P3.11 TOWARDS GENERALIZED BOUNDARY CONDITIONS IN DISORT

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1. INTRODUCTION

The Discrete Ordinate Radiative Transfer Code (DISORT) (Stamnes et al., 1988, 2000) solves the transfer of monochromatic unpolarized radiation in a scattering, absorbing and emitting plane parallel medium, with a specified bidirectional reflectivity at the lower boundary. The latest version of the code uses the Linear Algebra PACKage (LAPACK) and supporting routines from the Basic Linear Algebra Subprograms (BLAS) for solving systems of simultaneous linear equations.

DISORT has been used by many researchers in the atmospheric science and other communities to calculate the transfer of monochromatic radiation. It has also become a kind of standard against which to compare other modeling results (Stamnes et al., 2000). The time consuming computation of the non-Lambertian lower boundary, however, has emerged as a kind of bottleneck for implementation in schemes, like line-by-line codes, that frequently call DISORT to do radiative transfer.

DISORT applies analytical corrections to improve the accuracy of intensity calculations. This feature requires accurate specification of the scattering phase function. This requirement, however, not always realized by users.

2. INTENSITY CORRECTION

Strongly forward-peaked scattering is treated in DISORT by the δ–M method (Wiscombe, 1977) in which the forward peak of the phase function is separated and approximated by a Dirac delta-function. The remaining part is represented by a truncated phase function that is expanded in a series of Legendre polynomials that is shorter than the series expansion of the original phase function. The procedure also requires the transformation (scaling) of the optical depth and single scattering albedo. The method has proven to be both accurate and efficient for flux computation, but it has been shown to introduce spurious oscillations around the true intensity (Nakajima and Tanaka, 1988).

To provide accurate intensity values with the short (truncated) series expansion of the phase function made possible by the δ–M method (and thus needing only low numbers of quadrature angles or "streams"), DISORT uses the Nakajima and Tanaka (1988) method. The method calculates accurate and approximate (as represented in DISORT by the truncated δ–M phase function) single and double scattered intensities, subtracts the approximate intensities from the δ-*M* intensities, and then adds back the accurate ones.

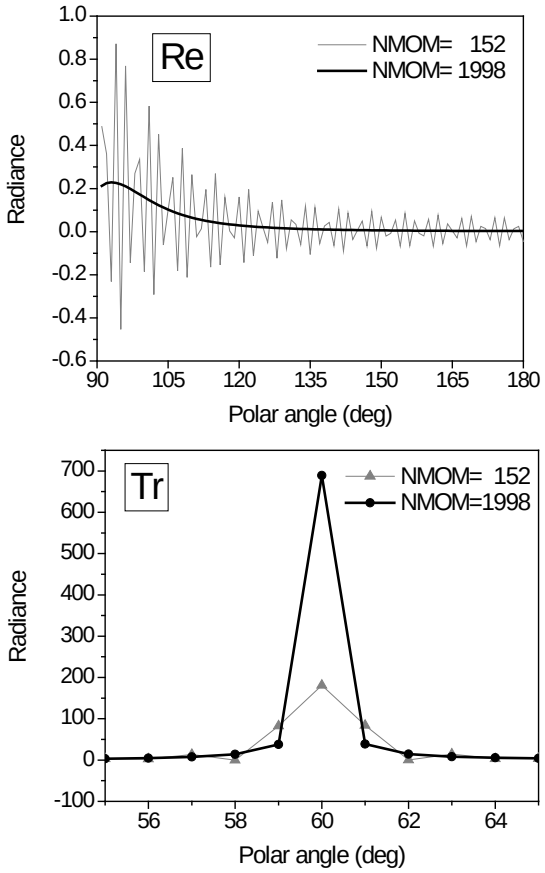

Figure 1. Reflected (top) and transmitted (bottom) radiances from a plane parallel ocean layer characterized by a highly anisotropic scattering phase function. The results are shown for two cases: (1) when all significant Legendre moments of the phase function expansion (NMOM=1,998) are used, and (2) when a shorter expansion is used (NMOM=152).

