating assuming a sinusoidal function from cloud base to cloud top with the constraint that the integrated amount of total heating is always constant and equal to the in-cloud heating obtained from the mean profile. Because sublimation causes a net cooling below cloud base, we apply a linear cooling downwards for 4 vertical grid points with the cooling being constant below those four points. The integrated total cooling of this idea profile is constrained to match the average value derived from the standard mixed-phase simulations.

The profile of the mean change in θ_{il} is shown in Figure 2 (a), a sample profile of the adjusted change in θ_{il} based on the limits of a chosen cloud top and base is given in Figure 2 (b). Using an idealized profile allows us to control the impact of precipitation on cloud base stabilization. For instance, when precipitation is weak we expect a small net warming of the liquid cloud and cooling of the subcloud region. However when ice precipitation is strong, a greater net warming of the liquid cloud, and cooling of the subcloud, is expected. Consequently, we can emulate the effects of strong precipitation and cloud-base stabilization if we multiply the mean change in the θ_{il} profile by a factor. We use values that range from 1.5 to 4, which cover a realistic range of the diabatic effects of precipitation. We then calculate the new profiles using the sinusoidal method based on the cloud base and top in the same way as the standard cloud-base stabilization explained above.

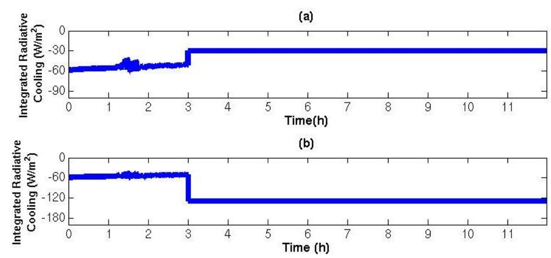

Figure 1. Time series of integrated in-cloud radiative cooling for simulations with (a) 30 W/m2 (b) 130 W/m2.

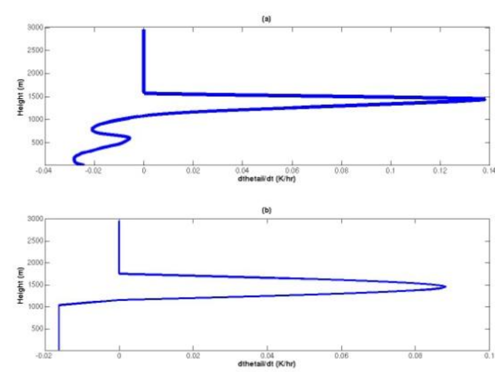

Figure 2. (a) Profile of the actual mean change in θ_{il} **(b)** A sample profile of the adjusted change in θ_{il} based on a chosen cloud top and cloud base.

Using these idealized profiles allows us to control the relative strength of cloud top longwave cooling, and the net diabatic effects of precipitation on cloud base stabilization. We use this method next to discuss the relative importance of cloud top radiative cooling and ice precipitation on the dynamics of mixed-phase clouds.

IV. EFFECTS OF ICE NUCLEATION AND CRYSTAL HABITS

The effects of ice nucleation and habits on liquid water paths in our standard simulations can be seen in Figure 3. Because of their lower effective density, dendrites have lower fall speeds and so can stay longer in the liquid portion of the cloud, thus depleting more liquid through the Bergeron process. Consequently, simulations with dendrites have much less liquid water compared to simulations with hexagonal plates. These results are consistent with Avramov and Harrington (2009), where diverse water paths were obtained for the same initial conditions but with different habit choices. Although different ice nucleation mechanisms also yield different water paths, the difference obtained through habits is much larger as long as the ice concentrations produced by the nucleation mechanisms are similar.

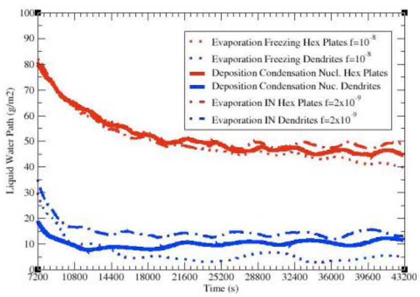

Figure 3. Domain-averaged liquid water path (g/m^2) as a function of time for dendrites and hexagonal plates with different mechanisms. All results use a 10-point running average in time.

