A Multiple Liquid and Ice Hydrometeor Species, Hybrid- Bulk/Bin, Three-Moment Microphysics Parameterization Scheme

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Abstract

A novel bulk microphysics scheme is introduced that combines what are now known to be essential and cutting-edge elements of microphysics schemes: a hybrid-bulk/bin framework, predictions for three-moments of the distribution, a shape discriminating parameterization, and multiple species for both liquid and ice hydrometeors. Recent work has shown that combining the benefits of using a three-moment shape-predicting scheme in conjunction with hybrid-bulk/bin for sedimentation results in a much improved time-evolving distribution shape. The use of hybrid-bin framework also results in more accurate conversions between species. The new microphysical parameterization has been incorporated into a fully three-dimensional, non-hydrostatic numerical model.

The most significant motivation for this work is to improve bulk species representation and species prediction and this requires the prediction of many ice and liquid species. Multiple ice crystal and mixed ice crystal habits are needed to simulate the many different type of events of ice and snowfalls that occur in nature. This includes a density variation for mid-size ice species - essential for accurate growth and differential sedimentation. Unlike many existing schemes, an intermediate drizzle category is included between cloud and warm rain. Also unique to this model is the use of additional rain categories to predict rain that results from A novel bulk microphysics scheme is introduced that combines what are known to be essential and cutting-edge elements: a hybrid-bulk/bin framework, predictions for results in a much improved time-evolving distribution shape. The use of hybrid-bin framework also results in more accurate conversions between species. The new microphysical parameterization has been incorporated into a fully three-dimensional, non-hydrostatic numerical model.

The most significant motivation for this work is to improve bulk species representation and species prediction and this requires the prediction of many ice and liquid species. Multiple ice crystal and mixed ice crystal habits are needed to simulate the many different type of events of ice and snowfalls that occur in nature. This includes a density variation for mid-size ice species - essential for accurate growth and differential sedimentation. Unlike many existing schemes, an intermediate drizzle category is included between cloud and warm rain. Also unique to this model is the use of additional rain categories to predict rain that results from other processes (such as melting). In all, 23 ice habits (13 are crystal habits, four of which are mixed habits) and sixSIX? five liquid habits are used.

The carefully-developed parameterizations in the scheme of those processes with the include complex auto auto-conversions within a between liquid species (cloud to drizzle to rain) and even more difficult conversions between different ice species. All of the microphysical processes in the new model make use of the latest physical bases found in the literature and use hybrid-bulk/bin parameterizations for their calculation. Every attempt has been made to keep the physics as consistent as possible with observations from laboratory or in situ observations.

Finally, the main advantage of the new parameterization is a means to avoid the huge memory and computational cost of bin-parameterization, while still representing the completeness of the physics that can be incorporated with bin-parameterization models.

1. Introduction

The primary purpose of the proposed microphysical parameterization scheme is to attempt to make up for some of the deficiencies that are found in many bulk-microphysical parameterization models (e.g., Lin et al. 1983; Ferrier 1994; Walko et al. 1995; Meyers et al. 1997; Milbrandt and Yau 2005a,b; Straka and Mansell 2005; Woods et al. 2007; Thompson et al. 2008). The following are some deficiencies common to many bulk-parameterization microphysics schemes that the proposed model does not have:

- the assumption that certain variables are constant over the entire size distribution (e.g., collection efficiency, mass weighted terminal velocity, mass weighted ventilation coefficient, particle density, and particle shape),
- the assumption that spheres are adequate for most particles, or that there are consistent densities and shapes of particles with observations (e.g., ice crystals),
- the assumption that the prediction of only one- or two-moments is usually sufficient to capture the essence of the microphysics of the atmosphere without some diagnostic equation or prediction of a distribution shape parameter,
- The prediction of too few species such that the model must be "tuned" on a case-by-case basis toward the assumed dominant species,
- The prediction without the Lagrangian timescale memory required for some improved accuracy with some physical processes such as auto-conversion as well as other conversions.

Bin-parameterization methods can alleviate some of the shortcomings listed above (e.g., Takahashi 1976; Lynn et al. 2005) but not all (Straka and Rasmussen 1997). Bin-parameterization models are also quite computationally intensive in terms of not only the microphysical gain and depletion terms, but particularly the advection, diffusion, and filtering of many the many scalars variables. Thus, a more practical approach is to have a bulk-parameterization model with many of the ice, liquid, and mixed-phase species as exist in nature and somehow incorporate the essence of the bin-parameterization models in the prognostic equations. At least for the time being bin-parameterization models are usually prohibitively expensive for operational forecasting and for very high-resolution simulations of larger storm systems (e.g, three-dimensional orographic precipitation, lake snow, convective precipitation). Though it should be noted that just recently Khain et al. (2010) used a bin-parameterization model with nested grids to simulate a hurricane using 3 km grid spacing in the inner nest and it only took eight days to complete on a computer with eight processors. Below, brief descriptions of existing parameterizations and their advantages and disadvantages are discussed. Then the ways in which the proposed model can improve upon previous limitations of bulk-parameterization models are presented. In addition, the importance of the use of multiple species in microphysical parameterizations is noted.

a. Bulk schemes

1) COLLECTION GROWTH ARTIFACTS

An important problem with bulk-parameterization models can be demonstrated with the simple example of raindrops collecting graupels, and graupels collecting raindrops. In bulk schemes, both can be sources for graupels of various densities, frozen drops, or embryonic hail based upon preset mixing ratio thresholds (e.g., Lin et al. 983) which seem somewhat *ad hoc*. Also, the bulk-parameterization integrations regardless of the approach (e.g., Wisner et al 1972, Minuzo 1990, Verlinde et al. 1990), permit raindrops to collect graupels and vice versa across the entire bulk-parameterization size spectra regardless of which is falling faster and, for three-body interactions, it can be shown that this results in over-depletion of the source categories and overproduction of the third body. One exception is the bulk scheme of Thompson et al (2008) where particles with smaller mean size are prevented from collecting larger particles. Errors are also introduced by using a single bulk fallspeed for each distribution (larger particles falling too slow and smaller falling too fast) and a single collection efficiency between two interacting distributions. Another artifact is that single moment bulk-parameterization models allow particle number concentrations that are continuously collecting (e.g., graupel collecting cloudwater) to artificially increase (Straka et al. 2005) even though they should remain constant (Straka et al. 2007). In contrast, the bin-parameterization framework and hybrid-bulk/bin framework permits the shapes of the raindrops and graupels to vary with size, collection efficiencies to vary with different sizes, and individual bins to have correct terminal velocities and geometric sweep-out rates with only faster-falling particles collecting slower falling resulting in more accurate collection rates. This differs than Thompson et al. (2008) where smaller particles are collected by larger particles.

