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MULTIRESOLUTION REYNOLDS STRESS ANALYSIS OF URBAN CANYON TURBULENCE

Cheryl L. Klipp
US Army Research Laboratory, Adelphi, Maryland
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1. Introduction

Turbulence near a solid surface is anisotropic, since motion perpendicular to the surface is limited by the surface. Motion at different spatial scales is impacted by the surface to differing degrees. As a consequence, the fundamental variances and directions for the turbulence vary not only with location within the canyon, but also with the scale of the motion being considered. Due the presence of multiple surfaces, many of which are vertical, the anisotropy in an urban setting is different than the anisotropy in an open setting (Klipp 2010b). Better urban turbulence dispersion modeling will require better characterization of the anisotropy, directions of greatest and least variance, and which spatial scales are likely to have greatest influence.

2. Data

The bulk of this paper will present results from University of Oklahoma's 16 meter tower on the south side of the Park Avenue canyon (OU2). The data are from a fairly typical day, during near neutral conditions a bit before sunrise, 0900 UTC, Sunrise 1120 UTC, Day 190, 9 July, 2003. Results from the intersections and one rooftop sonic are shown at the end.

The canyon is roughly 180 meters long from the center of the intersection on the west to the center of the intersection to the east. The canyon is 24 meters wide and approximately 50 meters tall, although building heights vary considerably (Allwine and Flaherty 2006). The tower is about 70 meters from the opening to the intersection to the west. Since the upstream wind direction is approximately 190°, the canyon winds flow from west to east during this hour.

Since the multiresolution analysis, described below, requires a power of two number of data points, this analysis uses 2^{15} data points, which represents 54.6 minutes of the 10 Hz data.

3. Scales of motion

Larger eddies carry more energy and contribute more to transport than smaller eddies. The smallest eddies are isotopic while the larger eddies are anisotropic. By definition, to calculate variances and covariances (fluxes) it is required to pick a fixed scale over which to calculate the average. Although an averaging time of 15 minutes may capture the full turbulent variances and covariances at an urban location, it does not tell us how much of the total variances and covariances are due to motions at different scales.

The multiresolution decomposition in Howell and Mahrt (1997) and further explained in Vickers and Mahrt (2003) is used here to break up one hour (54.6 min.) variances and covariances into sums of sub-variances due to different time scales. The time scale is then converted into a length scale by multiplying by the mean wind speed for that hour. For small scales, the assumptions of frozen turbulence hold and the scale can be considered an eddy size. This is not true for the larger scales. In some cases it is preferable to think of the larger scales in terms of the original time scales.

Figure 1, shows the multiresolution decomposition for all six members of the Reynolds stress tensor for both the 1.5 m sonic and the 15.7m sonic. For the lowest level (Figure 1a), the along canyon component, $U'U'$, has a peak at the 70 meter scale. Due to the coarseness of the resolution of this method, it is not clear if this is representative of half the canyon length or the distance from the sonic to the edge of the canyon. The cross-canyon component, $V'V'$, has a peak at the 17.6 meter scale. Again, due to the coarse scale resolution, this peak may be related to the canyon width or the distance of the sonic from the canyon wall (16 meters from far wall, 8 meters to near wall). The vertical variance is very small, presumably suppressed by the street surface. Also of note is that the $U'V'$ component is quite significant indicating momentum being lost to the canyon wall. The unexpected positive value for $U'W'$ is probably indicative of momentum being lost to the tree leaves.

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* *Corresponding author address*: Cheryl Klipp, US Army Research Lab, attn RDRL-CIE-D, 2800 Powder Mill Rd., Adelphi, MD 20783. cheryl.l.klipp@us.army.mil

