2A.2 Production and flow of turbulence kinetic energy in convective boundary layers*

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Abstract

There are two frameworks within which we can discuss energy in convective boundary layers. The first is the one provided by the Reynolds-averaged Navier Stokes (RANS) equations, as interpreted by Osborne Reynolds in the later 19th Century. The other, much newer framework is that provided by complex dynamical systems. The latter emphasized energy flow through the whole boundary layer rather than the interpretation of local budgets of turbulence kinetic energy. It is argued that the two frameworks constitute two incompatible paradigms, since the first localizes physical causality while the second denies such a simple view. The reasoning applied by Reynolds to his interpretation of the RANS energy equations is examined and found to be faulty. This paper presents a model for energy flow in convective boundary layers from a dynamical systems perspective. This is done both at the whole-system level and, in finer detail, at the level of the individual patterns of motion, or eddies, within the system.

1 Introduction

In his book *Hydodynamics*, Lamb (1916) wrote that turbulence is "the chief outstanding difficulty of our subject". This remains true today, almost a century later. We know the governing equations but we can't solve them except by numerical integration, and even that remains impractical for flows with high Reynolds and Rayleigh numbers. Practical models of flows such as convective boundary layers (CBLs) therefore rely on a combination of empirical information, physical reasoning and intuition, constrained by whatever guidance can be obtained directly from the Navier-Stokes equations. To use these effectively we must have some kind of *understanding* of each kind of turbulent flow. This paper examines the sources of our understanding.

In the absence of rigorous theory, approaches to turbulence can be divided into two broad paradigms: Statistical Fluid Mechanics (SFM) and Complex Dynamical Systems (CDS). The first of these takes a fundamentally stochastic view of turbulence, al-

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beit with an interpretative overlay that often crosses over with the dynamics. It has the longer history, with origins in the work of Osborne Reynolds in the late 19th century, and with major development by Kolmogorov and his Russian school in the 1940's. It leads to the so-called Reynolds-averaged Navier-Stokes (RANS) equations, which equations have been used successfully to model many kinds of flows, albeit with this success tempered by a disconcerting lack of generality, so that methods must be specially tailored for each narrow class of flows.

The second paradigm, CDS, takes a fundamentally deterministic view of turbulence. It has a shorter history and several strands: one begins in the 1950s with the contributions of Alan Townsend on the forms of eddies, and another with the work of Edward Lorenz on chaotic flows. The third strand, the non-equilibrium thermodynamics, has yet to find its great originator in boundary-layer studies, but we note the work of Garth Paltridge who introduced his Maximum Entropy Production principle in the context of the global climate system. Unlike SFM, CDS has yet to produce any practical applications. Indeed, Alan Townsend and his successors would probably not even recognize that their contributions could be categorized in this way—not until we note that complex dynamical systems are all, by definition, spontaneously self-organizing and pattern-forming systems. Thus we can link the study of coherent structures and other recurring patterns of motion to the CDS paradigm.

The word *paradigm* is used carefully here, and to mean much as Kuhn (1970) would have it mean. That is, different paradigms imply not just different models, which may or may not be complementary, but different ways of understanding based on different fundamental concepts; these giving rise to different kinds of questions and different kinds of acceptable answers. Even the meaning of language can change between paradigms, so that dialogue can become difficult between the adherents of different paradigms. In this context we ask 'What is *shear production* of turbulence kinetic energy?' Is it, as the adherents of SFM would have it, the local passing of kinetic energy from the mean flow to the fluctuations; or could 'production' of energy apply only to the flow of energy into a system from sources external to that system—all else being simply the moving of energy around within a system and not 'production' at all—as might be maintained from a CDS standpoint.

The SFM paradigm has lead us to pursue a local, layer-by-layer understanding of boundary-layer mechanics, based on differential equations, with local energy budgets as a key component. One of its most successful products, the Monin-Obukhov similarity theory, is a product of exactly this kind of thinking. The CDS paradigm, on the other hand, demands a system-wide view, since all parts of the system mutually adjust to each other in such a way as to optimize transport. (We don't yet know quite what we mean by *optimize* here). Essential patterns of motion appear only when the governing differential equations are integrated, either naturally or digitally. CDS leads to different concepts—a 'surface friction layer', for example, defined in terms of the dominant patterns of motion found there rather than a 'surface layer' defined as the layer where surface values of fluxes are sufficiently close to their local values for those values to be useful in local modelling—and to a similarity theory based on the forms, energies and sizes of the various patterns of motion ('eddies') in the flow, all of which are integral properties (McNaughton et al., 2007; Laubach and McNaughton, 2009).

Paradigms ultimately survive or fail depending on how well their constructs and relationships correspond to those found in the real world, and on how well they lead to the asking of useful new questions. Internal contradictions can destroy a paradigm, but often not immediately since a new paradigm might be hard to find and, when found, be markedly less well developed and so less useful in the short term. It is the opinion of the present author that boundary-layer meteorology is now entering a phase where the shortcomings of the SFM paradigm, and the advantages and opportunities entailed in the CDS paradigm, are becoming apparent. The present discussion of energy production and flow in CBLs is intended to highlight this.

2 Energy flows in the CBL

We begin by taking a CDS perspective and look at the energy flows in a horizontally-homogeneous convective boundary layer (CBL). Energy flows are a central concern because energy must flow through any CDS if its patterns are to be maintained against the universal tendency to disorder. The energy flows in a CBL are shown schematically in Fig. 1, according to the description by Laubach and McNaughton (2009). Energy sources are indicated by the boxes which break the outline of the main box enclosing the CBL system. The top system boundary, though drawn schematically as a straight line, is the convoluted boundary that separates the quiet air above the CBL from the air within it, characterized by the presence of small, dissipating eddies. The surface friction layer (SFL), shown at the bottom of Fig. 1 as the layer from the ground up to the line marked z_s, usually displays substantial wind shear and higher levels of turbulence kinetic energy (TKE) than found in the rest of the CBL. By common consensus, it is this wind shear and proximity to the ground that gives rise to the intense eddying and high rates of dissipation found here. The flow schematic is partly geographically organized, making it clear that most of the energy dissipated within the SFL flows down from above and is not 'produced' within the SFL.

To demonstrate this we use the Reynolds expansion of the velocity field into ensemble-mean and fluctuating parts, so that

$$u = \overline{u} + u', \quad v = \overline{v} + v', \quad w = \overline{w} + w', \tag{1}$$

then apply the Reynolds averaging rules to obtain an expression for the one-dimensional flux of total kinetic energy, following Taylor (1952), then take its vertical divergence to obtain the equation

$$\frac{1}{2}\frac{\partial \overline{w(u^2+v^2+w^2)}}{\partial z} = \overline{w'u'}\,\frac{\partial \overline{u}}{\partial z} + \overline{u}\,\frac{\partial \overline{w'u'}}{\partial z} + \frac{\partial \overline{w'e}}{\partial z} \tag{2}$$

Here e is the turbulence kinetic energy (TKE), $e = (u'^2 + v'^2 + w'^2)/2$, and we have set $\overline{v} = \overline{w} = 0$, as befits a plane, horizontally-homogeneous flow in a CBL.