A measure of the strength of circulations, turbulent kinetic energy (TKE), is shown in Figure 4. Simulations that yield larger liquid water paths, such as simulations with hexagonal plates, end up having stronger circulations. Note that while nucleation certainly influences growth and therefore dynamics, habit has a greater impact on TKE. This result is important since many studies assume that ice nucleation dominates the microphysical impact on mixed-phase clouds (e.g. Harrington and Olsson, 2001; Fridlind et al., 2007). We next investigate the processes that influence the TKE using two representative simulations with deposition-condensation nucleation for dendrites and hexagonal plates.

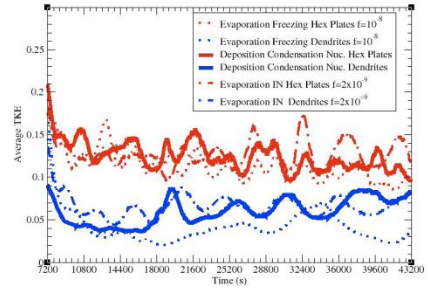

Figure 4. Average Turbulent Kinetic Energy as a function of time for dendrites and hexagonal plates with different ice nucleation mechanisms. All results use a 10-point running average in time.

Since we turned off the surface fluxes in these simulations, the main dynamic influence on cloud circulations is entrainment and cloud top radiative cooling, which produces TKE through buoyancy. As a result less radiative cooling means less buoyant production of TKE, or weaker circulations. Cloud-integrated radiative cooling (Figure 5) is a measure of the total infrared cooling of the cloud, and therefore of the total amount of buoyancy produced. The reduced LWP of the simulations with dendrites produces much less radiative cooling compared to hexagonal plates, and it appears that this is the main reason for the dynamic differences between simulations with different habits. However, ice precipitation can also influence the dynamics of the cloud layers.

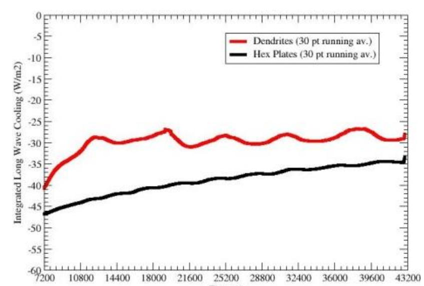

Figure 5. Domain-averaged, vertically-integrated longwave cooling in W/m^2 as a function of time with different habits for deposition-condensation nucleation. All results use a-30point running average in time.

Precipitation influences cloud dynamics indirectly through cloud base stabilization. Ice particles grow large enough by vapor diffusion to fall out of the liquid layer leading to a net latent heating of the liquid cloud. This falling ice then sublimates in the subcloud layer leading to a cooling of the region beneath cloud base. The net result of precipitation, then, is to stabilize the cloud layer with respect to the subcloud (e.g. Nicholls, 1984; Stevens et al., 1998).The stabilization effect is shown in Figure 6, where we plot the domain-averaged potential temperature (θ) as a function of height at the initial and 6th hour of the simulations. Note the relatively strong stabilization that is produced during the simulation.

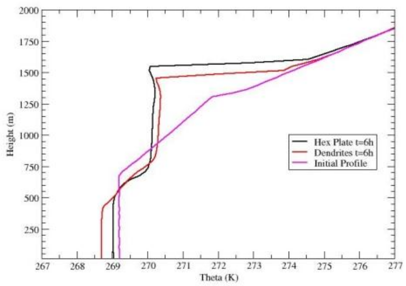

Figure 6. Domain-averaged θ (K) as a function of height (m) with different habits for deposition-condensation nucleation at initial and 6th hours of the simulations.

Stabilization at cloud base is much stronger in simulations with dendrites because of the rapid growth, and low fall-speeds of the dendrites. Therefore, the more substantial cloud-base inversion in θ leads to a significant drop in TKE. Our results are consistent with Harrington et al. (1999), where dendrites lead to increased ice formation, growth and weaker circulations.