2) EVAPORATION, SUBLIMATION, AND MELTING ARTIFACTS

Another important problem is demonstrated with the example of evaporation, deposition or melting of particles. With the simple multi-moment, pure bulk-parameterization, larger sizes than would naturally evaporate (or sublimate) occurs using the familiar slope preserving approach for number concentration change (Ferrier 1994). Moreover for growth by vapor condensation (or vapor deposition) single-moment bulk-parameterization models artificially increase number concentration (Straka et al. 2005; Straka et al. 2007). With a bin parameterization or hybrid-bulk/bin parameterization model is mixing ratio and number concentration adequately re-distributed from larger to smaller hydrometeors for evaporation, sublimation or melting or conserved for condensation, deposition or freezing.

b. Multi-moment bulk schemes

Milbrandt and Yau (2005a, b, hereafter MY05) are the first authors to have successfully implemented the prediction of a third moment into a bulk-parameterization model to diagnose shape parameters of hydrometeor size distributions. They showed that by predicting reflectivity, for instance, that the shape of the gamma distribution could be diagnosed, which improved the realism of time evolving size distributions. In the three-moment bulk-parameterization scheme the total number concentration of hydrometeor species N_{Tx}, total mixing ratio for a hydrometeor species Q_x, and the reflectivity of a species Z_x, are predicted and the shape parameter ν_x is found from iteration. In contrast, for the MY05 version of the two-moment scheme, the ν_x is preset to a constant or diagnosed from an equation found from previous three-moment interactions and only N_{Tx} and Q_x, are predicted (and Z_x is diagnosed from those). The two-moment scheme with a diagnostic ν_x also can produce very reasonable results that are sometimes better than three-moment results compared to a bin model (MY05) at least for fallout.

c. Bin model schemes

Bin multi-moment microphysical parameterization models are often considered the type of parameterization most able to represent, for example, rain distribution evolutions in rain clouds. They have bins representing the spectrum of drops from very small cloud droplet sizes (D = 2 -

10 mm) to larger raindrops (D = 4 to 8 mm) for bin parameterizations of raindrop development. Each bin is usually is usually exponentially larger than the previous size / mass bin owing to the wide spectrum of liquid water drops that are possible, which ranges over three orders of magnitude. For liquid water drop sizes, bins often will increase by 2, 21/2, 21/3, or 21/4 times the previous size bin over perhaps 36, 72, or 144 bins. A shortcoming of bin models is the excessively large computation resources needed to make use of them in large three-dimensional models. At a minimum, number concentration is predicted with these schemes, though mixing ratio and reflectivity can be predicted or calculated as well. Considering number concentration with mixing ratio prediction improves the bin parameterization results against using just number concentration by limiting anomalous spreading of the distribution against analytical test problems of particles collecting other particles (REF NEEDED?).

d. Hybrid-bulk/bin schemes

The traditional hybrid-bulk/bin microphysical parameterization model that was pioneered by Feingold et al. (1998) is one in which water contents and number concentrations from a two-moment bulk-parameterization model distribution size-spectra are cast in a bin-parameterization model and grown/depleted by various processes. After all the computations are made for water contents and number concentrations, the bin-parameterization model results are recast in the two-moment bulk-parameterization water content and number distribution size spectra for improved computational efficiency in other parts of the model such as advection, diffusion and filtering. This was done for cloud droplets and drizzle (rain) by Feingold et al. (1998). The three-moment hybrid-bulk/bin parameterization is proposed here for a model which incorporates many ice and liquid water hydrometeor species with microphysical all the microphysical processes such as collection, evaporation / condensation, sublimation / deposition, melting / freezing, conversion, and sedimentation.

By predicting a third moment to track the evolving shape parameter in a one-parameter gamma distribution, it should be possible to be able to preserve the resulting size distributions, which might otherwise be “lost” with lower number moment schemes that use a hybrid-bulk/bin parameterization technique. It will be shown that a scheme similar to Feingold et al.’s (1998) hybrid-bulk/bin parameterization two-moment (number concentration and water content) for fallout becomes far superior when three-moments are used (number concentration, water content and reflectivity). Thompson et al. (2008) used the hybrid-bulk parameterization for some one-moment collection processes.

The present work is considered to be to some degree a proof-of-concept model for a complex hybrid-bulk/bin parameterization. An important limitation is to realize that the hybrid bulk/bin-parameterization model will not produce the same results as a pure bin-parameterization model because some information is still lost when converting back and forth between the bulk-parameterizations for advection, turbulence, filtering etc., and bin-parameterizations for the microphysical computations. Another possible limitation (which follows from Milbrandt and Yau 2005a,b) is that it is unclear whether prediction combination of zeroth-moment (number concentration) and third-moment (water content) with the sixth moment (reflectivity) really gives a shape parameter that respects all of the particle interactions that have occurred over the time-step [Milbrandt and McTaggert-Cowwen 2010 (this volume)]. Best-fit procedures exist (e.g., Freer and McFarquhar 2006) could be used to evaluate the accuracy of the new shape parameter (diagnosed from the predicted reflectivity).

e. Importance of multiple liquid and ice species

In an effort to address the "too weak stratiform regions" and "too intense convective regions" (e.g., McFarquhar et al. 2006) that may exist due to incorrect growth rates present in single moment schemes with only one ice crystal habit, several ice crystal habits have been added for capturing the diversity of ice crystal habits within thunderstorm anvils for hydrometeor growth and radiation physics. Of particular importance are the differing self-collection processes, depositional growth rates, and riming rates for each of 13 cloud ice habits herein. Having different species (or habits) is important because some, like dendrites, interact to produce snow aggregates more readily than other crystals (see Cotton et al. 1986 for a brief summary).

Woods et al. (2007) already has shown significant sensitivities in model simulations of IMPROVE cases by varying the rate of a single cloud ice species in a bulk Lin et al. (1983) style scheme by allowing the crystals to take on the properties of the growth characteristics of different habits in different parts of the cloud. The proposed model herein additionally allows initiation and prognostication of different various crystal types that can be simultaneously advected, diffused, and filtered, compete for depositional and riming growth, and also collect one another producing snow aggregates. Additionally, by using specific criteria to determine the collection size thresholds and efficiencies (Wang 2004), something possible only in a bin or hybrid-bulk/bin parameterization scheme, the collection (riming) rates are reduced to more realistic values. Of course, this means that some of the bulk microphysical history should be better represented with more ice species. This is not only true for ice, but also for liquid categories.

Important to representing different ice and liquid species is having a fully consistent framework where the mass-diameter relationships can be easily represented as well as the fall velocity-diameter relationships. Many users of microphysics schemes may not realize that the particle shape is hardly ever consistently represented (McFarquhar and Black 2005) for collection, vapor deposition, etc.

The specific species included in the new parameterization are discussed in Section 2, the foundational equations used in the parameterization in Section 3, the representation of various microphysical processes in the new parameterization in section 4, and the summary in section 5.

2. Hydrometeor species included in the new three-moment hybrid bulk/bin scheme

The liquid hydrometeors discussed in this parameterization are cloud water CW (typically range from 0-82 microns in diameter), drizzle DZ (typically range from 82-500 microns), and warm rain RW (typically range from 500-8000 microns). In addition to warm rainwater there are big drops BD that from nearly or completely melted hail, melt rainwater MW from melted snow aggregates, graupels, and frozen drops, and shed rainwater DW from melting hail from graupel and hail from frozen drops. In traditional bulk schemes, these all would be combined with warm rainwater into a single rainwater category. However, by separating these species, one can additionally obtain some history of the microphysics processes that are occurring. For instance, one can determine what fraction of rain reaching the ground resulted from melting of various species of ice versus other processes.

Another feature of the liquid water aspect of the proposed model is the inclusion of drizzle, which owes to the difference in sizes between cloud droplets and rainwater. Most models, except for perhaps the CSU-RAMS model, do not have an intermediate size particle size for liquid water between cloud drops and raindrops. The CSU-RAMS model does in a way in that it has large cloud drops of 50-100 micrometers, which, however, is different than this model that uses a true drizzle category. The large cloud drops are used for growth on ultra giant cloud condensation nuclei (CCN) in CSU-RAMS. If drizzle particles grow larger than 500 microns they are transferred into the warm rainwater RW species.