Since both $\overline{u'w'}$ and $\partial\overline{u}/\partial z$ are negative in the SFL, the divergence of the downwards flux of kinetic energy is positive, contributing energy to the local turbulent mo-

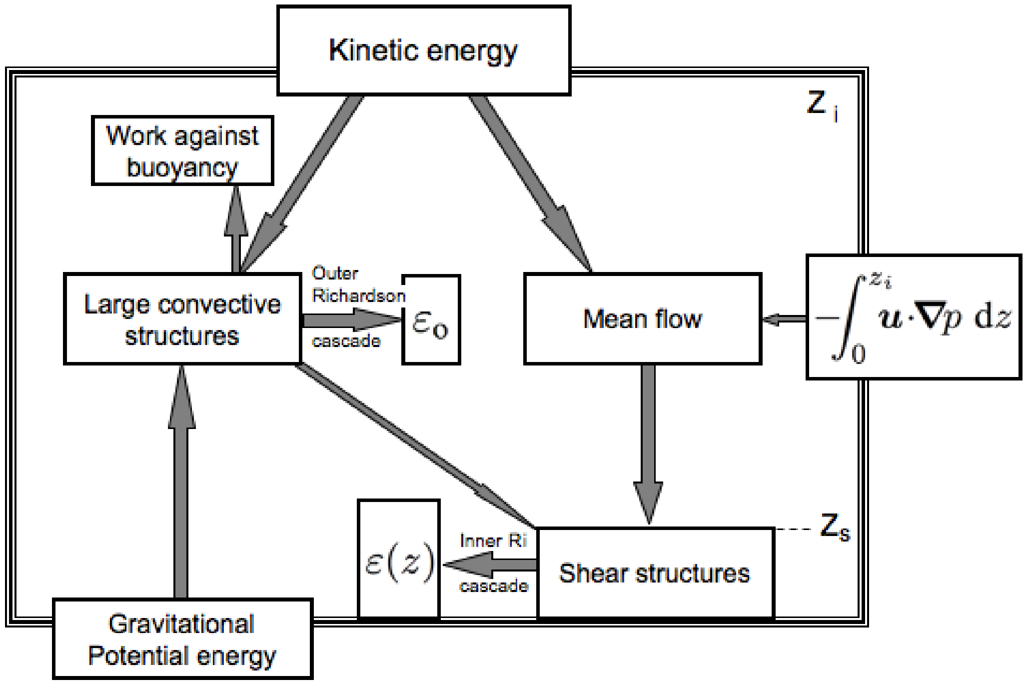

Figure 1: Energy flows in a growing convective CBL, as described by Laubach and McNaughton (2009). The CBL is a thermodynamically open system, with external energy sources indicated by boxes which break the outline of the system we call the CBL. Every part of the CBL is dynamically connected, so local processes cannot be understood except within the context of the flow as a whole.

tions. Experiments show that at heights below $-0.5L$, where L is the Obukhov length, the first term on the right of (2) is usually the largest; the second term is small because $\overline{u'w'}$ is approximately constant with height; and the third term is usually small compared to the first (Li et al., 2008). Fig. 1 predicts the same thing since only a small fraction of the energy introduced as work done by the imposed pressure gradient is actually done within the SFL. Equation (2) is therefore consistent with Fig. 1.

Let's take stock of what we have just done. Equation (2) is a formally correct statement, based on the definitions of kinetic energy and of ensemble means of the velocity components, as introduced by Kolmogorov and his Russian school (Monin and Yaglom, 1971). The manipulations required employ Reynolds' averaging rules, which are themselves formally correct for ensemble averages. The equation entails no new physics—not even that implicit in the Navier-Stokes equations. It is therefore purely an accounting equation, and it is necessarily correct for a horizontally-homogeneous flow. Interpretation of $\overline{w'u'}\,\frac{\partial \overline{u}}{\partial z}$ as the largest term on the right of (2) relies on empirical information, but again this requires no new physics. Our interpretation should be as correct within the CDS paradigm as it is within the SFM paradigm, since both paradigms respect the primacy of experimental evidence. We might say that there could be no possible argument with this interpretation, but we would be wrong since all the major textbooks (Lumley and Panofsky, 1964; Monin and Yaglom, 1971; Wyngaard, 2010) have a different explanation, and that explanation lies within the SFM paradigm.

3 Production of turbulence kinetic energy

The textbooks interpret $-\overline{w'u'}\,\partial\overline{u}/\partial z$ as the local transfer of kinetic energy from the mean flow to the turbulent fluctuations, and they describe this process as *shear production* of TKE. Their argument has two parts: the first is a formal development to produce the Reynolds-averaged equations for the budgets of mean- and turbulent kinetic energy based on the Navier-Stokes (RANS) equations; and the second part is an interpretation based on physical arguments. Both parts trace back to the work of Reynolds (1895), though Reynolds employed volume rather than ensemble averaging so that rigor was not achieved in his formal development.

The modern textbooks all use ensemble averaging so their derivations are all formally correct. Beginning with the Navier-Stokes equations and making substitutions, using the Reynolds decomposition of field variables into mean and fluctuating parts, then manipulating the results with the aid of Reynolds' rules for the behaviours of the averaged variables, they obtain equations which, for a horizontally-homogeneous flow, can be represented as

$$\frac{\partial E}{\partial t} = \ldots + \overline{w'u'}\,\frac{\partial \overline{u}}{\partial z} \tag{3}$$

for the kinetic energy of the mean flow and

$$\frac{\partial e}{\partial t} = \ldots - \overline{w'u'}\,\frac{\partial \overline{u}}{\partial z} \tag{4}$$

for the kinetic energy of the fluctuations. In these equations "..." represents terms that differ between the two equations. The term $\overline{w'u'}\,\partial\overline{u}/\partial z$ appears on the right of both (3) and (4), being added in (3) and subtracted in (4). It comes from the expansion of the nonlinear advection terms in the Navier-Stokes equation, and it signifies a formal coupling of the two equations, so that neither can be solved independent of the other. So much is achieved purely by definition and formal manipulation, so (3) and (4) are unexceptionable as accounting equations. Accurate measurements of the various terms will satisfy them exactly.

The textbooks, however, go further and interpret these terms as representing the physical transfer of kinetic energy from the mean part of the flow to the fluctuating part. This explanation has become part of the SFM paradigm since the textbooks do not distinguish between the formal aspects of (3) and (4) and their interpretation. But this creates a problem because we now have two interpretations of $-\overline{w'u'}\,\partial\overline{u}/\partial z$, one from (2) and the other from (3) and (4), and the first of these is, as said above, unexceptionable. The second interpretation, that the TKE is created locally rather than transported in from somewhere else, must be wrong. The error must lie in the interpretation since the equations themselves are rigorously derived.

