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1. INTRODUCTION

An understanding of how boundary layer flows interact with topography and surface cover is important in a range of applications. Specific examples include the interpretation of eddy covariance observations in 'non-ideal' (i.e. most) locations, the prediction of site specific and near-surface meteorology using numerical weather prediction (NWP) models, the parameterisation of sub-grid scale topographic effects and surface exchange processes within NWP and air quality modelling. Simple descriptions of these flows, which are nevertheless, comprehensive, reasonably accurate, robust and have well understood error characteristics would be of enormous value in these applications (because of time and resource constraints) and so have motivated a quest for analytic solutions.

Analysis of the turbulent flow over gentle topography is well established. It was initially based on analytical approaches such as linearised perturbation theory (Hunt et al. 1988), for the flow over rough hills, but increasingly resorted to numerical modelling. The impact of gentle topography on the flow through and transport from a deep but uniform canopy has been successfully incorporated into both the analytical (Finnigan and Belcher 2004; Poggi et al. 2008) and numerical approaches (e.g. Ross and Vosper 2005; Patton and Katul 2009). The resulting insights have proved qualitatively robust when compared to wind tunnel (Section 4), flume (Poggi and Katul 2007) and some field studies (e.g. Zeri et al. 2010). There remain, however, notable differences between the magnitude and phasing of some flow features predicted by the analytical models, the numerical approaches and the observations especially in the case of 'narrow' topography (e.g. Ross and Vosper 2005).

The impacts of topography with horizontal length scales which are long relative to hill height and canopy length scales (see Section 3) can often be satisfactorily treated using existing approaches and adequate resolution or are

found to play a secondary role relative to other variations in the meteorological forcing or surface conditions. The effects of topography with short length scales, however, cannot be modelled accurately (without great computational expense) yet it is these small scale features that are more likely to be neglected when analysing any particular problem or site. We seek, therefore, to advance understanding of how the neutrally stratified boundary layer flow over a homogenous canopy responds to gentle topography, with the specific aim of identifying the causes of the theoretical-observational differences for 'narrow' topography.

2. MATERIALS

We use data from two sources to aid the analysis. First are time-averages of the flow obtained from the Large Eddy Simulations (LES) of Patton and Katul (2009). These two simulations considered the role of 'sparse' and 'dense' canopies in determining the flow response to sinusoidal terrain. They seek to replicate in detail the flume experiments of Poggi and Katul (2007). Second are the neutrally stratified flow data from the wind tunnel experiment of Finnigan and Hughes (2008). The model canopy used was identical to the 'Tombstone' canopy of Raupach et al. (1986), which provided a reference (flat surface) flow whose turbulence characteristics are well understood.

Theoretical predictions have been obtained from a numerical implementation of the analysis of Finnigan and Belcher (2004) (henceforth FB04), and the extension of this theory to generalised gentle 2-dimensional topography (Harman and Finnigan 2010). Further extensions to this theory are discussed next. For consistency, the wind tunnel and synthetic data have been rotated into the same curvilinear (potential flow) co-ordinate system as used in the theory. For the gentle topography considered, the flow obtained is then very close to that which would be obtained by rotation into a streamline or planar fit co-ordinate system for most locations across the hill.

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3. ANALYSIS AND RESULTS

The response of a neutrally stratified turbulent boundary layer to two dimensional topography covered by a plant canopy is effectively governed by one velocity scale, the friction velocity u_* , and four length scales. These are the hill height, H , the hill length, L , the canopy height, h_c , and the canopy adjustment length, L_c , which measures the ability of the canopy to exert drag and determines the aerodynamic roughness of the canopy. The analysis of FB04 applies to cases where $H/L \ll 1$, $L_c/L \ll 1$ and h_c/L_c is large enough that a large majority of the aerodynamic drag is absorbed by the canopy but small enough that the vertical motion induced by the within-canopy response to the topography remains small. In practice these last two conditions are somewhat restrictive (Ross and Vosper 2005).