The general ice water hydrometeors species include ice crystals (13 habits or species), snow aggregates SW, frozen drizzle FZ, three graupel categories GL, GM, and GH, frozen raindrops FW, and four hail categories HG, HF, HS, HW. The 13 crystal habits are frozen cloud droplet CZ (produced by homogeneous freezing at temperatures between -30 °C and -50 °C; Demott et al. 1994) bullet rosettes BR, columns CN, hollow columns HC, dendrites DN, needles ND, plates PL, sectors SC, side planes SP, column crystals with plate growth CP (i.e., capped columns), column crystals with dendrite growth CD, dendrites with plate growth DP, and plates with dendrite growth PD. The existence of cloud droplets at quantities of 1×10^{-6} kg/kg is used to indicate supersaturation with respect to liquid water. It will be discussed below that because rime mass and deposition mass are both predicted for each of these crystal habits and larger ice species, the actual shape and fallspeed is allowed to change from the "pristine" conditions that they are initialized as.

Frozen drizzle FZ particles (mean density of 900 kg m^{-3}) result from the freezing of drizzle and either fall out or become graupel or frozen drops embryos. Frozen drops FW (mean density of 900 kg m^{-3}) result from the freezing of any of the rainwater species by assuming that there are ice nuclei in the water drops. Not enough information exists to consider other types of freezing such as raindrop breakup and other phenomena (Cotton 1972b). There are three graupel species based on density. The graupel species are low-density graupel GL (mean density of 200 kg m^{-3}), medium-density graupel GM (mean density of 400 kg m^{-3}), and high-density graupel GH (mean density of 600 kg m^{-3}). As each of these species has a particular drag coefficient and different densities, each has a particular terminal velocity for a given size. This influences the differential sedimentation of the various graupel species (see Straka and Mansell 2005), as well as the riming densities of the graupel species, the latter of which is a function of terminal velocity, cloud droplet size and temperature.

Finally there are the four hail species. These include hail from graupel HG (mean density of 700 kg m^{-3}), and hail from frozen drops HF (mean density of 900 kg m^{-3}). These two species have size ranges from 9-19 mm and are considered to represent small to marginally severe hailstone sizes. Then there are the two large hailstone species, including large solid hail HW (mean density of 900 kg m^{-3}), and large spongy hail HS (mean density of 900 kg m^{-3}). These two species have size ranges of 19-51 mm that meet criteria for severe to very severe hail.

3. Equations in the new microphysical scheme

There are three prognostic variables related to three different moments predicted in this model. These are total number concentration, mixing ratio, and reflectivity, which are related to the zeroth, third and sixth moments, respectively. The purpose of using three different moments is to be able to diagnose the shape parameter at each grid point for each hydrometeor species following MY05a, b as explained in the next section. This adds greater freedom for the model to represent real cloud processes that might occur in the atmosphere as shown in solutions of tests compared to analytical solutions and other comparisons by MY05a, b. They show that the best solutions among single-, double-, and triple-moment schemes are obtained by using a set of predictive equations for these moments. Also predicted are the rime collected, vapor deposited as ice, the mean cloud water to which a particle is exposed, and the time of exposure or parcel age (Straka and Rasmussen 1997). Predicting this age helps improve autoconversion rate accuracies as suggested by Cotton (1972) and Straka and Mansell (2005). Separate prediction of rime and vapor deposition for ice is done significantly differently than, but in the same spirit as, that shown in Morrison and Grabowski (2008).

The collection of these prognostic equations are given in terms of their symbols, respectively, which are total number concentration $N_{T,m}$, mixing ratio Q_m, reflectivity Z_m, liquid water on larger ice particles $Q_{l,m}$, rime collected $Q_{r,m}$, vapor deposited as ice $Q_{d,m}$, the mean cloud water to which a particle is exposed $\overline{Q_{cw,m}}$, and parcel age τ_m used to find $\overline{Q_{cw,m}}$. Note the terminal velocity is $V_{t,Q}$ or $V_{t,N}$ and the mixing coefficient is K_h.

$$\frac{\partial N_{T,m}}{\partial t} = -u_i \frac{\partial N_{T,m}}{\partial x_j} + \frac{\partial}{\partial x_i}\left(K_h \frac{\partial N_{T,m}}{\partial x_i} \right) + \frac{\partial \left(\overline{V}_{t,N} N_{T,m} \right)}{\partial x_3} + SN_{T,m} \tag{1}$$

$$\frac{\partial Q_m}{\partial t} = -\frac{1}{\rho}\frac{\partial \rho u_i Q_m}{\partial x_j} + \frac{Q_m}{\rho}\frac{\partial \rho u_i}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\rho K_h \frac{\partial Q_m}{\partial x_i} \right) + \frac{1}{\rho}\frac{\partial \left(\rho \overline{V}_{tQ} Q_m \right)}{\partial x_3} + SQ_m \tag{2}$$

$$\frac{\partial Z_m}{\partial t} = -u_i \frac{\partial Z_m}{\partial x_j} + \frac{\partial}{\partial x_i}\left(K_h \frac{\partial Z_m}{\partial x_i} \right) + \frac{\partial \left(\overline{V}_{t,Z} Z_m \right)}{\partial x_3} + SZ_m \tag{3}$$

$$\frac{\partial Q_{l,m}}{\partial t} = -\frac{1}{\rho}\frac{\partial \rho u_i Q_{l,m}}{\partial x_j} + \frac{Q_{l,m}}{\rho}\frac{\partial \rho u_i}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\rho K_h \frac{\partial Q_{l,m}}{\partial x_i} \right) + \frac{1}{\rho}\frac{\partial \left(\rho \overline{V}_{t,Q_{l,m}} Q_{l,m} \right)}{\partial x_3} + SQ_{l,m} \tag{4}$$

$$\frac{\partial Q_{r,m}}{\partial t} = -\frac{1}{\rho}\frac{\partial \rho u_i Q_{r,m}}{\partial x_j} + \frac{Q_{r,m}}{\rho}\frac{\partial \rho u_i}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\rho K_h \frac{\partial Q_{r,m}}{\partial x_i} \right) + \frac{1}{\rho}\frac{\partial \left(\rho \overline{V}_{tQ} Q_{r,m} \right)}{\partial x_3} + SQ_{r,m} \tag{5}$$

$$\frac{\partial Q_{d,m}}{\partial t} = -\frac{1}{\rho}\frac{\partial \rho u_i Q_{d,m}}{\partial x_j} + \frac{Q_{d,m}}{\rho}\frac{\partial \rho u_i}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\rho K_h \frac{\partial Q_{d,m}}{\partial x_i} \right) + \frac{1}{\rho}\frac{\partial \left(\rho \overline{V}_{tQ} Q_{d,m} \right)}{\partial x_3} + SQ_{d,m} \tag{6}$$

$$\frac{\partial \overline{Q_{cw,m}}}{\partial t} = -\frac{1}{\rho}\frac{\partial \rho u_i \overline{Q_{cw,m}}}{\partial x_j} + \frac{\overline{Q_{cw,m}}}{\rho}\frac{\partial \rho u_i}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\rho K_h \frac{\partial \overline{Q_{cw,m}}}{\partial x_i} \right) + \frac{1}{\rho}\frac{\partial \left(\rho \overline{V}_{tQ} \overline{Q_{cw,m}} \right)}{\partial x_3} + S\overline{Q_{cw,m}} \tag{7}$$

$$\frac{\partial \tau_m}{\partial t} = -u_i \frac{\partial \tau_m}{\partial x_j} + \frac{\partial}{\partial x_i}\left(K_h \frac{\partial \tau_m}{\partial x_i} \right) + \frac{\partial \left(\overline{V_{tQ}} \tau_m \right)}{\partial x_3} + S\tau_m , \tag{8}$$

where the subscript m is the hydrometeor species index, t is time, x_i or x_j are the Cartesian directions, u_i are the Cartesian velocities, and ρ is the density of air. The S terms are the source and sink terms summarized in section 2 below.