A careful reading of the textbooks shows that the arguments used in favor of the *shear production* interpretation of $-\overline{w'u'}\ \partial\overline{u}/\partial z$ all trace back to the work of Osborne Reynolds himself (Reynolds, 1895), so we must revisit that paper if we are to understand where the standard SFM interpretation of *shear production* comes from.

3.1 Reynolds' paper of 1895

Reynolds' paper (Reynolds, 1895) was not given an easy passage by its editor and reviewers. We know this because the editorial correspondence on it has now been made available by the Royal Society of London (Jackson and Launder, 2007). Lord Rayleigh was then editor of the *Philosophcal Transactions of the Royal Society*, and he selected Sir George Stokes—he of the eponymous Navier-Stokes equations—as reviewer and, for a second opinion, Horace (later Sir Horace) Lamb, a mathematical physicist and author of the highly-respected book *Hydrodynamics* (Lamb, 1916). It would be hard to imagine three scientists better qualified to evaluate a paper of this nature. Unfortunately they found Reynolds' writing to be obscure, and they could not understand his arguments. Rayleigh, acting on the recommendation of his reviewers, decided to accept the paper for publication, though he based this decision on Reynolds' high standing rather than on any endorsement of his arguments.

Two things puzzled his reviewers. One was Reynolds' pernickety discussion of averaging methods and his insistence that they limited the generality of his conclusions to flows with linear mean velocity gradients, or to steady flows. The other was the substantial space that Reynolds devoted to discussing how the kinetic energy of eddies is converted into heat energy. Perhaps Reynolds' reasons for this concern have never been fully understood, yet Reynolds saw this as a central issue.

In a section of his paper, apparently added in response to the reviewers' comments, Reynolds (1895) re-asserted the importance of these matters. He wrote:
"*My object in this paper is to show that the theoretical existence of an inferior limit to the criterion follows from the equations of motion as a consequence*
(1) Of a more rigorous examination and definition of the geometrical basis on which the analytical method of distinguishing between molar-motions and heat-motions in the kinetic theory of matter is founded; and
(2) Of the application of the same method of analysis, thus, definitely founded, to distinguish between mean-molar-motions and relative-molar-motions where, as in the case of steady-mean-flow along a pipe, the more rigorous definition of the geometrical basis shows the method to be strictly applicable, and in other cases where it is approximately applicable."[R5]
where the notation [Rx], here and elsewhere, indicates that the source is numbered section 'x' of Reynolds' paper.

Though Reynolds' writing style is characteristically sinuous and obscure, we understand that he was looking for an explanation of why a laminar flow becomes turbulent at a particular value of his [Reynolds] number. He sought this explanation in the conditions necessary for energy to pass from the mean flow to the turbulent fluctuations. To do this he needed a reliable method for distinguishing between the mean flow and the fluctuations, and for this he appealed to the known method for distinguishing between molar (i.e. bulk) and molecular (heat) motions of a fluid. He was particularly interested in this because he saw this process as analogous to the transfer of kinetic energy from mean motion to the turbulent fluctuations. To follow this argument we begin with his ideas on molar- and molecular-motions.

3.1.1 Molar- and molecular-motions

Reynolds (1895) asked how the kinetic energy of eddies is finally transformed to heat energy. He understood that energy is passed down from larger to smaller eddies and eventually to dissipation as heat, and he was familiar with the kinetic theory of fluids, having made important contributions to this subject in his own work on the escape of molecules through surfaces having surface tension. Thus Reynolds well understood that, at the finest scales, fluids are not continuous but are divided into molecular parts, and that heat energy could be interpreted in terms of the mean kinetic energy of the molecules. He also knew that a continuum model for a molecular fluid could be created by taking an integration volume large enough to average out the erratic molecular motions within it, then assigning the mean velocity over that volume to a point at its centre of gravity. By making successive integrations over neighboring volumes one could define the molar velocity as a smooth function of position. In this Reynolds was up with, or ahead of, other scientists of his time since the molecular nature of fluids was not generally acknowledged until 1905, when Einstein's paper on Brownian motion was published.

This is not to say that Reynolds possessed a modern knowledge of small-scale processes in fluids. For example, he didn't know that the smallest eddies have sizes comparable to the Kolmogorov microscale, so he considered the case where the smallest eddies have sizes comparable to the mean free paths of molecules. The molar velocity field can then be curved within the minimum averaging volume needed to smooth the molecular motions. This means that molar and molecular motions cannot be perfectly separated by the volume averaging method. However, Reynolds believed that a strict separation must be possible. This is a pivotal point in Reynolds' argument, so we quote him directly:

'*The only known characteristic of heat-motions, besides that of being relative to the mean-motion, already mentioned, is that the motions of matter which result from heat are an ultimate form of motion which does not alter so long as the mean-motion is uniform over the space, and so long as no change of state occurs in the matter. In respect of this characteristic, heat-motions are, so far as we know, unique, and it would appear that heat-motions are distinguished from the mean-motions by some ultimate properties of matter.*" [R11]

Here Reynolds uses the term *mean-motion* to indicate a volume mean over the local molecular motions, while we use his alternative name, *molar-motion*, for the same thing.

Since there must be an absolute distinction between molar- and molecular-motions, Reynolds argued, there must be some mechanism able to maintain that distinction throughout the flow. Reynolds didn't know what that mechanism might be, but he was certain it must exist, and he called it the "*discriminative cause*" of scale separation. Since energy must be transferred from the molar- to the molecular-motions, Reynolds reasoned, there must be a physical mechanism able, at each point in the flow, to bypass the discriminative cause and so effect that transfer; he called this mechanism the "*cause of transformation*". Thus, for Reynolds, the molar- and molecular-motions were physically real and separate things, but able to exchange energy at each point in the flow by some unknown mechanism.

While Reynolds believed that molar- and molecular-motions are different "*ultimate properties of matter*", we would say that they are merely different *representations* of the same underlying reality. Molecules (or something even more fundamental) are the stuff of all flows and the kinetic model, which *represents* a fluid as a collection of small, perfectly-elastic molecules, can be used to describe its behavior at very fine scales comparable to the mean free path of the molecules. At coarser scales the same flow, made of the same molecules, can be well *represented* as a continuum in which velocities are continuous field variables governed by differential equations. Either representation may be used conveniently at scales between the Kolmogorov micro-scale and the scale set by the molecular mean free path since, in 1866, James Clerk Maxwell had shown that the Newtonian viscosity of a fluid—a continuum property—is a property predicted by the kinetic model of gases. No physical cause of transformation is required to transfer energy from one *representation* of the flow to the other.

Since Reynolds was wrong in his belief that molar- and molecular- motions are physically distinct states of motion, he was also wrong in his belief that some kind of physical "*discriminative cause*" is necessary to prevent molar- and molecular scales from overlapping, and wrong again in his deduction that some physical "*cause of transformation*" is needed to explain the transfer of kinetic energy from molar to molecular motions. The key point hidden in Reynolds' argument is that this description must apply everywhere in the flow, which is to say, locally.

