16A.3 CLASSIFYING AND ANALYZING TORNADO-LIKE VORTICES FAR FROM AXISYMMETRY

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1 INTRODUCTION

Much of our understanding of tornado structure and dynamics is based on analysis of vortices that are near axisymmetry. Theoretically it reduces a three-dimensional problem to a two-dimensional one (or even one-dimension since cylindrical symmetry often prevails). In field or laboratory observations it allows wind-fields to be inferred from a much more limited set of velocity measurements than would be required far from axisymmetry. Moreover the axisymmetric limit is still sufficiently rich to illustrate and study much of the complex dynamics of vortices that is seen in general. Nonetheless, better understanding and analysis methods for vortices far from axisymmetry would be extremely useful. Vortices in the field can deviate significantly from the axisymmetric limit, complicating their categorization even when the dynamics is basically that of the axisymmetric case. More fundamentally, while all of the important dynamics found in the axisymmetric limit is also important in the general case the converse is not true: important new dynamics can arise because of the asymmetry that has no counterpart in the symmetric case.

As part of a longstanding effort to better understand the near-surface behavior of tornadoes we have conducted numerous large-eddy simulations (LES) with both realistic and idealized boundary conditions, often generating examples exhibiting asymmetric behavior in different guises. In this work we consider some aspects of the classification and analysis of tornado-like vortices away from the axisymmetric limit, gleaned from study of some of these LES sets. In the simulations the asymmetric behavior can arise from asymmetries imposed through the lateral boundary conditions, asymmetric forcing within the domain (e.g., surface friction acting on a translating vortex), or through vortex formation driven by internal convergence or shear instability. The cases can be inherently time varying or quasi-steady. Here we emphasize an important subclass: the embedding of a concentrated vortex within a larger-scale vortex circulation for different configurations. This category is of relevance for analyzing both tornadoes embedded within mesocyclones and secondary vortices embedded within tornadoes. The high-resolution LES model and simulation procedures are ones we have used extensively in previous work (e.g., Lewellen et al. (2008) and references therein). The code allows for the inclusion of multiple debris species interacting with the airflow but, for simplicity, debris is not considered here.

The general problem of analyzing asymmetric vortices is much more complex than its axisymmetric counterpart and remains far from solution. Nonetheless some general features can be determined allowing different cases to be categorized, and modest quantitative progress can be made with different techniques in some limits. We will touch upon some examples here with a detailed treatment to appear elsewhere.

2 ASYMMETRIC VORTEX EXAMPLES

Figs. 1-6 provide a sampling of the variety of asymmetric vortices we have encountered within simulations, visualized through nested iso-surfaces of hydrodynamic pressure (i.e., that part of the pressure arising from fluid motions, the hydrostatic part not included). All are in the class of a concentrated smaller-scale vortex (or vortices) embedded within a larger-scale vortex. Asymmetric vortices outside of this class that we have simulated but are not illustrating here include quasi-steady vortices forced by inflow far from axisymmetry and vortices with large imposed translation velocities over rough surfaces. The concentrated vortices of figs. 1,3,4 and 5 are approximately quasi-steady, though with the exception of fig. 5 they are not stationary with respect to the large-scale vortex. The case of fig. 2 is quasi-steady on the large-scale but the concentrated vortices are cyclically time dependent: forming, strengthening and weakening within one rotation period of the large-scale vortex. The example in fig. 6 is from an asymmetric corner-flow collapse case, and is constantly evolving.

As seen in the figures and described in the captions, a large variety of results are encountered for the

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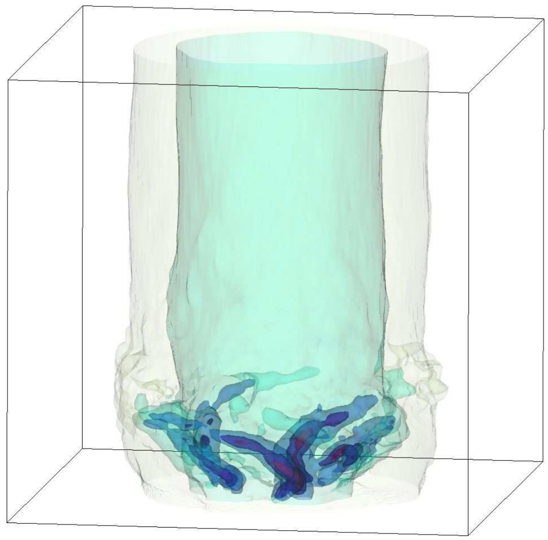

Figure 1: Secondary vortices within a high-swirl cornerflow. The secondaries wind upward anticyclonically (clockwise) and rotate about the central axis cyclonically.

vortex axis shape (including both left and right handed winding), axis motion (fully evolving, stationary or rotating with or opposed to the large-scale rotation), and embedding location (e.g., near-central within the large-scale vortex or well off axis). In the quasi-steady cases these qualitative features are governed largely by the basic cornerflow structure of the larger-scale vortex and the nature of the vorticity source for the concentrated vortex, as will be discussed below.