V. SENSITIVITY STUDIES

We have seen that both radiative cooling and precipitation-induced stabilization significantly affect cloud dynamics, and it appears that these are the two main factors that drive the dynamics of mixed-phase cloud layers in the Arctic. We do note, however, that entrainment also plays a role in the dynamic evolution of these layers, and entrainment effects will be examined in future work. In this section, we attempt to isolate the individual effects of radiative cooling and precipitation-induced stabilization. We present simulation results obtained using these idealized processes in which ice crystals are assumed to be spheres and ice nucleates through evaporation freezing nucleation. Later stages of this study will include a similar analysis using simulations with other habits and nucleation mechanisms.

5.1 Effect of Radiative Cooling

Here we examine the effect of increasing integrated in-cloud radiative cooling from 30 W/m^2 to 130 W/m^2 using the standard cloud-base stabilization (in other words, the un-modified profiles derived from the standard simulations). To understand the cloud dynamics response to increasing radiative cooling, we examine the magnitude of the horizontal (Figure 7) and vertical components (Figure 8) of the circulations. As radiative cooling is increased, circulations become much stronger and this affects the extent of the cloud layer (Figure 9). Although circulation depth is limited below the liquid base by cloud-base stabilization, circulations can extend below cloud base to some degree as radiative cooling is increased. This result is important because it gives an insight into the magnitude of radiative cooling that is needed to prevent the decoupling of the cloud layer from the subcloud layer. If the layers can be recoupled, then moisture and aerosol can be resupplied to the layer, which could either increase or decrease the layer's longevity. For instance, if recoupling supplies more vapor, then precipitation-drying of the layer can be reduced. However, if more ice nucleating aerosol are added to the layer during recoupling, then precipitation could be enhanced potentially causing cloud collapse.

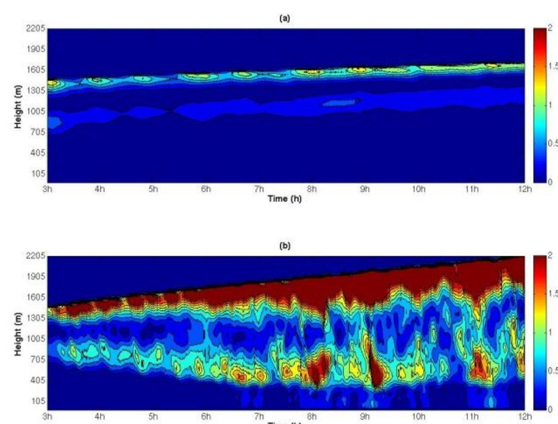

Figure 7. Domain averaged profile of u'u' in time for constant latent in cloud heating and below cloud cooling with constant radiative cooling of **(a)** 30 W/m^2 and **(b)**130 W/m^2.

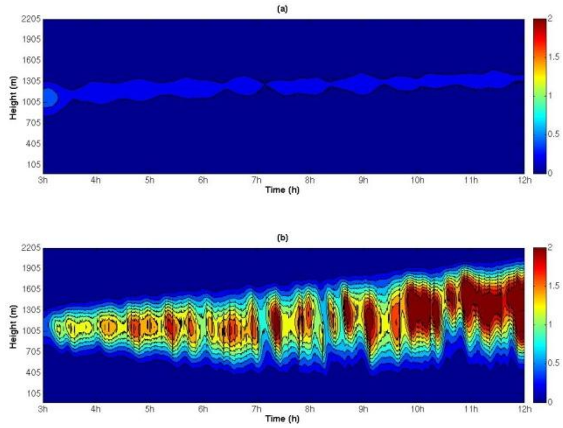

Figure 8. Domain averaged profile of w'w' in time for constant latent in cloud heating and below cloud cooling with constant radiative cooling of **(a)** 30 W/m^2 and **(b)**130 W/m^2.