The three data sets we consider satisfy the gentle topography criterion, $H/L \ll 1$. However, the Tombstone experiment and sparse canopy LES have $L_c/L \approx 1$, implying that the hill is 'narrow', while the dense canopy LES has $h_c/L_c \approx 1$, a case where we would expect significant vertical velocities to be observed. The three cases therefore provide realistic tests to ascertain how far outside its regime of applicability the analysis of FB04 can be taken.

Figures 1a) and 2a-b) show a comparison between the predictions from FB04 and the two sets of data for a 'sparse' canopy. The analytical theory provides predictions of the perturbations from a nominal reference wind speed profile. For the purposes of comparison the predicted perturbations have been added onto the reference (unperturbed) wind speed profile as would be generated over level terrain covered by the same canopy. Necessary parameter values, to determine the reference profile, are obtained from the upstream data (the wind tunnel experiments) or from the streamwise averaged flow (the LES). It is clear from the figures that the analytic model overestimates the magnitude of the hill-induced perturbations to the mean wind speed, especially so within the canopy. When comparing to the LES simulation (Fig. 2) this over estimate leads to the prediction of a region of reversed mean flow not present in the simulations or the associated flume study (Poggi and Katul 2007). Such regions of reversed flow often coincide with significant advective transport terms in the mass balance of passive scalars, such as CO_2 (Katul et al. 2006) and hence affect ecosystem exchange estimates significantly.

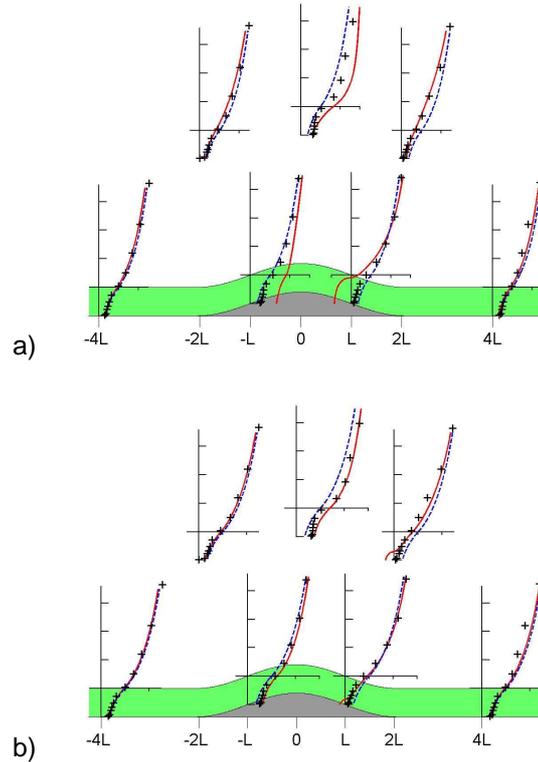


Figure 1. Comparison between observed (crosses) and predicted (solid red) wind speed profiles for the Tombstone canopy surface. a) Predictions using FB04; b) with revisions as described in the text. Blue-dashed line gives the (upstream) reference profile.

3.1 Pressure perturbation scaling

FB04 decomposes the perturbations to the flow over topography into a series of thin layers (a consequence of the gentle hill approximation) before formally matching the layers. Each layer is characterised by an approximation to the Reynolds-averaged Navier Stokes equations, in curvilinear co-ordinates, as appropriate to each layer.

Driving the velocity and stress perturbations is the hydrodynamic pressure perturbation, Δp , generated by the hill. Δp is determined by the (inviscid) flow in the outermost layer – the outer region. Due to the gentle hill condition the pressure perturbation is approximately constant with height and provides the external forcing for the lower layers. For sinusoidal terrain of height H and hill length scale L the pressure perturbation is

$$\Delta p(x) = -\frac{H\pi}{4L} U_m^2 \cos\left(\frac{x\pi}{2L}\right) \quad (1)$$

with $x = 0$ the location of the hill crest and U_m the wind speed at the *middle-layer height*, h_m , which is itself dependent solely on the canopy roughness and L (Hunt et al. 1988).