When using the Nakajima and Tanaka (1988) intensity correction it is imperative to provide the full

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phase function Legendre expansion, not just the Legendre coefficients for the truncated phase function. This is because accurate single and double scattered intensities can only be recovered from the full phase function expansion. Some users do not realize this important requirement. The consequence of using less than the required series expansion is illustrated in Figure 1.

Reflected and transmitted intensities (radiances) from a plane parallel ocean layer characterized by a highly anisotropic scattering phase function (Mobley, et al. *2002*) requiring 1,998 Legendre polynomials were calculated. (The Legendre moments were provided by A. Kokhanovsky and V. P. Budak.) Optical depth and single scattering albedo were unity and 0.99, respectively. The intensity of incident parallel beam at the top of the water layer was unity; polar angle cosine of the incident beam was 60°. A non-reflecting surface was assumed below the water layer. The calculations were done with DISORT using 152 computational polar angles and 152 Legendre polynomials in the truncated phase function and in the phase function used to calculate the single- and double-scattered intensities. The resulting intensities (labeled NMOM=152) are shown as a function of the polar angle in Figure 1. Note how badly the reflected intensity oscillates (top panel of Figure 1) around the correct solution (labeled NMOM=1998) that was calculated by providing all 1,998 significant phase function moments of the polynomial expansion. The NMOM=152 result of transmitted intensity also oscillates (bottom panel of Figure 1); in addition, the transmitted intensity is severely underestimated in the direction opposite to the sun (forward direction).

3. LOWER BOUNDARY

DISORT can accurately treat the non-Lambertian surface reflection characterized by a general and realistic bidirectional reflectance distribution function (BRDF). The formulation of the lower boundary allows for a BRDF that depends on the incident polar angle μ', reflected polar angle μ, and azimuth angle difference ϕ-ϕ' between the incident and reflected directions. By this formulation DISORT assumes that the surface BRDF is symmetric to the principle. To separate the Fourier components needed in DISORT, the bidirectional reflectivity ρ_d is expanded into a Fourier cosine series with $2N$ terms, where $2N$ is the number of computational polar angles ("streams"). Because the BRDF is assumed to be symmetric to the principle plane the expansion will only contain cosine terms.

$$\rho_d(\mu,\phi;-\mu',\phi') = \rho_d(\mu,-\mu',\phi-\phi') = \sum_{m=0}^{2N-1} \rho_d^m(\mu,-\mu')\cos m(\phi-\phi') \qquad (1)$$

To maintain high accuracy the coefficients of the Fourier expansion ρ_d^m are calculated from the defining equation, in which the integral is evaluated by an N_g-point Gaussian quadrature with weights w_k.

$$\rho_d^m(\mu,-\mu') =$$

$$(2-\delta_{m0})\frac{1}{\pi}\int_0^{\pi}\rho_d(\mu,-\mu';\phi-\phi')\cos m(\phi-\phi')\,d(\phi-\phi'), \qquad (2a)$$

$$\approx (2-\delta_{m0})\frac{1}{\pi}\sum_{k=1}^{N_g} w_k\,\rho_d(\mu,-\mu';\Delta\phi_k)\cos m\Delta\phi_k \qquad (2b)$$

$$m = 0, \ldots, 2N-1.$$

Here δ_{m0} is the Kronecker delta.