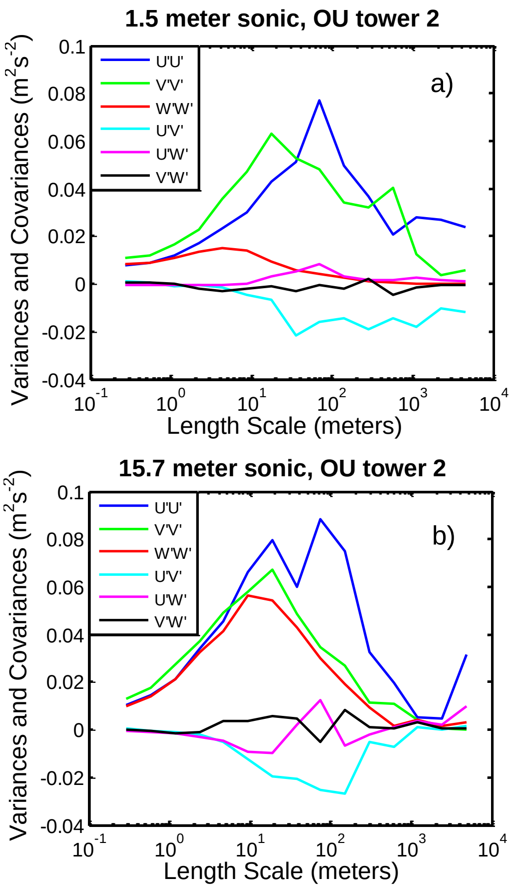

Figure 1: Multiresolution spectra of the variances and covariances of the three components of the wind vector for the University of Oklahoma mid-canyon tower.

For the highest sonic on the tower (Figure 1b), all three variances show peaks near the 20 meter scale, probably related to the canyon width. The along-canyon component also peaks at about the 76 meter scale. As at the lower level, significant momentum is lost to the canyon wall and U′W′ behaves in unexpected ways. This highlights the problems with traditional stress calculations for locations inside the roughness sublayer (Klipp 2008).

At both levels, for the smallest scales the variances are nearly equal to each other and the covariances (fluxes) are nearly zero. This is consistent with the definition of isotropic turbulence.

4. Anisotropy

The Reynolds stress tensor is real and symmetric and can be diagonalized. For isotropic turbulence, the three eigenvalues will be equal. The degree and manor in which they are not equal can be parameterized and plotted as described by Banerjee *et al.* (2007). They scale the eigenvalues by dividing by the trace then decompose the resulting matrix into three basis matrices and corresponding coefficients:

$$\begin{pmatrix} \frac{\lambda_B}{\text{trace}} & 0 & 0 \\ 0 & \frac{\lambda_M}{\text{trace}} & 0 \\ 0 & 0 & \frac{\lambda_S}{\text{trace}} \end{pmatrix} = C_3 \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix} + C_2 \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} + C_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

where the coefficients range in value from 0 to 1 and are constrained to sum up to 1. They can be plotted on a barycentric map, Figure 2. Perfectly isotropic turbulence, $\lambda_B = \lambda_M = \lambda_S$, will map at the C_3 vertex at the top of the triangle. Turbulence with $\lambda_B = \lambda_M$ and $\lambda_S = 0$ will plot at the bottom left vertex, C_2. Turbulence with $\lambda_B > 0$ and $\lambda_M = \lambda_S = 0$ will plot at the bottom right vertex, C_1.

If $\lambda_B = \lambda_M > \lambda_S$, the turbulence will plot along the left side: pancake-like turbulence. If $\lambda_B > \lambda_M = \lambda_S$, the turbulence will plot along the right side: cigar-like turbulence. These describe the axisymmetry of the variances, not eddy shapes.

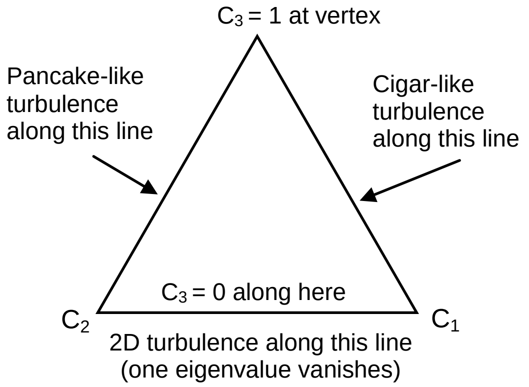

Figure 2: The barycentric triangle plot of turbulence anisotropy.

5. Anisotropy of each scale

Since the matrix elements of the Reynolds stress tensor are variances and covariances and each variance and covariance can be written as a sum of sub-values, each corresponding to a scale of motion (Figure 1), the Reynolds stress tensor can now be written as a sum of sub-tensors, each corresponding to a different scale of motion. Each of these sub-tensors is real and symmetric and can be diagonalized and analyzed to determine the degree and type of anisotropy of each scale.