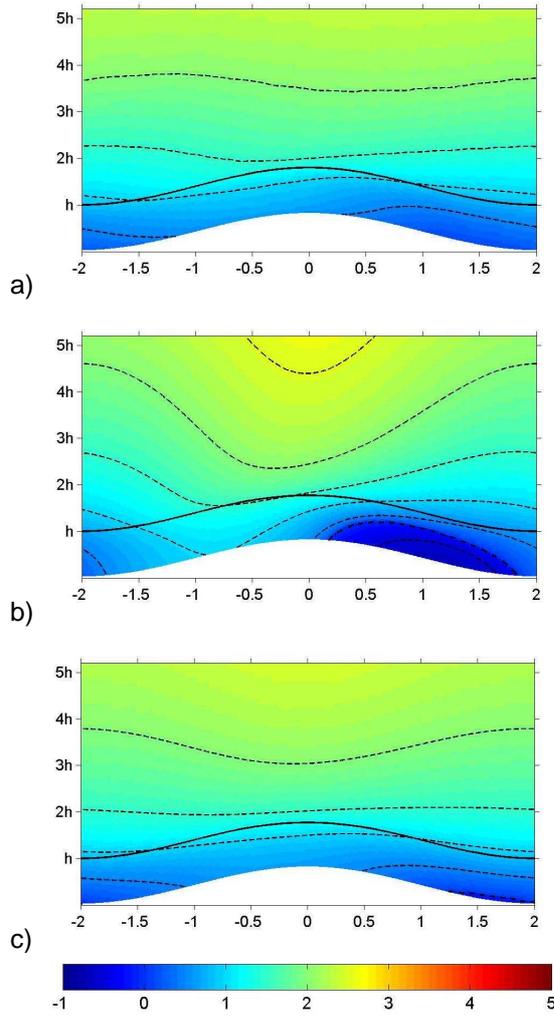


Figure 2. Comparison between the LES (a), predictions from FB04 (b), and with the revisions as described in the text (c) for the sparse canopy simulation. Colours and contours show $U(z)/U_h$, where U_h is the reference wind speed at canopy top. The thicker dash-dotted contour marks lines of $U(z)=0$. Horizontal ticks mark every $x = 0.5L$.

A more complete analysis of the flow response would include the feedback between the induced flow and the pressure perturbation through the different thin layers, and would allow for any vertical variation in the pressure perturbation. By incorporating the interaction between the flow responses, specifically the vertical velocity induced by the hill, in the different layers Belcher and Wood (1996) show that (to the next level of approximation) the pressure perturbation is given by

$$\Delta p(x) = -\frac{H\pi}{4L} U_m^2 \cos\left(\frac{x\pi}{2L}\right) \left(1 + \frac{h}{2L} \frac{\pi}{m}\right)^{-1} \quad (2)$$