3.1.2 Mean- and eddy-motions

Believing strongly in the physical distinction between molar- and molecular motions and, in consequence, that he had firmly established the local existences of the *discriminative cause* and the *cause of transformation*, Reynolds argued for an analogous distinction between the mean and fluctuating parts of the molar flow. He noted that if the *discriminative cause* and *cause of transformation* depend on properties of matter which affect all modes of motion, distinctions in periods must exist between mean motions and fluctuations, and a transformation of energy must take place from the one to the other, as between the molar- and molecular-motions [R11]. He then argued that proof of the analogy should be sought in experiment [R12].

Reynolds' evidence for this analogy is presented in just one paragraph of his forty-two page paper. He appeals to his earlier pipe experiments (Reynolds, 1883) and argues that energy must be passed from the mean flow to the fluctuations, since it is
"*by which transformation the state of eddying-motion is maintained, notwithstanding the continual transformation of its energy into heat-motions*"[R13].

This is sound physics, and exactly what we would say from a CDS viewpoint for the flows with which Reynolds was familiar, but it applies in an integral sense, to the flow as a whole, rather than in the local sense required by Reynolds.

Reynolds, however, was confident of his arguments and concluded:
"*We have thus direct evidence that properties of matter which determine the cause of transformation, produce general and important effects which are not confined to the heat-motions.*" [R13]
In effect, Reynolds transferred his *belief* in the necessary physical separateness of molar- and molecular motions to the relationship between the mean-motion and the fluctuations, transferring also his belief that there must exist a local cause of transformation to pass energy from the mean motion to the fluctuations. The word *local* is key here. Reynolds had transferred his integral argument down to local scale.

Reynolds then went on to derive his famous RANS equations for the mean and fluctuating parts of the kinetic energy (his equations (17) and (19); summarized here as (3) and (4)). These are differential equations, so they apply locally at each point in the flow. He noted the terms $\overline{w_i' u_j'}\ \partial \overline{u}_i / \partial x_j$ was added to the one and subtracted from the other, where we have used the Einstein summation convention for the general case. Reynolds identified this term with his *cause of transformation*, arguing:
"*These terms which thus represent no change in the total energy of mean-motion can only represent a transformation from energy of mean-mean-motion to energy of the relative-mean-motions.*" [R16]
or, in the simplified terminology used here, transformation of energy from the mean flow to the fluctuations. He uses the word 'transformation', not 'transfer', because his analogy had lead him to believe that mean motion is a different state of motion to the fluctuations, held separate by a discriminative cause. In this way, Reynolds implicitly transferred his integral arguments on energy transfer from the mean flow to the fluctuations down to local scale.

Reynolds' error becomes clear when we consider flows other than the pipe and stream flows with which he was familiar. His flows were driven by uniform body forces (pressure and gravity) while other flows can be driven from their boundaries by the action of traction forces or, as in Fig. 1, by the entrainment of faster fluid through an open boundary. Energy introduced at boundaries is then redistributed throughout the flow, and this redistribution is described partly by the terms $-\overline{u_i' u_j'}\ \partial \overline{u}_i / \partial x_j$ or, in our horizontally-homogeneous CBL, by $-\overline{w' u'}\ \partial \overline{u} / \partial z$.

Though the comments presented above are clearly critical of Reynolds' arguments, they are not critical of Reynolds' understanding that it is necessary to establish the local physical distinctness of the mean flow and fluctuating parts of the flow if the one part is to pass kinetic energy locally to the other. Unfortunately, and no doubt partly because of the obscurity of Reynolds writing, the importance of this point was not understood by his editor and reviewers from the Royal Society of London, or, it seems, by later researchers and textbook writers. Reynolds had implicitly localized the ideas of cause and effect in turbulent flows, and this localization became a central but unquestioned part of the SFM paradigm. It is to this localization of cause and effect that the CDS viewpoint raises its strongest objection.

Of course much has changed since 1895, and Reynolds' own averaging methods are not the ones described in modern textbooks. Ensemble means now replace Reynolds' volume means. This solves the problem of defining mean profiles in flows where the largest eddies span the flow, and it makes rigorous Reynolds' rules for dealing with averages. The RANS equations then become formally rigorous. The terms with $\overline{w'u'}\ \partial\overline{u}/\partial z$ in (3) and (4) remain as coupling terms, so that the one equation cannot be solved without the other. However, the new statistical interpretation of these equations is quite agnostic as to whether any physical distinction can or should be made between the mean and fluctuating components of velocity, and it cannot provide any interpretation of the kind developed by Reynolds. Reynolds' interpretations have simply been carried forward from his original paper to more modern works.

4 Energy flow in shear turbulence

Though we have found Reynolds' local interpretation of $-\overline{u_i'u_j'}\ \partial\overline{u}_i/\partial x_j$ to be flawed, we may still ask whether there any physical processes we can associate with this expression. Many flows have regions with both velocity gradients and momentum fluxes, and these are often regions of intense turbulence. Examples are the turbulent wall-bounded shear layers that form whenever a fluid of sufficient velocity passes over a rigid surface, and the turbulent mixing layers that form at the interface between streams of fluid moving with different velocities.

A good example of the former is the surface friction layer shown at the bottom of Fig. 1. A good example of the latter is the clear-air turbulence formed high in the troposphere on the margins of jet streams. The sheared inversion layer often found at the top of the CBL in Fig. 1 is not a mixing layer, even though the mean conditions might appear to be favorable, because there are no reports that the kinds of turbulent structures found in mixing layers are also found there, and because turbulence levels there are not elevated above those found in the bulk of the CBL (Kaimal et al., 1976; Pino et al., 2003).

Thus we have three examples where $\overline{u_i'u_j'}$ and $\partial\overline{u_i}/\partial x_j$ are both non-zero, so $-\overline{u_i'u_j'}\ \partial\overline{u_i}/\partial x_j$ is significant, but each has distinct characteristics. Below we look at the particular eddy structures found in each of these, and at the energy flows that maintain them. Mixing layers are the simplest, so we start there.

4.1 Plane mixing layers

A mixing layer is the turbulent layer that forms when two streams of fluid having different velocities 'rub against' each other. The mixing layers examined in laboratory studies are usually created by bringing together two parallel flows which have been generated on either side of a plane splitter plate. The mixing layer is the turbulent region that grows between the two laminar streams. The imposed conditions create an inflected mean velocity profile measured normal to the flow, and so create a classic Kelvin-Helmholtz instability in the flow. This leads to the roll-up of transverse vortices which span the mixing layer. Bernal and Roshko (1986) and Moser and Rogers (1991) present images of the roll-up of these large structures, and their subsequent breakdown into the smaller eddies associated with the Richardson cascade towards dissipation.

The energy powering the roll-up of the transverse vortices is the kinetic energy entrained into the mixing layer as its width grows with distance downstream. That is, the energy entering from outside the system passes first to the largest and simplest structures, then on down to the smaller and less-well organized structures of the Richardson cascade, and finally to dissipation as random motions of the molecules. This energy flow is associated with an increase in entropy.