a. Hydrometeor related power laws and constants and variables

In this model the mass, terminal velocity, and density are all written as power laws with regard to diameter and are given, respectively as $M_x\left(D_x\right) = a_x D_x^{b_x}$, $V_{t,x}\left(D_x\right) = c_x D_x^{d_x}$, $\rho_x\left(D_x\right) = e_x D_x^{f_x}$, with constants a_x through f_x for each hydrometeor species denoted by subscript x provided in Table 1 as mentioned above. In addition ice crystal diameter and thickness are described using $D_x = g_x M\left(D_x\right)^{h_x}$, and , $H\left(D_x\right) = i_x D_x^{j_x}$.

b. Number spectral density function and some derived variables

A modified gamma distribution (two shape parameters ν_x and μ_x) noted by Flatau et al. (1989), and used by Cohard and Pinty (2000), and probably others in various forms, is employed,

$$n\left(D_x\right) = \frac{\mu_x N_{T_x}}{\Gamma(\nu_x)} \left(\frac{D_x}{D_{n,x}}\right)^{\mu_x \nu_x - 1} \frac{1}{D_{n,x}} \exp\left(-\left(\frac{D_x}{D_{n,x}}\right)^{\mu_x}\right). \tag{9}$$

where μ_x is the breadth parameter for species X, N_{Tx} is the number concentration, and the gamma function is Γ, and the shape parameter is ν_x.

The definition of the zeroth-moment given as

$$N_{T,x} = \int_0^\infty n\left(D_x\right) \, \mathrm{d}D_x , \tag{10}$$

where a general solution is possible in terms of partial gamma functions and the complete gamma function if $D_{min} = 0$ and $D_{max} = \infty$.

From the third-moment,

$$Q = \frac{D_{n,x}^{b_x}}{\rho_o} \int_0^\infty a_x \left(\frac{D_x}{D_{n,x}}\right)^{b_x} \frac{\mu_x N_{t,x}}{\Gamma\left(\upsilon_x\right)} \left(\frac{D_x}{D_{n,x}}\right)^{\left(\nu_x \mu_x - 1\right)} \exp\left(-\left(\frac{D_x}{D_{n,x}}\right)^{\mu_x}\right) d\left(\frac{D_x}{D_{n,x}}\right), \tag{11}$$

the characteristic diameter, D_n can be found from,

$$Q_x = \frac{\mu_x a_x N_{T,x} \Gamma\left(\nu_x\right)}{\rho \Gamma\left(\dfrac{b_x + \nu_x \mu_x}{\mu_x}\right)} D_{n,x}^{b_x} , \tag{12}$$

where a_x and b_x are constants and $D_{n,x}$ is given by

$$D_{n,x} = \left(\frac{\rho_o Q_x \Gamma\left(\dfrac{b_x + \nu_x \mu_x}{\mu_x}\right)}{\mu_x a_x N_{t,x} \Gamma\left(\nu_x\right)}\right)^{\left(1/b_x\right)} . \tag{13}$$

Finally, the mean volume diameter for spheres $D_{mass\_vol,x,}$ is given by

$$\bar{D}_{\text{mean vol,x}} = \left(\frac{6 Q_x (D) \rho}{\pi \rho_x (D) N_{t,x} (D)}\right)^{1/3} . \tag{14a}$$

On the other hand, the mean volume diameter for ice crystals and snow is given by the following, where c is the thickness (prism plane) and a is the radius of the width (basal plane),

$$\bar{D}_{\text{mean vol,x}} = \left(c a^2\right)^{1/3} . \tag{14b}$$

c. Diagnosis of the shape parameter

The equation for the prediction of reflectivity, dZ_x/dt, is given in a different form so as to make it possible to diagnose the shape parameter ν_x from $G(\nu_x)$ (note, μ_x is not included in this equation as at this time it is always set to 1),

$$\frac{dZ_x}{dt} = \frac{G(v_x)}{a_x^2}\rho^2\left[2\frac{Q_x}{N_{T,x}}\frac{dQ_x}{dt} - \left(\frac{Q_x}{N_{T,x}}\right)^2\frac{dN_{T,x}}{dt}\right] \quad (15)$$

from which v_x can be diagnosed. This form of the equation is used in association with collection, diffusion, and melting processes. The second kind of source for reflectivity is one such as nucleation where

$$\frac{dZ_x}{dt} = -G(v_x)\left(\frac{\rho}{a_x}\right)^2\left(\frac{dQ_y}{dt}\right)^2\left(\frac{dN_{T,x}}{dt}\right)^{-1}. \quad (16)$$

The last kind of source occurs when one habit becomes another such as Bigg freezing, where drizzle freezes into frozen drizzle or rain freezes into frozen drops,

$$\frac{dZ_x}{dt} = -\left(\frac{\rho_x}{\rho_y}\right)^2\frac{dZ_y}{dt}. \quad (17)$$

The reflectivity can be written as,

$$Z_x = \frac{(5+v_x)(4+v_x)(3+v_x)}{(2+v_x)(1+v_x)(v_x)}\frac{(\rho Q_x)}{a_x^2 N_{T,x}}, \quad (18)$$

from which $G(v_x)$ can now be defined as,

$$G(v_x) = \frac{(5+v_x)(4+v_x)(3+v_x)}{(2+v_x)(1+v_x)(v_x)}. \quad (19)$$

The expression $G(v_x)$ is found after the sum of many iterations with an $x = g(x)$, bisection, linear interpolation, or some other iteration scheme. For efficiency, the use of several lower-order polynomials for various ranges of v_x, as used by MY05a,b, can be attempted.

d. Hybrid-bulk/bin algorithm

At the beginning of a time step, with the hybrid-bulk/bin-parameterization, a bulk-parameterization spectrum of particles assuming a two-parameter gamma distribution is transformed into bin-parameterization space. Next, the physical processes can be computed in the bin-parameterization space. Then sedimentation fluxes are calculated appropriately for each bin. The bin-parameterization solutions are summed together to produce a new bulk-parameterization spectrum using the two-parameter gamma distribution (a hybrid-bulk/bin parameterization distribution) with a consistent shape parameter.

The diameters D_x of the particles involved in any microphysical process are defined to exponentially increase in size according to the following for 48 (or any number) bins sizes as follows,

$$D_x = D_{\min}\exp\left(\frac{[l-1]}{xjo}\right) \ \textit{for } l = 1, nl, \quad (20)$$

where D_{min} = 5 microns, l is an index number for the bin, nl is the number of bins, and xjo = 4.5.