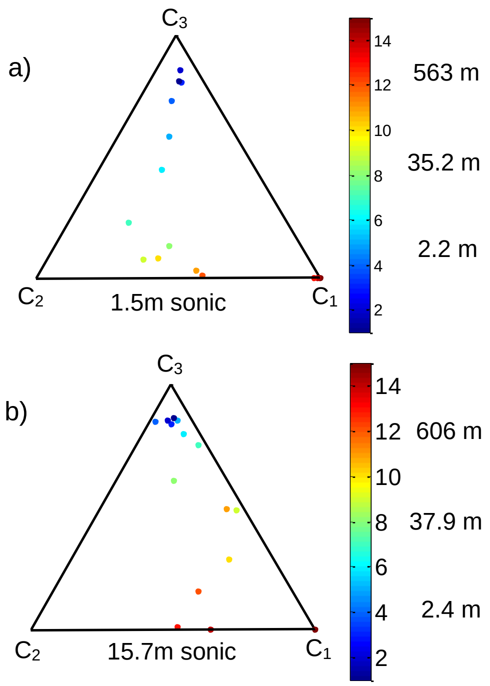

Figure 3: Barycentric plots of the turbulence anisotropy for all 15 scales for mid-canyon OU tower 2 at a) 1.5m and b) 15.7m. The numbers beside the color bar indicate the number of data points for that scale in powers of two, i.e. 8 means 2^8 data points. This is 10 Hz data.

In Figure 3, the smallest scales (blue shades) are near the C_3 vertex indicating that turbulence at these scales is close to isotropic. The largest scales (red shades) are two dimensional, one eigenvalue nearly vanishes. The mid-range scales (greenish shades) are neither isotropic nor extremely anisotropic. These scales are where turbulent kinetic energy is at a maximum (Figure 1). This behavior is also different than found at an open location, CASES99 (Klipp 2010b). This method of plotting anisotropy is more sensitive to deviations from isotropy than the method of Choi and Lumley (2001) (Banerjee *et al.* 2007).

6. Directions of the eigenvectors

In addition to the relative magnitudes of the eigenvalues (the fundamental variances), the directions of the eigenvectors associated with the largest and smallest eigenvalues are also of interest for atmospheric dispersion, since these are the directions of most and least dispersion. In laboratory flows, the eigen-directions are nearly aligned with the streamwise, cross-stream and wall normal coordinates except for a 17° rotation about the cross-stream axis (Hanjalic and Launder 1972). This is found in some open area atmospheric data (Klipp 2008, Klipp2010b), but not in the urban canyon data.

When examining the eigenvector directions, it is best to keep some caveats in mind. Since the smallest scales are nearly isotropic, the distinction between the smallest and largest eigen directions is not very clear since the eigen values are nearly identical. For nearly perfectly pancake-like axisymmetry, the two largest eigen values are nearly identical and a small error can affect which is chosen as the largest. Similarly for nearly pure cigar-like axisymmetry, the two smallest eigen values are nearly identical and which is chosen by the software to be the smallest is highly sensitive to small errors in the measurements.

In addition to the above caveats, the best method to calculate error bars for these analyses have not been determined yet. It will need to combine the measurement uncertainties inherent in sonic anemometers and the propagation of these errors in the multiresolution decomposition and anisotropy analysis, as well as the sensitivity of both the multiresolution decomposition and anisotropy analysis to small changes in the input numbers.

In Figure 4, the direction of the largest variance is plotted relative to the direction of the mean wind direction. This direction varies considerably with scale. At the smallest scales, the direction of largest variance is nearly perpendicular to the mean wind and becomes nearly aligned with the mean wind at mid-sized scales. At larger scales, the direction varies from one level to another.

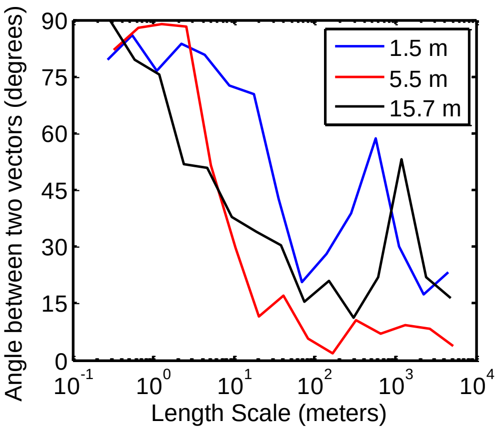

Figure 4: Angle between the mean wind direction and the eigenvector associated with largest eigen value, the direction of largest variance, at the mid-canyon OU tower 2.

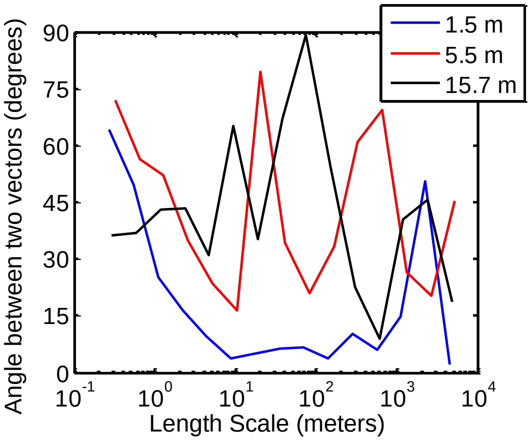

Figure 5: Angle between vertical and the eigenvector associated with the smallest eigenvalue, the direction of least variance, at the mid-canyon OU tower 2.