4.2 Sheared capping inversions

The velocity profiles often found up through sheared capping inversions at the tops of CBLs are rather like the inflected velocity profiles found across plane mixing layers. Despite this, sheared inversion layers are not plane mixing layers, even though the term $-\overline{w'u'}\ \partial\overline{u}/\partial z$ in Reynolds' TKE equation can be substantial (Pino et al., 2003). The reason is that capping inversion layers are not isolated systems like plane mixing layers, sandwiched between two laminar flows, but part of the larger system we call the CBL. We have no evidence of transverse roll structures there. Instead the CBL grows by the engulfment of rather large volumes of air from above the CBL. Thus velocity spectra from the entrainment layer (Kaimal et al., 1976) show just a single peak at the scale of the largest, CBL-spanning eddies, followed by a long inertial subrange.

Other information is hard to interpret because it is usually averaged horizontally along a transect crossing and re-crossing the interface between the CBL system and the free atmosphere above. To support the meagre experimental information we appeal to the argument that we would not expect to find any special shear structures in this region, since different structures of similar scale cannot coexist in such non-linear systems. Here the width of the capping inversion($\sim z_i/3$) is of the same scale as the CBL itself, z_i. This is why no box for 'entrainment structures' in the energy flow diagram, Fig. 1.

In Fig. 1 the kinetic energy entrained from above the CBL divides into two steams: one on the left and the other on the right of the diagram. One part goes to the largest convective structures and the other to the mean flow. The mean velocity can be defined by volume integration over the depth of the CBL, and its physical significance can be found in the momentum budget for the mean flow. The diagram is schematic, and we note that the height integral of the pressure work is from 0 to z_i, meaning that some of the work by the horizontal pressure gradient, is done within the SFL, so some energy is produced there. Consistent with this, the energy of the largest eddies also extends over the full depth of the CBL, so the mean flow both fully occupy the CBL. Fig. 1 does not imply any physical separation of the mean and fluctuating parts of the flow at finer scales since these are multiply connected by interactions at all scales. That is, Fig. 1 does not imply a Reynolds-style local decomposition of the flow into mean and fluctuating parts.

On the left of Fig. 1 both the entrained kinetic energy and the gravitational potential energy combine to power the largest-CBL-spanning polygonal convection cells or streamwise roll vortices, according to conditions. This energy then passes directly from these to the outer Richardson cascade and thence to dissipation. This flow from larger to smaller structures, and from more organized to less-organized structures is the same as that which occurrs in plane mixing layers, though the particular large structures are different in the two flows. A small part of the energy is transferred to the SFL as the large convective eddies 'rub along the ground'. Within that layer the pathway to dissipation is not quite so simple. We will deal with it next.

4.3 Surface friction layers

The energy flowing into the SFL, principally from above, go first to the largest structures, of z_s scale where z_s is the depth of the SFL, but from there only a small fraction flows directly to the Richardson cascade and dissipation. The rest follows a more complicated route. The reason is that the turbulence in the the SFL must also transport momentum down to the ground. The z_s-scale eddies in the SFL cannot transfer momentum directly to the ground in one step because their vertical motions are blocked by the ground, so their motions must be nearly horizontal near the ground. Momentum transfer must occur in stages.

At each stage momentum is transferred from the larger eddies, which are attached eddies because blocking by the ground directly affects their dynamics, to smaller but more numerous attached eddies, and the process must repeat down through the scales until the final eddies of this sequence are small enough to transfer momentum directly to the ground or its vegetation. The momentum flux wu, must be accompanied by a kinetic energy flux, wu^2, some of which must go to the smaller, well-organized momentum-carrying structures. On average there must also be an excess of kinetic energy at each stage, because this transfer is down a velocity gradient, and this excess is transferred to the Richardson eddies and so to dissipation. In this way kinetic energy flows down-scale at each stage, but some goes to the well-organized eddies and some to the less-organized eddies. Overall, momentum is transported down to the ground, and the accompanying kinetic energy is all dissipated.

The physical structures and mechanisms involved have been hard to establish. Hairpin vortices are well known from analyses of physical experiments (Hommema and Adrian, 2003; Hutchins et al., 2005) and computer simulations (Lin, 2000; Wu and Moin, 2009). Some experimental studies have identified strong ejections of fluid from near the ground (Corino and Brodkey, 1969; Hagan and Kurosaka, 1993), and simulation studies have show that these originate between the legs of hairpin vortices (Lin, 2000; Adrian, 2007). Thus we know of something called the 'ejection-sweep ' mechanism, but there is no generally-accepted model for what this is or how it transports both momentum and kinetic of energy down towards the ground.

Before sifting the evidence we should note some of the limitations of tracer experiments in real flows and of visualizations in computed flows. Our brains are powerful pattern-recognition machines, but we must take care about what those patterns represent. Tracer experiments show the movements of marked fluid parcels, not the transfers of energy and momentum that we are concerned with here, and if an air parcel is not marked by a tracer then that air parcel will not be seen, whatever its importance to the dynamics of transport. Conversely, a sequence of motions connected by the passage of momentum and energy may not be connected by any continuous movement of introduced tracer: without pressure transport the tracer may not keep up. In the digital world, if a visualization algorithm is not tuned to show some critical structure in simulation results then that structure will not be seen, no matter how clearly the chosen discriminator may show up other recurrent features of the flow.

We can give examples of the care that must be taken. The famous hairpin packets of Head and Bandyopadhyay (1980) were visualized using smoke released near the floor of a wind tunnel, so they trace the movements of air parcels labelled at the wall and travelling in a direction opposite to that of the energy and momentum which are our focus. The observed smoke patterns were undisturbed by any vigorous ejections. Ejections can be marked by tracers, but only if placed where they can be picked up. (Hagan and Kurosaka, 1993) traced ejections by carefully bleeding ink from fine tube set slightly above the ground and between legs of hairpin vortices. A general problem is that tracer experiments are poorly suited to tracking exchanges from larger to smaller eddies since this would cause excessive dispersion of the tracer.

Digital examples of interpretative problems are also plentiful. Wu and Moin (2009) visualized forests of hairpins near the wall of a simulated flow, but all are of the same scale; we must attribute this to both the small Reynolds number of the flow—a major limitation of tracer and visualization experiments—and the size-selectivity of the detection algorithm rather than to any restriction on the sizes of hairpins in high-Reynolds-number flows. Visualizations employ algorithms that are usually biassed towards detecting long-lived structures and against transient structures because the latter may be fewer in a single snapshot and require more specific detection algorithms to capture them. Digital techniques can, in a sense, do anything, but what is done is in the hands of the programmer. For these reason, understanding and targeted looking usually come before seeing and confirmation.