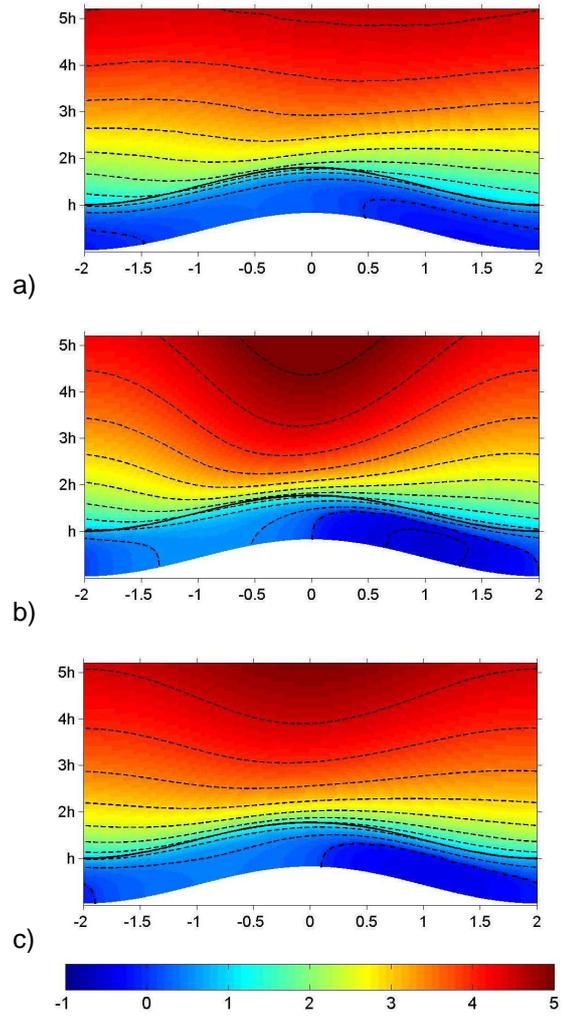


Figure 3. As Figure 2 but for the dense canopy simulation. Horizontal ticks mark every $x = 0.5L$.

Eq. (2), like Eq. (1), produces a pressure perturbation in phase with the hill. Smaller additional terms can also be included which provide a small out-of-phase component to $\Delta p(x)$. Eq. (2) for Δp represents a noticeable decrease in the magnitude of the pressure perturbation for the cases considered here. The inclusion of a vertically varying pressure perturbation, however, leads to an analytically intractable problem (a system of 2 dimensional nonlinear partial differential equations).

3.2 Advection within the canopy

The analysis of FB04 makes a series of approximation to the equations of motion in the different layers considered for the parameter regime of applicability. Within the canopy this results in the shear stress divergence, pressure gradient and canopy drag terms being retained

but the inertial (or advection) terms being neglected. However, for cases where L_c/L or h_c/L_c are not small the advection terms are not necessarily negligible. Poggi et al. (2008) address this issue by retaining the advection terms at the expense of the shear stress divergence term. However this is an equally questionable approximation. It is, however, possible to obtain analytical expressions which maintain all four components, through the incorporation of the continuity equation. The inclusion of the advection terms leads to flow perturbations within the canopy which are smaller in magnitude and shifted downstream compared to previous predictions.

Figures 1b) and 2c) show the revised predictions when both the modified pressure perturbation (2) and the impact of the advection terms are included. For these sparse canopies, the agreement between the predictions and observations/synthetic data is much improved. Some phasing issues remain, especially above the canopy. Analysis of the relative impacts indicates that the majority of the improvement above the canopy results from the changed pressure perturbation whereas within the canopy the inclusion of the advection terms is the more important of the two extensions.

3.3 Deep canopies and streamline curvature

The extensions to the FB04 analysis proposed herein have enabled a reasonable agreement between observations and predictions to be reached in the case of a sparse canopy on gentle terrain. However, Figure 3 shows less satisfactory agreement in the case of a deep, dense canopy. Specific areas of notable disagreement include the magnitude, extent and positioning of the reverse flow within the canopy, the general over prediction and incorrect depth scale of the above-canopy wind speed perturbations and, more subtly, the disagreement in the vertical tilt to the flow perturbations within the canopy. The majority of these discrepancies can, again, be traced to errors in the prescribed pressure perturbation and induced vertical velocities (Ross and Vosper 2005). Attempts to use the extended analysis of Belcher and Wood (1996) to correct this difference, however, result in an analytical system which is unphysically sensitive to the exact parameter values used. Furthermore, this attempt does not correct the depth scale or tilt issues in the predicted perturbation.

There is, however, another potential element missing from the analysis to date. The analyses employ a curvilinear co-ordinate system, with the mean flow vector rotated as appropriate.