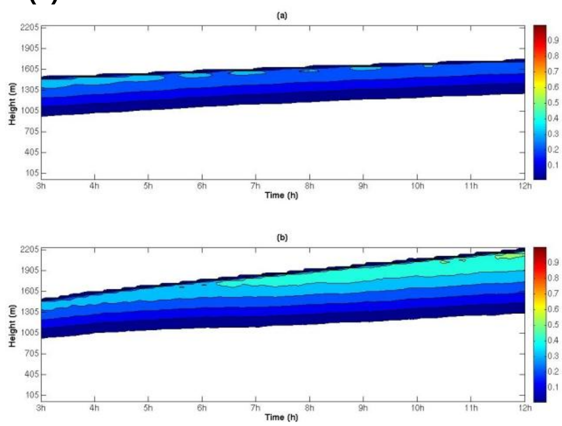

Figure 9. Domain averaged profile of liquid water content in time for constant latent in cloud heating and below cloud cooling with constant radiative cooling of **(a)** 30 W/m^2 and **(b)** 130 W/m^2.

5.2 Effect of Ice Production and Precipitation

Results in the previous subsection were based on diabatic precipitation effects derived from the standard simulation. In order to explore the effect of ice formation and precipitation on cloud dynamics, we apply a stronger precipitation induced cloud-base stabilization as explained in section III. We do this to emulate the diabatic effects that increasing precipitation rates may have on the net stabilization of the cloud layer, and hence on the dynamics. Naturally, then, the result of increasing the idealized precipitation rate is strong cloud base stabilization. This implies that significant in-cloud ice production, which causes net latent heating of the liquid layer, and significant sub-cloud ice precipitation, which leads to a latent cooling, produces much stronger cloud base stabilization. Effects of increasing the net cloud base stabilization on the strength of the circulations are seen in Figure 10 for horizontal component of the winds and Figure 11 for the vertical component of the winds. Comparing Figures 7 and 8 with Figures 10 and 11, circulation strengths are reduced significantly as the cloud base stabilization is increased. Figure 12 shows the cloud layer for both simulations.

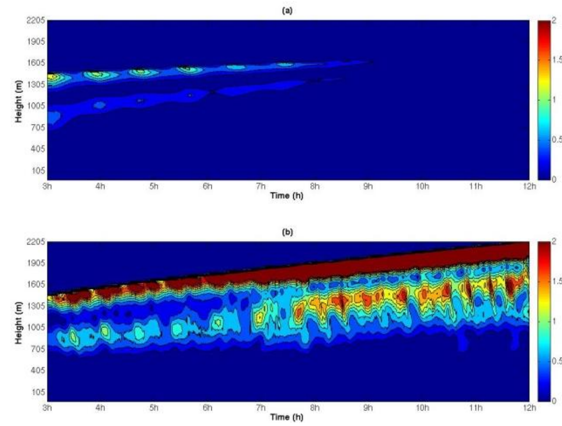

Figure 10. Domain averaged profile of u'u' in time for strong cloud base stabilization with constant radiative cooling of **(a)** 30 W/m^2 and **(b)** 130 W/m^2.

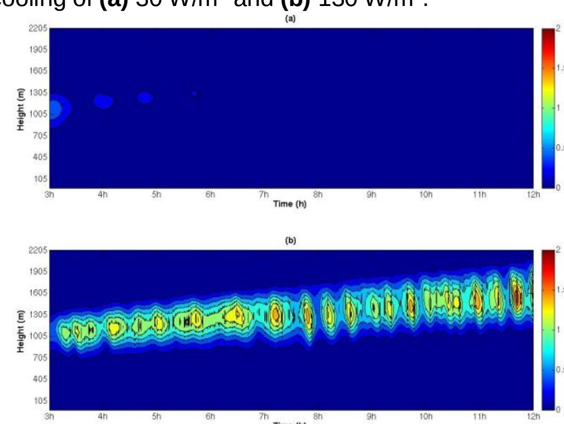

Figure 11. Domain averaged profile of w'w' in time for strong cloud base stabilization with constant radiative cooling of **(a)** 30 W/m^2 and **(b)** 130 W/m^2.