These comments show there is plenty of scope for an independent approach to understanding momentum and energy transport down through the SFL. The TEAL model discussed here (McNaughton, 2004), is consistent with the results of visualization experiments, though it was not directly deduced from them. It was developed from an hypothesis of a particular underlying eddy mechanism—the Theodorsen ejection amplifier (TEA) mechanism—named to honor the discoverer of hairpin vortices. The Appendix gives some previously unpublished arguments to further support this model. Here the intention is simply to describe the energy and momentum transport pathways down through the SFL, using the the TEAL model to show how these might work.

The archetypal TEA structure is a disturbance with a summersaulting action that grows, once properly initiated, into an ideal laminar and logarithmic flow. It begins with an ejection of fluid, squirted from the ground and angled at 45° back into the flow. The oncoming flow is deflected over and around this temporary jet, curling over it to form a vortex with a hairpin-shaped core. Shear rotates the transverse arm of this vortex, causing it to grow and confine the fluid within the hairpin's arc, which fluid then escapes as a squirt, upwards and backwards into the flow. This produces a second, larger ejection. This basic sequence can be called an ejection amplifier, and a series of such structures can form a self-similar, upscale cascade. Further information is given in McNaughton (2004). The trailing arms of the hairpin vortices will create new shear zones, but these are too complicated to visualize in a simple thought experiment.

In real wall-bounded shear flows TEA structures exist only in distorted forms since the growing cascades will collide and jostle for space as the cascades grow. Most TEA-like (TEAL) structures at each cycle will become so distorted that they do not generate a next cycle. These terminal TEAL structures then pass their energy on to Richardson eddies. This results in a population of TEAL structures whose size distribution and energies are consistent with the statistical properties of the energy-containing range of the turbulence found in deep neutral SFLs (McNaughton, 2004). The energy passed to the Richardson eddies varies inversely with z^3, but these eddies, once produced, are swept along within larger eddies so production of Richardson eddies and their final dissipation may not match at each level. In particular, fine-scale Richardson eddies are found in higher concentrations in the warm plumes of rising air (Khalsa, 1980), which carries some of them above the SFL, where dissipation is enhanced (McNaughton et al., 2007), as suggested in Fig. 1. Our concern is to show how the TEAL mechanism, with its basic upscale development, can carry momentum and energy downwards.

Consider first a structure that is the final product of a TEAL cascade that has grown to span the full width of the SFL. As it develops, such a structure conveys faster air from the top of the SFL downwards towards the ground as a sweep around the outside of its hairpin-shaped core, and it conveys low- or negative-velocity air outwards in the ejection from its core. At this stage of its development it is driven by the velocity shear across the whole SFL. Its effect is to increase the velocity, and so the local velocity shear, near the ground and to reduce it above. This increased shear near the ground favors the upwards growth of many smaller TEAL cascades within its footprint, which cascades initiate continuously at the ground. Momentum and energy are both passed downwards from the larger to the smaller TEAL structures in this way. This process then continues down to the next stage, and the next until momentum is transferred directly to the ground. Thus while there is structural continuity in the upscale TEAL cascades, the momentum passes down discontinuously from larger TEAL structures to new cohorts of smaller TEAL structures at each stage. Tracer experiments simply cannot tag this process. Excess kinetic energy passes to detached Richardson eddies at each stage.

Other structural models may yet provide similar explanations, and much remains to be done in this area. Whatever the case, it seems that the eddy structures found in wall-bounded shear layers are specific to such layers, and substantially different to those found in mixing layers. Though the correct model may not be our TEAL model, the model embodies general principles of momentum and energy transfer, and of entropy production, that must be observed by any successful model.

5 Conclusions

We have discussed the production and flow of turbulence kinetic energy in the CBL using two quite different paradigms. The first, the Statistical Fluid Mechanics (SFM), includes the interpretations developed by Reynolds (1895) and adopted in the mainstream of boundary layer meteorology ever since. Its principal effect has been to provide a means of understanding such flows based on the localization of cause and effect. In SFM 'shear production' of TKE represents the transfer of kinetic energy from the mean flow to the fluctuations. SFM leads to the RANS equations, which have widely been used to model boundary-layer flows, and to the many studies of local budgets of turbulence energy in boundary layers. However, SFM also leads to internal contradiction, and we have traced this back to an error in Reynolds' original arguments for the localization of cause and effect in turbulent flows.

We may call the other paradigm the Complex Dynamical Systems (CDS), though this name collects together some lines of work that have not previously gone under this name. Complex dynamical systems are pattern-forming systems, and CDS models take a fundamentally deterministic, albeit chaotic view of turbulence. CDS has a shorter history and has yet to find any prominent place in the development of practical models. This paradigm leads to an emphasis on energy flows, which have been little studied in Boundary-layer meteorology because of the dominance of the SFM paradigm. A schematic diagram is presented describing energy flows in a convective boundary layer. This outlines the energy flows from one kind of eddy structure to the next, and eventually to dissipation as heat. The CDS paradigm denies the possibility of any strictly local interpretation of processes in turbulent flows, including the local transfers of kinetic energy from the mean to the fluctuations. Furthermore, we find no physical process that corresponds to 'shear production' as defined using Reynolds interpretation of SFM.

We conclude that SFM, and so the RANS equations as usually interpreted, are fundamentally flawed as a guide to understanding physical processes in turbulent flows such as convective boundary layers. CDS, on the other hand, offers a more reliable framework for understanding boundary-layer flows, but it is at yet poorly developed and has not yet lead to practical models. Further research may well change this.

A Appendix

Here we present some further arguments in support of the TEAL model, and provide some information that should assist in generating the strongest possible evidence for it—digital simulation of a self-similar TEA cascade. This information is not critical to the arguments developed in the main part of the paper, but this appendix provides a convenient opportunity to record some developments since the last publication on this subject by McNaughton (2004).

A.1 Simulating the TEA cascade

McNaughton (2004) describe an attempt to simulate a TEA cascade by McNaughton and Blundell (2003). Here we provide extra information that might assist others in carrying that project forward. It is also of theoretical interest.

McNaughton and Blundell (2003) considered the growth of a disturbance in a frictionless flow with a logarithmic velocity profile. Their initial disturbance was an 'attached' jet (ejection) of fluid released into the flow through a port in the wall. Viscosity could be neglected since it does not affect the dynamics of large eddies. Euler's equation governs this flow, and was written as

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\,\boldsymbol{u} = -\boldsymbol{\nabla} p \tag{5}$$

The flow also obeyed the continuity equation

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = -\boldsymbol{\nabla} p \tag{6}$$

The wind profile was set to be logarithmic everywhere before the initial ejection was introduced. This profile is written here in differential form because this makes its self-similar form more obvious, and with free slip at the ground the absolute velocity is arbitrary; this and the upwind and downwind boundary conditions were given by

$$\frac{\partial u_1}{\partial x_3} = \frac{u_s}{x_3};\;\; u_2 = u_3 = 0 \text{ at } x \to \pm\infty \tag{7}$$

The lower boundary was impermeable, so that

$$u_3 = 0, \quad \frac{\partial p}{\partial x_3} = 0 \;\text{ at } z = 0 \tag{8}$$

The initial disturbance McNaughton and Blundell (2003) was an ejection of fluid, squirted at 45° back into the flow (as observed in experiments), with equal withdrawal of fluid through ports on either side of the ejection port. This initial ejection induced a downstream vortex with a hairpin-shaped core, as expected, and this disturbance evolved to produce a larger ejection from between the arms of the hairpin. This supported the TEA hypothesis. However this simulation could not be considered a proof of existence of a self-similar cascade of the kind sought. By the time the second ejection appeared the disturbance had grown to such an extent that the constraining effects of the domain walls were substantial, and numerical dissipation had robbed the flow of a significant amount of energy. Available computer resources limited both the resolution and domain size in this simulation. The result was an encouraging pilot study, but not more.