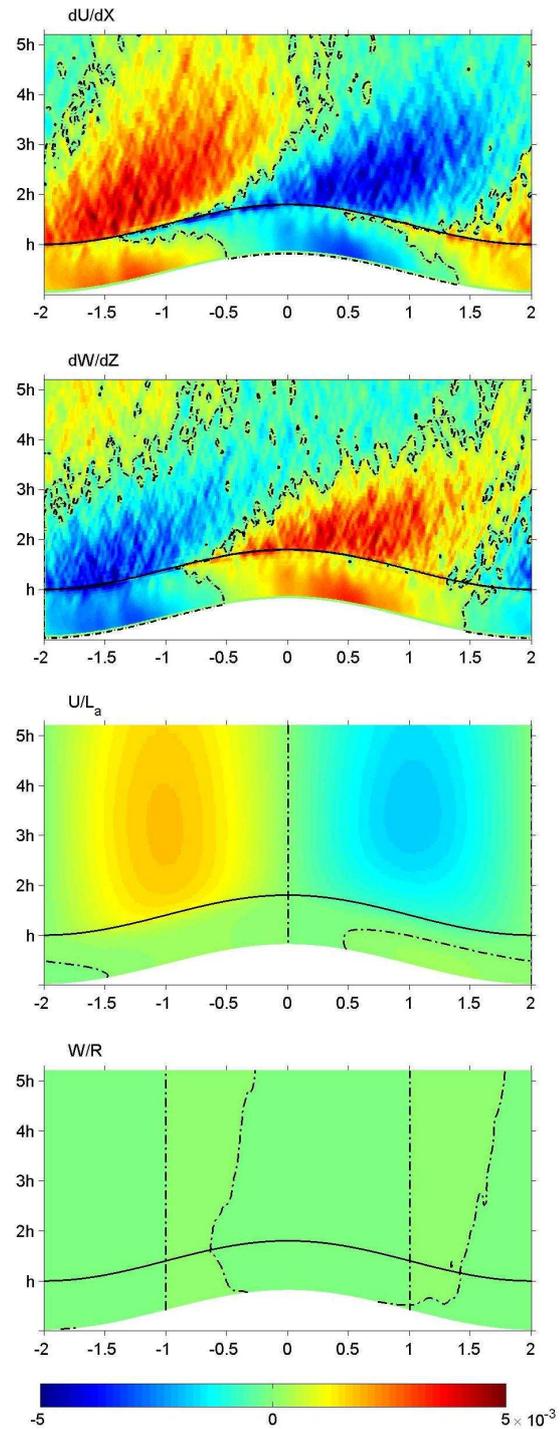


Figure 4. The four terms of the continuity equation in the curvilinear co-ordinate system (Eq 3) from the LES (dense canopy case). Dash-dotted line gives the zero contour.

In such co-ordinate systems, the continuity equation and equations of motions have to be rewritten to include the effects of streamline curvature. For the long length scale topography envisaged in the original FB04 analysis the additional terms that result can be neglected on

grounds of magnitude. However for the short length scale topography of interest here this may not be the case. In a 2-dimensional streamline co-ordinate system, (Finnigan, 1983) the continuity equation is given by

$$\frac{\partial U}{\partial X} - \frac{U}{L_a} + \frac{\partial W}{\partial Z} - \frac{W}{R} = 0 \quad (3)$$

where L_a and R are the local radii of curvature of the X and Z (curvilinear) co-ordinate lines respectively. Figure 4 shows how these four terms vary in the dense canopy LES. For the majority of locations the usual balance between the $\partial U/\partial X$ and $\partial W/\partial Z$ remains, however the U/L_a term attains values between 15-25% of these gradient terms at some locations. This suggests that the 'co-ordinate rotation' terms cannot justifiably be neglected for short length scale topography. However if this is the case then the underlying linearised perturbation theory, upon which the predictions are made, needs to be revisited because including these terms would fundamentally alter the analytical form of the predicted perturbations to the wind field both within and above the canopy.

4. CONCLUSIONS

There are a range of applications that require simple, yet comprehensive and accurate knowledge of the flow and transport characteristics of how boundary layer flow interacts with topography and surface cover. The extended linearised perturbation theory (Finnigan and Belcher 2004) has been fundamental in advancing our understanding of how such interactions vary qualitatively between rough and canopy-covered terrain and with canopy density. However the quantitative agreement between the predictions and observations or high resolution numerical simulations, which is required for confidence in the application of the theory within other studies, is less than satisfactory. Further simple extensions to the theory, presented here, have substantially improved the agreement, especially in the case of sparse canopies on hills of short length scale. However, there clearly remain difficulties when applying the theory to deep, dense canopies, when the vertical motion induced by the terrain and the canopy is dynamically significant. Further progress is likely to require a novel combination of LES and Reynolds-Averaged Navier Stokes model simulations, successful parameterisation of those results and analytical advances as well as wind tunnel and field observations.

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