The next step is demonstrated for number concentration using (9) to transform the bulk-parameterization spectra into bin-parameterization space, and stored in terms of D_n,

$$N_x(D_x) = n_x(D_x)dD_x = \frac{\mu_x N_{T,x}}{\Gamma(v_x)}\frac{1}{D_{n,x}}\left(\frac{D_x}{D_{n,x}}\right)^{v_x\mu_x - 1}\exp\left(-\left[\frac{D_x}{D_{n,x}}\right]^{\mu_x}\right)dD_x, \quad (21)$$

which for one discretized bin in bin-parameterization space becomes,

$$N_x\left(D_x(i)\right)=n_x\left(D_x(i)\right)\Delta D_x=\frac{\mu_x N_{T,x}}{\Gamma(\nu_x)}\frac{1}{D_{n,x}}\left(\frac{D_x(i)}{D_{n,x}}\right)^{\nu_x\mu_x-1}\exp\left(-\left[\frac{D_x(i)}{D_{n,x}}\right]\right)\frac{D_x(i)}{j_o}\qquad(22)$$

where $j_o = 4.5$ is a variable that controls the spacing of the bins. The values of

$$\Delta D_x(i)=\frac{D_x(i)}{j_o}\qquad(23)$$

are used in (22) for number concentration over size interval D_x to $D_x+\Delta D_x$ (Farley and Orville 1986; Farley 1987). For the hybrid bulk/bin-parameterization, the initial mixing ratio can be defined in terms of number concentration for a bin summed over the interval describing a single bin,

$$Q_x(i)=\frac{N_x(i)m_x(i)}{\rho},\qquad(24)$$

where $Q_x(i)$ is the mixing ratio of bin number i, and ρ is the density of air. To transform back to bulk-parameterization space from the bin-parameterization space with the hybrid bulk/bin parameterization, N_x, Q_x, and Z_x are written as the sum of bins of number concentration $N_x(D_x(i))$ and mixing ratio $Q_x(i)$. For example, the transformation equation from bin space to bulk parameter space for total number concentration is found from (24),

$$N_{T,x}=\sum_i n_x\left(D_x(i)\right)\Delta D_x=\sum_i\frac{\mu_x N_{T,x}}{\Gamma(\nu_x)}\frac{1}{D_{n,x}}\left(\frac{D_x(i)}{D_{n,x}}\right)^{\nu_x\mu_x-1}\exp\left(-\left[\frac{D_x(i)}{D_{n,x}}\right]^{\mu_x}\right)\frac{D_x(i)}{j_o}.\qquad(25)$$

It should be noted that a two-moment version of the hybrid bulk/bin parameterization scheme modified from Feingold et al. (1998) is being used in the CSU-RAMS-2001 model as described by Cotton et al. (2003) and for riming of ice crystals by Saleeby and Cotton (2008). After all calculations are made in bin-parameterization space, they are summed using a one parameter gamma distribution similar to that given above to transform back to bulk-parameterization space.

4. Representation of microphysical processes

a. Cloud, drizzle, and rain formation

1) AEROSOLS AS NUCLEATION AGENTS FOR CLOUD DROPLET NUCLEATION

With a more detailed hybrid-bulk/bin-parameterization model, it seems appropriate to do away with saturation adjustment schemes that have been used by Tao et al. (1989), Cohard and Pinty (2000), Gilmore et al. (2004a), Straka and Mansell (2005), and Siefert and Beheng (2006) as well as most other models. However, we have found that the model with explicit condensation/evaporation of cloud drops (and deposition and sublimation of ice crystals) is nearly identical to the Tao et al. (1989) saturation adjustment scheme solution without having to resort to many small time-steps to integrate the diffusion equation.

A maximum supersaturation can be expressed (Cohard and Pinty 2000) by

$$S_{v,W\max}^{k+2}\,{}_2F_1\left(\mu,\frac{k}{2},\frac{k}{2}+\frac{3}{2};-\beta S_{v,W\max}^2\right)=\rho_L\frac{\left(\psi_1 W\right)^{3/2}}{2kc\pi\rho_w\psi_2 G^{3/2}B\left(\frac{k}{2},\frac{3}{2}\right)}.\qquad(26)$$

where the following are defined as,

$$\psi_1(T,P) = \frac{g}{R_d T}\left(\frac{\varepsilon L_v}{c_p T} - 1\right), \tag{27}$$

$$\psi_2(T,P) = \left(\frac{p}{\varepsilon e_s(T)} + \frac{\varepsilon L_v^2}{R_d T^2 c_p}\right), \tag{28}$$

$$G(T,P) = \frac{1}{\rho_L}\left(\frac{R_v T}{\psi e_s(T)} + \frac{L_v}{k_a T}\left(\frac{L_v}{R_v T} - 1\right)\right)^{-1}, \tag{29}$$

where c, k, β and μ are the four activation spectrum coefficients (Cohard et al. 1998). The functions ${}_2F_1$ and B are Gauss' Hypergeometric function and the Beta function, respectively. Then a value for activated N_{CCN} can be obtained (Cohard et al. 1998) using an iteration technique (Cohard and Pinty 2000),

$$N_{CCN} = cS_{v,W\max}^{k}\,{}_2F_1\left(\mu, \frac{k}{2}, \frac{k}{2}+1; -\beta S_{v,W\max}^2\right) \tag{30}$$

This equation has different values of c and k than Twomey's (1964) expression. The utility of (30) has been discussed by Cohard et al. (1998) as having four activation coefficients, which can express various aspects of aerosols that are involved in the nucleation of cloud droplets. This makes it possible to include the some of the aspects of activation size spectrums, chemical compositions, and solubility into the equation for heterogeneous nucleation (30) of aerosols into cloud drops. The maximum number of cloud droplets that can be nucleated is $1.5\text{x}10^9$ m^{-3}.

2) AUTOCONVERSION OF CLOUD DROPLETS TO DRIZZLE AND DRIZZLE TO WARM RAINWATER

As mentioned in the introduction, the separate drizzle category allows for the calculated number concentration rate of drizzle particles to be used directly and we do not have the problem of "artificially seeding" mature rain distributions later in a cloud simulation (artificially reducing the mean size of particles) and therefore it is not necessary to impose the size-preserving rain number concentration formulas of Carrio and Nicolini (1999) or Milbrandt and Yau (2005b).

Though hardly used any more, the Kovetz and Olund (1969) quasi-stochastic collection scheme is a simple, efficient, and mass-conserving interpolation method to study the collision-coalescence of particles. This scheme is similar to many others in that it only approximates the stochastic collection equation (Scott and Levin 1975). Although newer modern schemes are less diffusive (e.g., Tzvion et al. 1987), the Kovetz and Olund (1969) scheme is somewhat desirable as it is simple, efficient, and works well for cloud-to-drizzle autoconversion. More accurate methods probably will be incorporated into the current model in the future.

Cloud droplets enter the drizzle spectrum at 82 micrometers and the rain spectrum at 500 micrometers after sufficient broadening of the drizzle distribution. It has been show in our own tests and by Scott and Levin (1975) that the Kovetz and Olund (1969) scheme is able to reproduce these rough diameter limits. The approximate quasi-stochastic collection equation used by Kovetz and Olund (1969) is written such that

$$\begin{aligned} N_T(r_i, t+\Delta t) = N_T(r_i,t) &+ \sum_{n=1}^{i-1}\sum_{m=n+1}^{i} B(n,m,i)P(n,m)N_T(r_n,t)N_T(r_m,t) \\ &- \sum_{n=1}^{M} P(i,n)N_T(r_i,t)N_T(r_n,t) \end{aligned}, \tag{31}$$

where *P(n, m)* is the coalescence probability for particles with radii r_n and r_m. The term B is an exchange coefficient to move particles from one bin to another and is given by,

$$B(n,m,i)=\begin{cases}\left(r_n^3+r_m^3-r_{i-1}^3\right)/\left(r_i^3-r_{i-1}^3\right) & for\ r_{i-1}^3 \le r_n^3+r_m^3 \le r_i^3 \\ \left(r_{i+1}^3-r_n^3-r_m^3\right)/\left(r_{i+1}^3-r_i^3\right) & for\ r_i^3 \le r_n^3+r_m^3 \le r_{i+1}^3\ . \\ 0 & for\ r_n^3+r_m^3 \le r_{i-1}^3 \\ & or\ r_{i+1}^3 < r_n^3+r_m^3 \end{cases} \tag{32}$$

This parameterization for *B(n, m, i)* conserves the mass of total water.