There are good reasons to press on with this line of investigation since we know that a disturbance introduced at a wall will grow, and that growth will become self-similar. It remains to discover the form of that growing, self-similar disturbance. The arguments for these properties are as given below.

A.1.1 Disturbances do grow

Levinski and Cohen (1995) have considered the similar problem of a disturbance growing in a laminar flow with a linear velocity profile. Rather than attempt a full solution, they considered the behavior of the fluid impulse integral, which obeys a linear equation despite the nonlinearity of the Euler equation. They found that the impulse integral—a vector quantity—becomes inclined at 45° back into the flow, in keeping with experimental observations of the angle of hairpin vortex heads and ejections, and they concluded that "*any inviscid two-dimensional plane shear flow is unstable to a three-dimensional localized disturbance*". Indeed they showed that the growth of the impulse integral of the disturbance becomes exponential after some time. Translated to a logarithmic flow, this means that an isolated disturbance initiated at the wall will grow linearly at longer times.

A.1.2 They have self-similar form

The symmetry properties of the Euler equation show that this growth will be self-similar, either as steady growth or as a self-similar inverse cascade. This can be seen by transforming (5) - (8) into the coordinate system defined by

$$\boldsymbol{\xi} = \frac{\boldsymbol{x} - \boldsymbol{x}_0}{u_s(t - t_0)}, \quad \phi = \ln\left(\frac{t - t_0}{\tau}\right) \qquad (9)$$

where $(\boldsymbol{x}_0, t_0)$ are the coordinates and time of the origin of the disturbance at the wall, and u_s and τ are velocity and time scales. In these coordinates (5) - (8) can be written as

$$\frac{\partial \boldsymbol{u}}{\partial \phi} + (\boldsymbol{u} \cdot \boldsymbol{\nabla}_\xi)\, \boldsymbol{u} = -\boldsymbol{\nabla}_\xi\, p + \left(\boldsymbol{\xi} \cdot \boldsymbol{\nabla}_\xi\right) \boldsymbol{u} \quad (10)$$

and

$$\boldsymbol{\nabla}_\xi \cdot \boldsymbol{u} = -\boldsymbol{\nabla}_\xi\, p \qquad (11)$$

with

$$\frac{\partial u_1}{\partial \xi_3} = \frac{1}{\xi_3}; \;\; u_2 = u_3 = 0 \text{ at } \xi_1 \to \pm\infty \quad (12)$$

and

$$u_3 = 0, \quad \frac{\partial p}{\partial \xi_3} = 0 \;\text{ at } \xi_3 = 0 \qquad (13)$$

in which $\boldsymbol{u}$ is now normalized by the scaling velocity u_s and $\boldsymbol{\nabla}_\xi$ is the $\boldsymbol{\nabla}$ operator in $\boldsymbol{\xi}$ space.

These equations are analogous to the original set (5) - (8), except that a new linear term appears on the right of (10). This term acts as a body force opposing the radial spread of the disturbance down velocity gradients. It increases with distance from the origin of the disturbance, so it must eventually halt growth completely, and the disturbance will then assume either a steady form, i.e. onto a point attractor in phase space, or onto a cyclic attractor. The latter corresponds to a self-similar inverse cascade in the original co-ordinates.

We therefore have strong arguments to say that a self-similar cascade will develop if a suitable disturbance is initiated at the wall into a laminar flow with a logarithmic velocity profile. The problem is to find the form of that cascade, and a good place to start is with the hypothesis that it is a TEA cascade. The above coordinate transformation should prove useful in investigating numerical solutions since, by posing the problem in similarity variables, we avoid the computing problem of how to handle unlimited growth. The purpose of the initial ejection is to put the system in a state that lies within the basin of attraction of the proposed cyclical attractor. The time scale τ is arbitrary in (9), but since it determines the rate at which the axes grow, it should be tuned so that growth ceases after some time. Success in the simulation will be achieved when the disturbance stops growing and repeats itself at fixed intervals of ϕ.

A.2 TEAL cascades and drag reduction

A different kind of argument supporting the TEAL model is that the model explains how tiny amounts of polymer can reduce drag in flows over smooth surfaces (White and Mughal, 2008).

Certain polymers increase the extensional viscosity of fluids by several orders of magnitude while not displaying any viscoelasticity (Lu et al., 1997). One way to measure extensional viscosity is by the extra resistance observed when forcing such a solution through an orifice. A large extensional viscosity is measured by a reduced ability to squirt. Since ejections are essential parts of our TEAL mechanism for transmitting drag, and since ejections are squirts, adding polymers will increase the size at which the smallest ejections can develop strongly, and so the smallest scale at which TEAL structures can viably complete their cycles.

The size effect arises because the velocities of TEAL structures scale on u_*, and their cycle times on h/u_*, where h is the height of the structure itself. The rates of extension of ejections therefore increase as the size of the TEAL structures decrease towards the wall, so polymers which increase extensional viscosity have their greatest effect just where viscous drag begins to lose its dissipating effect on rotational structures. This increases the size of the smallest viable TEAL structures, and so reduces drag by thickening the irrotational wall layer of the flow.

More detailed studies confirm this locus of action of drag-reducing polymers. Polymer solutions released well above the wall have no effect until they reach the near-wall region (Wells et al., 1967), where they reduce drag. They act near, but not at the wall. This is confirmed by Tiederman et al. (1985), who show that polymers released at the wall have no effect while they remain in the viscous sub-layer, but have a substantial effect when they diffuse up to between 10 z^+ - 100 z^+ from a smooth wall. Here z^+ is height measured in wall units, u_*/ν. This is the source layer for the fluid found in smaller ejections, as observed by (Hagan and Kurosaka, 1993) who labelled ejections by bleeding tracer ink into this layer. Smoke bled into a flow from from beneath 10 z^+ travels essentially horizontally and is not disturbed by ejections (Kline et. al., 1967).

The above is the only viable account of how polymers reduce drag. It is different to that offered in a recent review by White and Mughal (2008), who suggest that viscoelasticity is the key property of effective polymers, and that these act to inhibit a proposed near-wall regnerating cycle with quasi-streamwise vortices—a limit cycle which has been simulated in constricted computational domains—but their dicussion falls short of full explanation. They say "*despite the capabilities of numerical simulations, how polymers disrupt the cycle remains open to interpretation*". Perhaps further effort to verify the TEAL model cascades will help resolve this important engineering question.