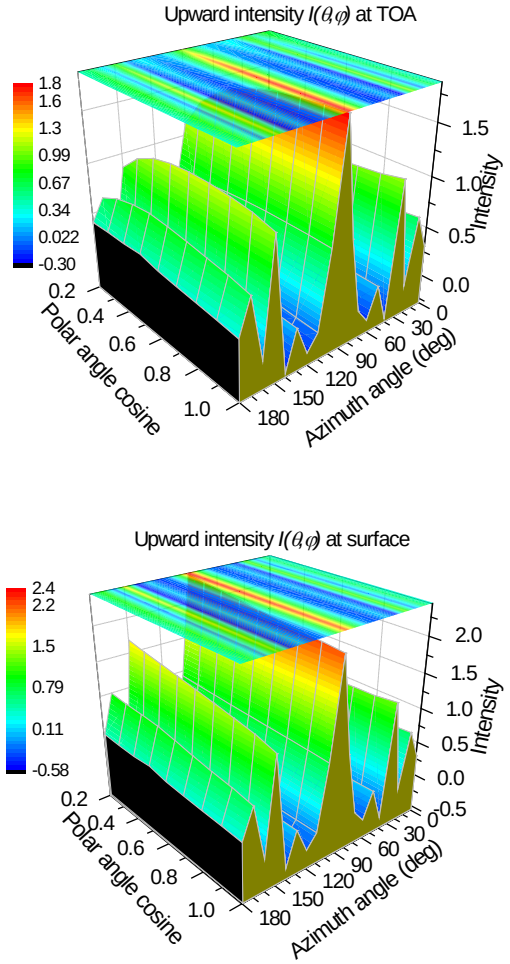

Figure 2. Example of incorrect intensities resulting from "undersampling" the integrand (using only a 50-point Gaussian quadrature) in Equation 2b.

Note that the integrand of Eq. 2a contains a highly oscillating cosine function whose frequency increases with the number of streams. In version 2 of DISORT this integral is evaluated using a fixed 50-point Gaussian

quadrature (N_g=50). This is usually sufficient for the frequently used value of N<16, but clearly "under-samples" the integrand and leads to errors when N is large. The situation is illustrated in Figure 2, which plots the upwelling intensities, as functions of the polar angle cosines and azimuth angles, at the top and the bottom of a scattering layer above a highly anisotropic surface. The intensities were calculated using 128 streams and N_g=50 in Equation 2b. Clearly, the intensities are incorrect; they widely oscillate as a function of the azimuth angle as a result of the error in the Fourier expansion of the surface bidirectional reflectance.

The bidirectional reflectance in this example is described by the model of Hapke (1993), and calculated from Equation 3. Its parameters and a plot of the reflectance are shown in Figure 3. In Equation 3 w is the single scattering albedo, α is the phase angle (π-ϕ), P is the phase function, B_0 and h are the amplitude and width of the shadow hiding opposition surge, respectively, and H is Chandrasekhar's H function that depends on angle and w.

$$\rho_d(\mu,\mu',\phi)=\frac{\pi}{\mu'}\,r(\mu,\mu',\alpha)=\frac{w}{4}\,\frac{1}{\mu+\mu'}\,*$$

$$\left[\left(1+\frac{B_0\,h}{h+\tan\dfrac{\pi-\phi}{2}}\right)P(\pi-\phi)+H(\mu)+H(\mu')-1\right] \quad (3)$$

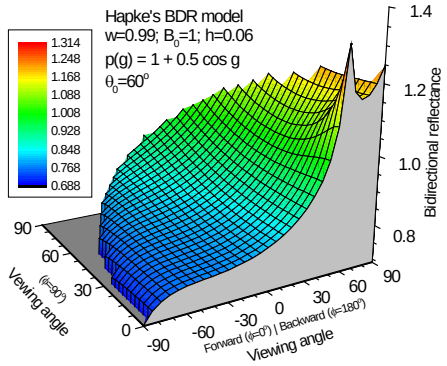

Figure 3. Three dimensional plot of Hapke's bidirectional model used to prescribe the lower boundary in the intensity calculations shown in Figures 2 and 4. Relevant parameters of the model are also shown.

In order to adequately sample the integrand in Equation 2a N_g should be at least twice as large as the number of streams. The intensities calculated with N_g=2*NSTR (256 in this example) are shown in Figure 4. These results now show a smooth variation with both polar angle cosine and azimuth angle, and the wild oscillations with azimuth seen in Figure 2 are gone. Note the presence of the "hot spot" at the polar angle cosine of 0.5 and azimuth angle 180 degrees in the plot of upward intensity at the surface. The "hot spot" is also noticable at the top of the layer.