Our results show that with strong stabilization it is harder for downdrafts to penetrate below the cloud base, and circulations are restricted to the cloud layer. This is consistent with Stevens et al. (1998) where the drizzle induced stabilization reduces the vertical extent of the downdrafts in liquid Stratocumulus clouds. Stabilization constrains the layer to deeper mixing and prevents the radiatively driven convection from reaching the surface. Although in our simulations we have a non-precipitating liquid cloud, we have the implied effect of ice precipitation hidden in the fixed profiles of the change in θ_{il}. In our simulations, we also see that even when the magnitude of the radiative cooling is increased, penetration below cloud base is small and quite limited in case of strong precipitation induced cloud-base stabilization. Harrington and Olsson (2001) also showed that in mixed-phase cloud simulations, the strength of the circulations is reduced significantly at times of strong ice precipitation, and the reduction is much stronger compared to liquid clouds.

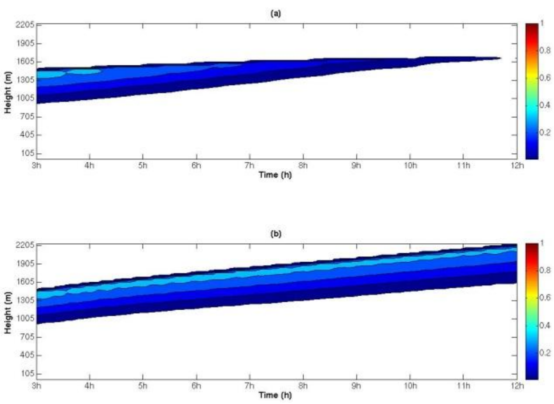

Figure 12. Domain averaged profile of liquid water mixing ratio (g/kg) in time for constant latent in cloud heating and below cloud cooling with constant radiative cooling of **(a)** 30 W/m^2 and **(b)**130 W/m^2.

VI. SUMMARY

The preliminary results of this study show that both ice nucleation mechanisms and ice crystal habits have significant impacts on cloud structure and dynamics. Moreover, we have seen that ice crystal habit has an impact on cloud dynamics and evolution that is as strong as the impacts of nucleation, and in some instances is even stronger. Similar to Avramov and Harrington (2009), we observe that water paths differ substantially with the choice of habits for the same initial conditions. We also examine how the dynamics of the cloud layer respond to the change in assumed habit, by looking at the amount of radiative cooling and cloud-base stabilization. Because of their lower density, dendrites stay longer in cloud and deplete more liquid through the Bergeron process compared to hexagonal plates. Dendritic crystals grow quickly but fall slowly, and so the amount of liquid is reduced in these simulations. The radiative cooling is therefore much reduced in these simulations and ice precipitation is stronger yielding stronger cloud-base stabilization. Both the reduced radiative cooling and stronger cloud-base stabilization act to decrease the strength of circulations in simulations with dendrites.

Because microphysics is closely tied to cloud dynamics, we also investigated the solitary impacts of radiative cooling and cloud base stabilization by fixing these processes so that their impacts remain constant, in an energetic sense, throughout the simulation. We have seen that increasing radiative cooling can allow circulations to extend below cloud base. However, ice production and precipitation from the cloud layer is also important as precipitation define the degree of stabilization at cloud base. It is harder for motion to penetrate below cloud base when the stabilization is strong. Similar to the drizzling stratocumulus of Stevens et al (1998), in our mixed-phase cloud simulations, a strong cloud base stabilization acts as a barrier to mixing, and limits the circulations to the cloud layer. This limitation may lead to a decoupling of the cloud layer from the surface. In their study, the feedback between surface fluxes and moistening through sublimation of ice below cloud-base eventually acts to produce cumulus like convection that re-couples the layers. However, we do not impose surface fluxes in our simulations and in our case the surface is at a much lower temperature. We will investigate the impact of surface fluxes to our simulations in future studies.

For future studies, we also plan to investigate each nucleation mechanism and habit, and understand how the dynamics (e.g. radiative cooling, cloud base stabilization and entrainment) differ for each simulation. Improving our understanding of the cloud microphysical and dynamical processes is not only crucial for accurate modeling of clouds, but may be significant for the accurate estimates of global energy budgets in future models.

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