The direction of least variance with respect to the sonic anemometer vertical direction is plotted in Figure 5. For the 1.5 m sonic, the direction of least variance is nearly vertical for most of the mid-sized scales. At the other elevations, these values vary considerably. Until error estimates are determined, it is not possible to know if these are real differences or just the result of error propagation. Although the 1.5 m direction of least variance is similar to open area directions of least variance (Klipp 2010b), the behavior of the largest variance direction is not quite the same, and the other sonics on the mid-canyon tower do not behave like the open area location at all.

7. Conclusions

Surfaces inhibit turbulent motion resulting in anisotropic turbulence especially in complex urban areas. That the flow is heavily influenced by building morphology is evident in the wide variety of characteristics at different locations in and around the street canyon (Figures 6 – 7) as well as the differences from open area characteristics (Klipp 2010b).

One conclusion is that urban canyon and intersection turbulence anisotropy is usually more cigar-like in its axisymmetry than open area anisotropy. There are places in the canyon, however, where the anisotropy is closer to pancake-like so broad generalizations are difficult to justify.

Future work will need to see how consistent these parameters are at a fixed location for different stabilities and wind conditions. Perhaps very small changes in winds could have a large effect due to the sensitivity of the anisotropy to boundary conditions. Also, due to the great variability from one location to another, similarity relationships for turbulence inside roughness sublayers will be difficult to generate.

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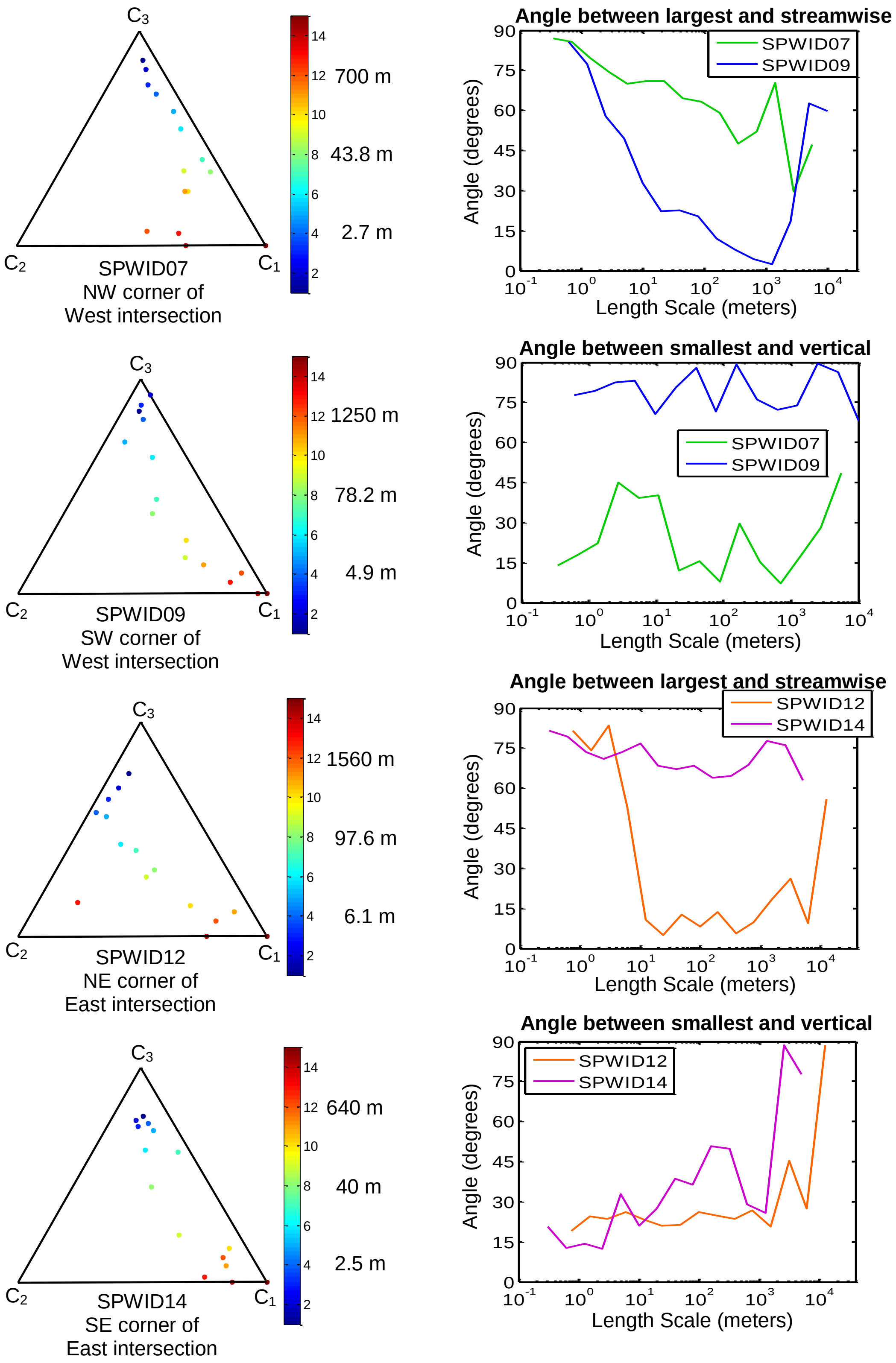