A brief illustration which compares the size distributions of the Kovetz and Olund (1969) scheme to Golovin's analytical solution and the Berry and Reinhardt (1974a,b) scheme is given below in Fig. (2) from Scott and Levin (1975, their Fig.7.9). They claim that the Kovetz and Olund (1969) solutions are not prohibitively erroneous and that neither the Kovetz and Olund, nor the Berry and Reinhardt scheme are perfect at representing the true quasi-stochastic collection process. In the first example, Golovin's analytical solution is compared to the Kovetz and Olund scheme. The peaks in the Kovetz and Olund scheme are slightly lower than with Golovin's solution, whereas, the tails at large sizes are very slightly longer indicating the known spreading of the particle spectrum by the Kovetz and Olund scheme. In the comparison with the Berry and Reinhardt (1974) scheme, which, however, conserves no moments, there is a larger difference between the solutions, with shallower peaks for the Kovetz and Olund (1969) scheme and more prominent undesirable spreading at 600 and 1200 seconds at the large drop tails.

b. Cloud ice formation

1) CRYSTAL NUCLEATION

As done by Seifert and Beheng (2005; SB05), the ice nucleation follows Reisner et al (1998) and various other authors. The model is used to make a number concentration prediction as follows

$$\frac{\partial N_{t,I}}{\partial t}=\begin{Bmatrix}\max\left[\dfrac{\left(N_{t,IN}-N_{t,I}\right)}{\Delta t},0\right] \\ otherwise\ zero\end{Bmatrix} \tag{33}$$

where $N_{t,IN}$ = 0.001exp(-0.639 + 12.96S_I). Reisner et al. (1998) limited the value of $N_{t,IN}$ by 10 times the result from the following equation given as $N_{t,IN}$ = 0.01 x exp(-min(T,246.15)-273.15), which owes to Fletcher (1962). They state there is an instability problem with the Meyers et al. (1992) scheme at very cold temperatures. The maximum number of ice crystal concentration herein is arbitrarily limited to the same number as the maximum number of cloud drops permitted, which as stated above as 1.5x10^9 m^{-3}.

2) HOMOGENEOUS FREEZING OF CLOUD DROPLETS

The number of cloud droplets that homogeneously freeze into frozen cloud drops in time Δt below T = –30 °C and T =-50 °C for each bin, is given as

$$\Delta N\left(D_i\right)_{freeze}=\left[1-\exp\left(-J_{ls}V\left(D_i\right)\Delta t\right)\right]n_{cw}\left(D_i\right)\Delta D_i\ , \tag{34}$$

where $J_{ls}=J_{ls0}$, J_{ls0} is the homogeneous freezing nucleation rate of pure water, and *V(D)* is the droplet volume for each bin associated with diameter D and width ΔD. This J_{ls0} was approximated in Demott et al. (1994) with work from Heymsfield and Sabin (1989) and Heymsfield and Milosevich (1993) as,

$$\log_{10}\left(J_{ls0}\right) = -606.3952 - 52.6611T_c - 1.7439T_c^2 - 2.65\times10^{-2}T_c^3 - 1.536\times10^{-4}T_c^4 \quad (35)$$

with units cm^{-3} s^{-1} and is noted by Demott et al. (1994) to be a good approximation between $-30°\mathrm{C} \geq T(°C) \geq -50°\mathrm{C}$. The total rate summed over the binned distribution is then given by

$$N\mathrm{FZ}_x = \frac{\sum_{i=1}^{48} \Delta N_{freeze}\left(D_i\right)}{\Delta t} \text{ and} \quad (36)$$

$$Q\mathrm{FZ}_x = \frac{\frac{1}{\rho}\sum_{i=1}^{48} \frac{\pi}{6}\rho_{liq} D_i^3 \Delta N\left(D_i\right)_{freeze}}{\Delta t} \quad (37)$$

For temperatures colder than –50°C, any remaining cloud droplets freeze.

3) PROBABILISTIC FREEZING OF LIQUID WATER DROPS

The probabilistic freezing of liquid drops forms high-density ice water particles (Bigg 1953; Wisner et al. 1972; Lin et al. 1983l Ferrier 1994; and others). The freezing of liquid drizzle is a source for the frozen drizzle category with a density of 900 kg m^{-3}. Similarly, the freezing of any species of rain is a source for frozen rain. As pointed out by Wisner et al. (1972) laboratory experiments suggest that this is perhaps a stochastic process, and a function of the temperature, volume of the liquid water particle, and number of ice nuclei that can activated in droplets or drops. Following Bigg (1953) and Wisner et al., an equation for the probability of freezing of a drop with volume V' and temperature T is given as

$$\ln\left(1-P\right) = B^{'}Vt\left\{\exp\left[A^{'}\left(T_o - T\right) - 1\right]\right\} \quad (38)$$

Following Wisner et al, the following equation for number of drops of diameter D that are frozen considering only differentials when only time t and N vary, the following results,

$$-\frac{d\left[n(D)dD\right]}{dt} = \frac{\pi B^{'} N D^3}{6}\left\{\exp\left[A^{'}\left(T_o - T\right) - 1\right]\right\} \quad (39)$$

The number freezing rate for each bin in the distribution is given by,

$$N_y FZ_{xi} = -\frac{dn(D_i)}{dt}\Delta D, \quad (40)$$

whereas the frozen particles are transferred to the appropriate bin in the frozen drizzle category and the unfrozen particles remain in the drizzle category. The total number of particles frozen would be given by

$$N_f FZ_L = \sum_{i=1}^{M} -\frac{dn\left(D(i)\right)}{dt}\Delta D, \quad (41)$$

although this formula is not actually used in the model since each bin is treated separately. Next a mass tendency equation for each bin is simply given as

$$Q_y FZ_{xi} = N_y FZ_{xi}\frac{\pi}{6}D^3\rho_x, \quad (42)$$

Here, the subscript x represents the liquid drops and y represents frozen drops. It is noted that Thompson et al. (2008) returned to the original Bigg (1953) paper and formulated a freezing rate based on the information therein.