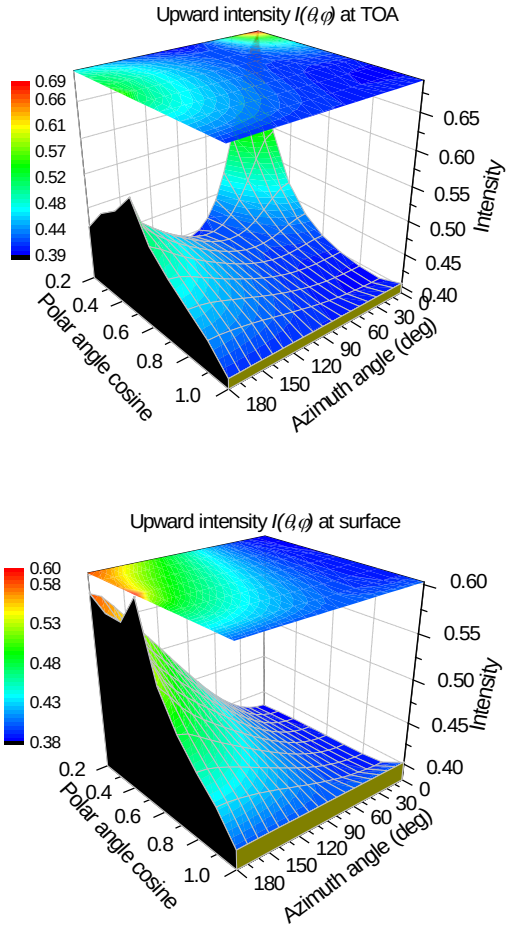

Figure 4. Same as Figure 2, but for the correct intensities calculated from a sufficiently sampled integrand (using a 256-point Gaussian quadrature) in Equation 2b.

In DISORT, Equation 2b is evaluated for sevaral combinations of the incident and reflected angles. These combinations are shown in Table 1. The total number of times Equation 2b is evaluated is $N_{total} = 2*N*N_g*(N^2 + N*N_p + N + N_p)$ where N is the number of streams per hemisphere, N_p is the number of user-defined upward directions, and N_g is the number of Gaussian quadrature points in Equation 2b. This can be a very large number. For example, when N=64, N_g=256, N_p=9, N_{total} is 155,484,160! This can be prohibitive, especially when DISORT is called a large number of times, for example to calculate intensities at many

different wavelengths as would be the case, for example, in a line-by-line code.

Fourier coefficient	Number of angles
$\rho_d^m(\mu_i, \mu_j)$	$N * N$
$\rho_d^m(\mu_i, \mu_0)$	N
$\rho_d^m(\mu, \mu_j)$	$N * N_p$
$\rho_d^m(\mu, \mu_0)$	N_p

Table 1. Number of direction pairs the Fourier expansion coefficients of the surface bidirectional reflectance is needed to be calculated in DISORT.

In addition, in version 2.0, calculations of the Fourier expansion coefficients of the surface BRDF was implemented somewhat inefficiently in that ρ_d was unnecessarily evaluated for each Fourier term. This has been changed in version 2.1. In this version, $\rho_d(\mu,-\mu';\Delta\phi_k)$ in Equation 2b is only evaluated once, for m=0, and kept in memory for use with m>0 (Figure 5). This leads to an almost five-fold increase in computational speed relative to that with version 2.0 (as shown in Figure 6). The execution time is further reduced by not calculating the BRDF Fourier coefficients when the surface is the same in repeated runs (version 2.1.1 result in Figure 6). In summary, in the latest version $\rho_d(\mu,-\mu';\Delta\phi_k)$ is only evaluated when m=0 or when the number of streams or the surface BRDF is changed (Figure 5).

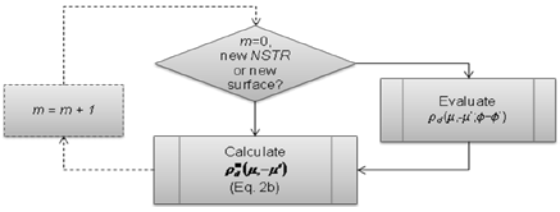

Figure 5. Flowchart of processing used in version 2.1.1 for calculating the Fourier expansion coefficients of the surface bidirectional reflectance.