References

Adrian, R. J.: 2007, 'Hairpin vortex organization in wall turbulence', *Phys. Fluids* 19, 041301.

Bernal, L. P., and Roshko, A.: 1986, 'Streamwise vortex structures in plane mixing layers', *J. Fluid Mech.* 170, 499-525.

Corino, E. R., and Brodkey, R. S.: 1969, 'A visual investigation of the wall region in turbulent flow', *J. Fluid Mech.* 37, 1-30.

Hagen, J. P., and Kurosaka, M.: 1993, 'Corewise cross-flow transport in hairpin vortices—the "Tornado effect"', *Phys. Fluids* A 5, 3167-3174.

Head, M. R., and Bandyopadhyay, P.: 1981, 'New aspects of turbulent boundary layer structure', *J. Fluid Mech.* 107, 297-338.

Hommema, S. E., and Adrian, R. J.: 2003, 'Packet structure of surface eddies in the atmospheric boundary layer', *Bound.-Layer Meteor.* 106, 147-170.

Hutchins, N., Hambleton, W. T., and Marusic, I.: 2005, 'Inclined cross-stream stereo particle image velocimetry measurements in turbulent boundary layers', *J. Fluid Mech.* 541, 21-54.

Jackson, D. and Launder, B.: 2007, 'Osborne Reynolds and the publication of his papers on turbulent flow', *Ann. Rev. Fluid Mech.*, 39, 19-35.

Kaimal, J. C., Wyngaard, J. C., Haugen, D. A., Cot, O. R., Izumi, Y., Caughey, S. J., and Readings, C. J.: 1976, 'Turbulence structure in the convective boundary layer', *J. Atmos. Sci.* 33, 2152-2169.

Khalsa, S. J. S.: 1980, 'Surface-layer intermittency investigated with conditional sampling', *Bound.-Layer Meteor.* 19, 135-153.

Kline, S. J., Reynolds, W. C., Schraub, F. A., and Runstadler, P. W.: 1967, 'The structure of turbulent boundary layers', *J. Fluid Mech.* 30, 741-773.

Kuhn, T.S.: 1970, *The structure of scientific revolutions.* 2nd Edition. The University of Chicago Press, Chicago.

Lamb, H.: 1916, *Hydrodynamics.* 4th Edition. University Press, Cambridge

Laubach, J. and McNaughton, K. G.: 2009, 'Scaling properties of temperature spectra and heat flux cospectra in the surface friction layer beneath an unstable outer layer', *Bound.-Layer Meteor.* 133, 219-252.

Levinski, V., and Cohen, J.: 1995, 'The evolution of a localized vortex disturbance in external shear flows. Part 1. Theoretical considerations and preliminary experimental results', *J. Fluid Mech.* 289, 159-177.

Li, X., Zimmerman, N., and Princevac, M.: 2008, 'Local imbalance of turbulent kinetic energy in the surface layer', *Bound.-Layer Meteor.* 129, 115-136.

Lin, C.: 2000, 'Local pressure transport structure in a convective atmospheric boundary layer', *Phys. Fluids* 12, 1112-1128.

Lu, B., Zakin, J. L., and Talmon, Y.: 1997, 'A non-viscoelastic drag reducing cationic surfactant system', *J. Non-Newt. Fluid Mech.* 71, 59-72.

Lumley, J. L. and Panofsky, H. A.: 1964, *The Structure of Atmospheric Turbulence*, Wiley, New York. 239pp.

McNaughton, K. G.: 2004, 'Turbulence structure of the unstable atmospheric surface layer and transition to the outer layer', *Bound.-Layer Meteor.* 112, 199-221.

McNaughton, K. G.: 2006, 'On the kinetic energy budget of the unstable atmospheric surface layer', *Bound.-Layer Meteor.* 118, 83-107.

McNaughton, K. G., and Blundell, R. E.: 2002, 'A model for the large-scale ramp structures observed in the atmospheric surface layer', In *15th Conference on Boundary layers and Turbulence*, Wageningen, Netherlands, 14-19 July 2002. American Meteorological Society, Boston, MA.

McNaughton, K. G., Clement, R. J. and Moncrieff, J. B.: 2007, 'Scaling properties of velocity and temperature spectra above the surface friction layer in a convective atmospheric boundary layer', *Nonlin. Process. Geophys.* 14, 257-271.

Monin, A.S., Yaglom, A.M.: 1971, *Statistical Fluid Mechanics* I, J.L Lumley Editor. MIT Press. 769pp.

Moser, R. D., and Rogers, M. M.: 1991, 'Mixing transition and the cascade to small scales in a plane mixing layer', *Phys. Fluids.* A 3, 1128-1134.

Pino, D., Vil-Guerau de Arellano, J., and Duynkerke, P. G.: 2003, 'The contribution of shear to the evolution of a convective boundary layer', *J. Atmos. Sci.* 60, 1913-1926.

Reynolds O.: 1883, 'An experimental investigation of the circumstances which determine whether the motion of water in parallel channels shall be direct or sinuous and of the law of resistance in parallel channels'. *Philos. Trans. R. Soc.* 174:93582

Reynolds O.: 1895, 'On the dynamical theory of incompressible viscous fluids and the determination of the criterion'. *Philos. Trans. R. Soc.* 186:123-164.

Richardson L. F.: 1920, 'The supply of energy from and to atmospheric eddies', *Proc. Roy. Soc. London A* 97, 354-373.

Stevens, B.: 2002, 'Entrainment in stratocumulus-topped mixed layers', *Quart. J. R. Met. Soc.* 128, 2663-2690.

Stull, R. B.: 1988, *An Introduction to Boundary-Layer Meteorology*, Kluwer, Dordrect. 666pp.

Taylor, R. J.: 1952, 'The dissipation of kinetic energy in the lowest layers of the atmosphere', *Quart. J. R. Met. Soc.* 78, 179-185.

Tiederman, W. G., Luchik, T. S., and Bogard, D. G.: 1984, 'Wall-layer structure and drag reduction', *J. Fluid Mech.* 156, 419-437.

Wells, C. S., and Spangler, J. G.: 1967, 'Injection of a drag-reducing fluid into turbulent pipe flow of a Newtonian fluid', *Phys. Fluids* 10, 1890-1894.

White, C. M., and Muhgal, M. G.: 2008, 'Mechanics and prediction of turbulent drag reduction with polymer additives', *Ann. Rev. Fluid Mech.* 40, 235-256.

Wu, X., and Moin, P.: 2009, 'Direct numerical simulation of turbulence in a nominally-zero-pressure-gradient flat-plate boundary layer', *J. Fluid Mech.* 630, 5-41.

Wyngaard, J.C.: 2010, *Turbulence in the atmosphere*, Cambridge University Press, Cambridge, 406pp.