c. Collection equations

For building look-up tables for hybrid bulk/bin models, power-law based terminal velocities $V_t(D_x)$ and $V_t(D_y)$ as described in tables (not shown) are used. In addition, The equations solved for various combinations of $D_{n,x}$, $D_{n,x}$ v_x ,v_y, μ_x, and μ_y are given above. A mixing ratio equation can be written as,

$$Q_x AC_y = \sum_1^{48}\sum_1^{48}\left[\begin{array}{c}\dfrac{0.25\pi M\left(D_x\right)E_{x,y}\mu_x\mu_y N_{T,x}N_{T,y}\left(D_x+D_y\right)^2\max\left[\left(V_{t,x}-V_{t,y}\right),0\right]}{\rho\Gamma\left(v_x\right)\Gamma\left(v_y\right)} \\ X\dfrac{1}{D_{n,x}}\dfrac{1}{D_{n,y}}\left(\dfrac{D_x}{D_{n,x}}\right)^{\mu_x v_x-1}\left(\dfrac{D_y}{D_{n,y}}\right)^{\mu_y v_y-1}\exp\left(-\left(\dfrac{D_x}{D_{n,x}}\right)^{\mu_x}\right)\exp\left(-\left(\dfrac{D_y}{D_{n,y}}\right)^{\mu_y}\right)dD_x dD_y\end{array}\right],\tag{43}$$

and a number concentration equation can be written as,

$$N_x AC_y = \sum_1^{48}\sum_1^{48}\left[\begin{array}{c}\dfrac{0.25\pi E_{x,y}\mu_x\mu_y N_{T,x}N_{T,y}\left(D_x+D_y\right)^2\max\left[\left(V_{t,x}-V_{t,y}\right),0\right]}{\Gamma\left(v_x\right)\Gamma\left(v_y\right)} \\ X\dfrac{1}{D_{n,x}}\dfrac{1}{D_{n,y}}\left(\dfrac{D_x}{D_{n,x}}\right)^{\mu_x v_x-1}\left(\dfrac{D_y}{D_{n,y}}\right)^{\mu_y v_y-1}\exp\left(-\left(\dfrac{D_x}{D_{n,x}}\right)^{\mu_x}\right)\exp\left(-\left(\dfrac{D_y}{D_{n,y}}\right)^{\mu_y}\right)dD_x dD_y\end{array}\right].\tag{44}$$

These equations are completely general in that they work for both self collection and collection between different species. Note that for mixing ratio that an equation for y collecting x is found by replacing *M(D_x)* with *M(D_y)* and $\max\left[\left(V_{t,x}-V_{t,y}\right),0\right]$ with $\max\left[\left(V_{t,y}-V_{t,x}\right),0\right]$. As a result, it is possible to get a more accurate estimate of two particles with quasi-similar, spectrum mean terminal velocities, than by other methods suggested by Wisner et al. (1973), Murakami (1990), Mizuno (1990), or even Verlinde et al. (1993).

1) COLLECTION EFFICIENCIES

All collection efficiencies (E*x,y*) between any two species *x* and *y* have been parameterized into two-dimensional lookup tables with the collector particle on one axis and the collectee particle on the other (Cooper et al. 1997; and Wang 2004), with diameters exponentially or otherwise spaced, values needed simply are obtained by bi-linear interpolation.

2) RIME STORAGE ON CRYSTALS, SHAPE CHANGES, AND ASSOCIATED VERTICAL FLUX ADJUSTMENTS

In this model, the amount of rime accreted by the ice is prognosed to allow for more realistic particle shape and density adjustments. Because of this, the fall speed for hydrometeors in each bin may be adjusted to a more realistic value. In this version of the model, a simple linear interpolation between an assumed ice crystal fall speed with no rime and a fully rimed crystal (graupel) is used. When fully rimed, the particle is converted to a graupel particle. The fraction of rime amount for each particle in a bin is then given by

$$F_i = \min\left(\frac{X_{actual\_rime,i}}{X_{crit\_rime,i}},1\right)\tag{45}$$

Once the prognosed rime that is stored on each crystal within a bin exceeds the critical threshold given by the above formula, then Fi=1 and the low-density graupel fallspeed equation is used for the bin. If there were no rime, then Fi=0 and the particles in the bin would fall as their original unrimed state. The weighting function can be expressed simply as

$$V_T\left(D_i\right) = c_x\left(D_i\right)_x^{d_x}\left(1-F_i\right) + c_{graupel}\left(D_i\right)^{d_{graupel}} F_i \,. \tag{46}$$

g. Breakup of raindrops

The breakup of rain drops plays an important role in describing the hydrometeor distribution in the real atmosphere and can lead to the so called Marshall-Palmer distribution or negative-exponent distribution in the mean over time. However, two to four modes in the distribution can develop in as little as five to ten min owing to break up by particles 0.5 to 2 mm in diameter colliding with larger particles. The most common type of breakup is near head on collisions producing sheet breakups, followed by filament breakups, disk breakups, and bag breakups (Low and List 1982). Aerodynamic breakup is rare (RY89, PK97) as few if any drops ever get large enough for this mechanism to operate. Presently, it is not known how to easily model the distribution modes owing to these types of breakup in a bulk parameterization model. Therefore, we follow the simple formulation by Verlinde and Cotton (1993) to describe the breakup. For warm rain as large as 5mm and for melting frozen drops, melting graupel, etc, up to 9 mm this parameterization does not always perform in a meaningful fashion. This parameterization limits the value of D_{mean_vol} to around 900 microns, and does so based on adjusting the collection efficiency for distributions with D_{mean_vol} > 600 microns. The application of the parameterization is an adjustment to the collection efficiency in the self –collection calculation. The efficiency used to modify the self-collection equations is given by

$$E_{xx} = \begin{cases} 1 & D_{mr} < 900\,microns \\ 2-\exp\left(2300\left[D_{mr}-D_{cut}\right]\right) & D_{mr} > 900\,microns \end{cases} \tag{47}$$

For example, $E_{x,x}$= 1.0 for $D_{m,x} < 6\text{x}10^{-4}$ m, then as the mean drop size increases, $E_{x,x}$ decreases to 0.0 at $D_{m,x} = 9\text{x}10^{-4}$ m. At sizes larger than $9\text{x}10^{-4}$ m the efficiency exponentially becomes more negative implying quick breakup. For example, with $D_{m,x} = 1\text{x}10^{-3}$ m $E_{x,x} = -0.51$, with $D_{m,x} = 1.1\text{x}10^{-3}$ $E_{x,x} = -1.16$, with $D_{m,x} = 1.2\text{x}10^{-3}$ $E_{x,x} = -1.97$. These numbers show the quick breakup of large drops and produce a number concentration source for rain. Breakup of particles other than rain is not permitted. Melting aggregates that are to a large extent liquid (> 50 %) might break up too, but these particles become redefined as melt rain in the model and then breakup can occur if they are large enough.

h. Vapor diffusion

Vapor diffusion of a water particle is based on the sub- or super-saturation of a particle relative to its environment, along with its phase, shape, size, and a variable that is a function of temperature and pressure. The vapor diffusion of liquid water particles is called evaporation and condensation and can occur at temperatures both above and below 273.15K. A saturation adjustment is used for vapor condensation/evaporation of cloud droplets and ice crystals. It is the mode for evaporation of all other liquid water particles and sublimation for ice particles.