Figure 6 plots the timing results from running the three versions (2.0, 2.1, 2.1.1) for a test case ten times. The test case was characterized by beam intensity π with an angle of incidence of 60° entering a slab of optical depth 0.2 and single scatter albedo 1.0. The phase function was that of Haze L model. The lower boundary was Hapke's BRDF with w=0.19. Intensities were calculated with 128 streams.

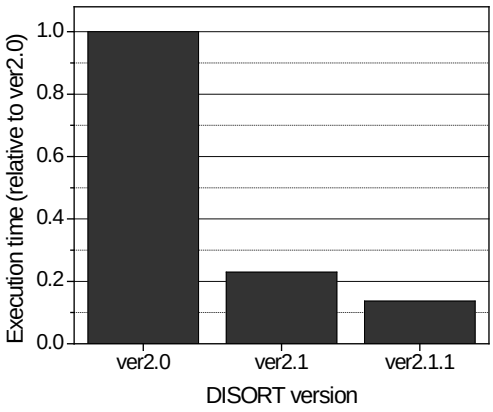

Figure 6. Relative execution times of the three different versions of DISORT as run for the test case described in the text.

Equation 2b calls for the evaluation of the cosine function for multiples of the same azimuth angle. This can usually be time consuming using the compiler's intrinsic function when the number of streams (and thus m) is large. The execution time can be further reduced by evaluating the cosine function for using the addition theorem of trigonometry (ATT), instead. The ATT requires evaluation of both the cosine and sine of the azimuth, and thus initially (for only a few streams) it is expected to be slower than using the intrinsic cosine function. However, after calculating the cosine and sine of the azimuth angle for m=1, those for m>1 are calculated using only additions and multiplications, which are faster to perform than calculating the cosine with the intrinsic function. The process starts by calculating and saving the cosine and sine of the azimuth angle for m=1 (C_1 and S_1, respectively) using the intrinsic functions. For m>1 the cosine and sine of $m\Delta\phi_k$ (C_m and S_m) are then calculated using the following equations:

$$C_m = C_{m-1} * C_1 - S_{m-1} * S_1$$
$$S_m = S_{m-1} * C_1 + C_{m-1} * S_1 \qquad (4)$$

In each step, C_m and S_m are saved and used in the next step. (Of course, only C_m is needed in equation 2b.)

To evaluate the gain in speed when using ATT instead of the intrinsic cosine function, $\cos m\Delta\phi_k$ was evaluated for increasing number of computational streams ($NSTR = i^2$; i=1,…,10) . The calculation for each $NSTR$ was repeated a million times. The reduction in execution time is shown in Figure 7 that plots the ratio of execution times from using the compiler's intrinsic cosine function and the method of ATT as a function of the number of streams ($NSTR$).

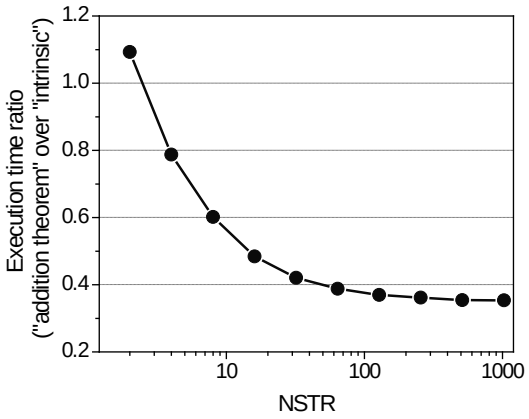

Figure 7. The ratio of execution times of evaluating $m\Delta\phi_k$ from using the compiler's intrinsic cosine function and from the method of ATT as a function of the number of streams (NSTR).

It is clear that the ATT method can significantly reduce the execution time for large number of streams; it is already faster even for four streams. (In these tests the Intel Fortran compiler was used.)

4. REFERENCES

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