Figure 6: Barycentric plots and angles of eigenvectors for four sonics in the intersections to the East and West of the Park Avenue canyon. They are about 8 meters above street level. See Allwine and Flaherty 2006 for details of placement. Plots are similar to Figures 3 – 5 above.

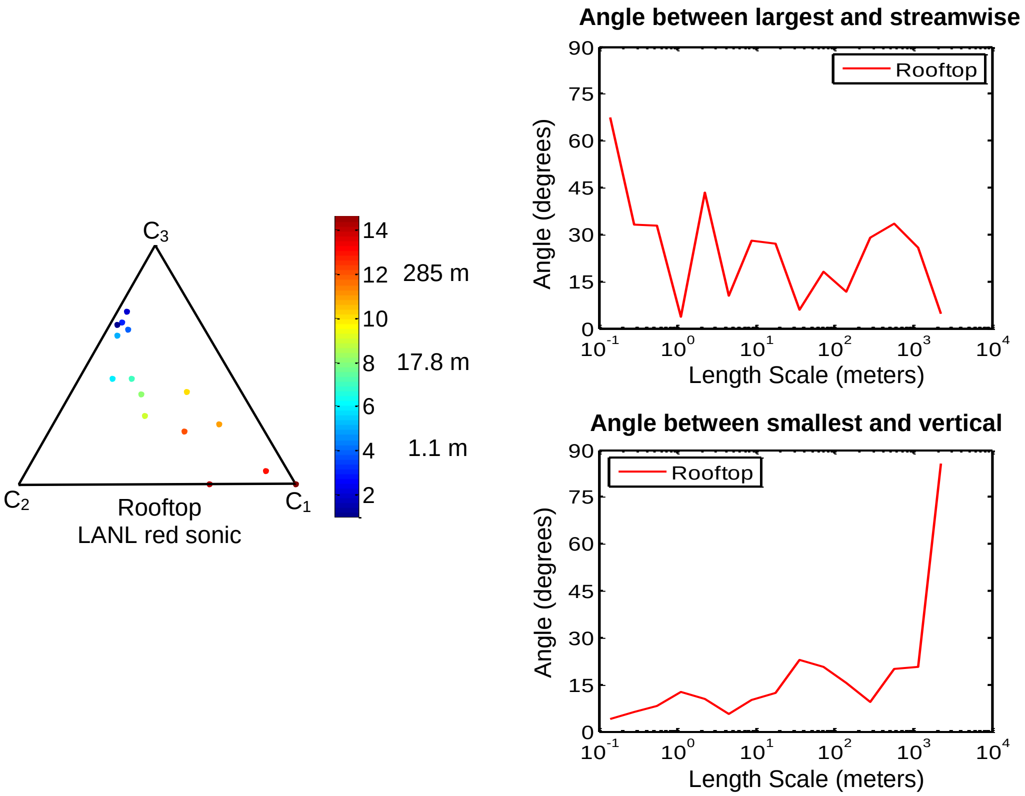

Figure 7: Same as Figure 6 but for a sonic located on the roof of a building at the corner of Park Avenue and North Broadway Avenue. It is 3.7 meters above the roof and 47.7 meters above street level. This location is very complex. The roof levels of nearby buildings vary greatly and there is a utility room next to the tower. Note that the smallest scales are not as close to isotropic as the other locations. This was true for all the hours of day 190 (9 July). See Allwine and Flaherty 2006 for details of placement.