1) LARGER LIQUID PARTICLES

For liquid particles, the basic equation that is solved for evaporation and condensation is based on a modification of Byers (1965),

$$\frac{dM(D)}{dt}=\frac{2\pi D\left(S_l-1\right)F_v}{\left[\frac{L_v^2}{K_aR_vT^2}+\frac{1}{\rho\psi Q_{sl}}\right]}, \tag{48}$$

where is the ventilation coefficient F_v is given by

$$F_v=\left(0.78+0.308S_c^{1/3}R_e^{1/2}\right)\ \text{for}\ \left(S_c^{1/3}R_e^{1/2}\right)>1.4\ \text{(larger particles)} \tag{49a}$$

$$F_v=\left(1.0+0.108\left(S_c^{1/3}R_e^{1/2}\right)^2\right)\ \text{for}\ \left(S_c^{1/3}R_e^{1/2}\right)<1.4\ \text{(smaller particles)} \tag{49b}$$

The values condensation and evaporation are given as

$$Q_xCD_v=\max(Q_xCE_v,0.0), \tag{50}$$

and

$$Q_xEV_v=\min(Q_xCE_v,0.0). \tag{51}$$

For condensation on or evaporation of melting ice and snow aggregates the value of Q_{sl} is given by that at 273.15K and S_l is computed from this value and that of Q_v. With number concentration, the evaporation of liquid drops is given by (52-56). Note that there is no number change for evaporation from melting ice and condensation. To accommodate the mass transfer with mass gain and loss owing to vapor diffusion and melting processes, the constraint is that the mass must be conserved as expressed by,

$$\int_0^\infty n(M)dM=\text{constant}, \tag{52}$$

where accommodations need to be made for complete evaporation or melting. The general form of the vapor diffusion gain and loss transfer equation is given as,

$$\frac{\partial n(M)}{\partial t}=-\frac{\partial}{\partial M}\left[n(M)\frac{dM}{dt}\right]. \tag{55}$$

where, *n(M)* is the number of particles of mass *M*. One of the most commonly used schemes in the 1970's was the Kovetz and Olund (1969) scheme. Generally, the starting place is to compute the diffusion growth *dM/dt* . Then, the following can be written using index *J* to indicate the bin to which a droplet belongs, and the mass of droplets *M(J)* within that bin to predict an intermediate value of *M*,

$$M'(J)=M(J)+\Delta t\left(\frac{dM}{dt}\right)_J. \tag{56}$$

The new *n(J)* at $t=\tau+1$ is computed from the latest *n*(J)* by

$$n^{**}(J)=\sum_{J'}^{J}R\left(J,J'\right)n^*\left(J'\right), \tag{57}$$

with $R\left(J,J'\right)$ defined by,

$$R\left(J,J'\right)=\begin{cases}\dfrac{M'\left(J\right)-M(J-1)}{M\left(J\right)-M(J-1)} & for\ M\left(J-1\right)<M'\left(J\right)<M\left(J\right)\\ \dfrac{M\left(J+1\right)-M'(J)}{M\left(J+1\right)-M(J)} & for\ M\left(J\right)<M'\left(J\right)<M\left(J+1\right)\\ 0 & for\ all\ other\ cases\end{cases}. \tag{58}$$

Note that number concentration remains constant during condensational growth, and is lost only through the smallest category during evaporation.

2) LARGER ICE PARTICLES

Vapor diffusion of an ice water particle is based on the sub- or super-saturation of a particle relative to its environment, along with its phase, shape, size, and a variable that is a function of temperature and pressure. The vapor diffusion of ice water particles, which can include sublimation and deposition that can occur at temperatures that are only below 273.15K. The basic equation that is solved is

$$\frac{dM(D)}{dt} = \frac{4\pi C\left(S_i - 1\right)F_v}{\left[\dfrac{L_s^2}{K_a R_v T^2} + \dfrac{1}{\rho\psi Q_{si}}\right]}, \tag{59}$$

where C is the capacitance using the electrostatic analog (PK97), and the ventilation coefficient for snow aggregates (Rutledge and Hobbs 1984) is

$$F_v = \left(0.65 + 0.44 S_c^{1/3} R_e^{1/2}\right). \tag{60}$$

Ventilation coefficients for ice crystals are given in Table 2. Equations similar to those for larger water drops are used for frozen drizzle, graupel, frozen rain, and hail. Note that deposition and sublimation are written as

$$Q_x DP_v = \max(Q_x DS_v, 0.0), \tag{61}$$

and

$$Q_x SB_v = \min(Q_x DS_v, 0.0), \tag{62}$$

and again the number concentration change during sublimation is (there is no number concentration change for deposition) is given by the number of particles that completely sublime. The sublimation and deposition of ice crystals is based on whether the particle is columnar or plate like using the particles capacitance analog (Harrington et al. 1995; and PK97). The capacitance is given for plate and dendrite like crystals as

$$C_{disk} = D/\pi \tag{63}$$

for column and bullet like crystals as

$$C_{prolate} = \frac{A}{\ln\left(\left(a + A\right)/b\right)}, \; where\ A = \left(a^2 - b^2\right)^{1/2} \tag{64}$$

with a and b being the radii of a prolate spheroid, and for spheres C is given as

$$C_{spheres} = D/2. \tag{65}$$

i. Melting

Melting involves three processes, including thermal conduction, vapor diffusion, and sensible heat transfer. These are all incorporated in (66), which is from the heat budget equation for a sperical ice water particle. Thus, the melting equation is written as

$$\frac{\partial M(D_x)}{\partial t} = -\frac{2\pi D_x N_{t,x}\left[K_a\left(T - T_o\right) + \rho_o \psi L_v\left(q_v - q_{s,o}\right)\right]F_v}{L_f} - \frac{C_L\left(T - T_o\right)}{L_f}\left(\frac{dM(D_x)}{dt}\right)\Bigg|_{x\,AC_L}, \tag{66}$$

with

$$Q_x ML_m = \min(Q_x ML_m, 0.0) \tag{67}$$

The number concentration change owing to melting is given by the number of particles that totally melt in a time step as with vapor diffusion (52-56).

5. Summary

In this paper, a new microphysical parameterization is proposed. This version is complete for warm rain processes, for ice crystal nucleation and growth and for snow aggregate formation. In addition, it includes a description on the inclusion of three densities of graupel, frozen drops, hail from graupel, hail from frozen drops, spongy hail and very large hail and rain species associated with the melting of these. Some of the highlights of the microphysics parameterization as presented in this paper include the following:

- Hybrid-bulk/bin microphysics calculations including nucleation, diffusion, collection, autoconversion, conversions, freezing, melting,
- Prediction of one-, two-, or three-moments including liquid or ice water content (third-moment), number concentration (zeroth-moment), and reflectivity (sixth-moment), for each hydrometeor species,
- Shape parameter diagnosis from the three-moments mentioned in the bullet above,
- Drizzle as a separate hydrometeor species between cloud droplets and warm rain,
- Nine species or habits of ice crystals, plus four mixed species or habits of ice crystals,
- Frozen drizzle, snow aggregates, three densities of graupel, frozen drops, and four hail species to track origins (from graupel or frozen drops) and size (9-19mm and 19-51mm).

Some important improvements to be made in the very near future include:

- Inclusion of a liquid water storage term on/in ice structures,
- Improve the prediction of median drop size of shed of liquid drops from larger ice (hail) when melting or growing in a wet growth mode (Rasmussen and Heymsfield 1987),
- Use of growth rate equations for ‘a’ and ‘c’ axes of ice crystals to determine the ice crystal species that grow most rapidly / most slowly for a given temperature and supersaturation with respect to ice (Takahashi et al. 1991),
- Use of more accurate equations to define *dM(D)/dt* for large ice particles with various Reynolds number distributions (Rasmussen and Heymsfield (1987),
- Use of newer more consistent diameters, lengths, densities and volumes of ice crystals and snow aggregates,
- Use of more accurate melting physics parameterizations for snow aggregate (e.g., Szyrmer et al. 1999 and Heyraud et al. 2008),
- Addition of a more accurate quasi-stochastic equation may need to be used, such as that by Tzivion et al. (1987), for the autoconversion parameterizations.

In future papers, results will be presented from comparisons of numerical simulations of orographic and lake snows as well as deep convection with in-situ observations from aircraft and platforms on the ground, as well as from remote observations using polarimetric radar.

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