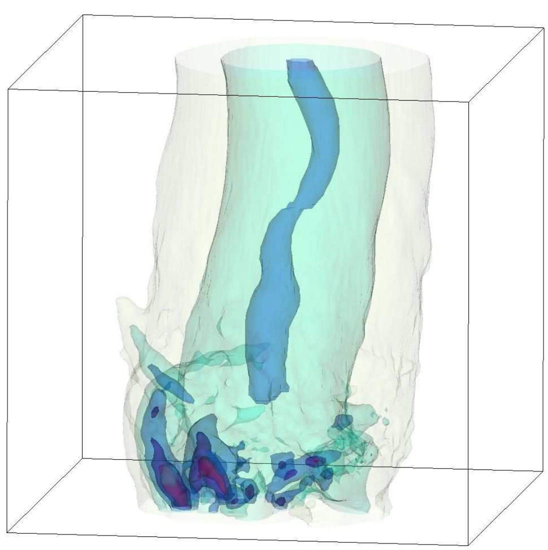

Figure 2: As in fig 1 but with a large translation velocity of the primary vortex to the right.

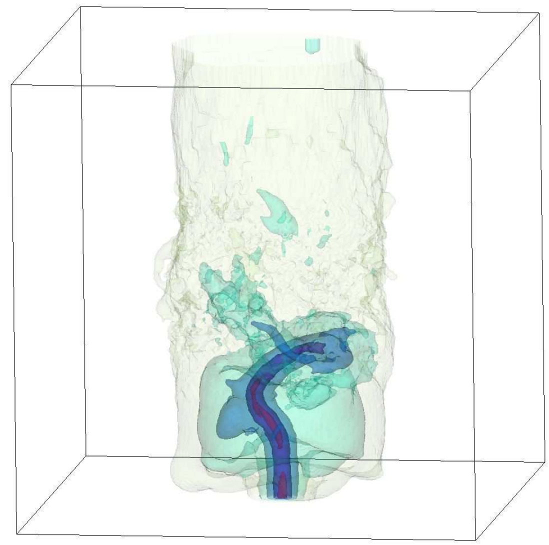

Figure 3: A concentrated small-scale vortex within a large-scale low-swirl cornerflow. The latter is driven by a high-angular-momentum inflow aloft with a low-angular-momentum inflow layer beneath, the former by addition of a thin layer of high angular momentum inflow just above the surface. The concentrated vortex winds upward anticyclonically and rotates about the central axis cyclonically.

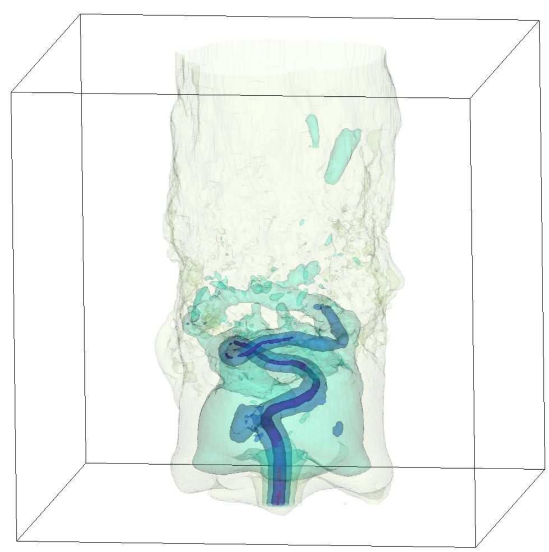

Figure 4: As in fig 3 but with the added thin high angular momentum inflow just above the surface of opposite sign. The concentrated vortex winds upward cyclonically, rotates about the central axis anticyclonically and about its own axis anticyclonically.

3 COMPLICATIONS OF ASYMMETRY

Vortices that are axisymmetric in the mean already exhibit a rich variety of structure complicating

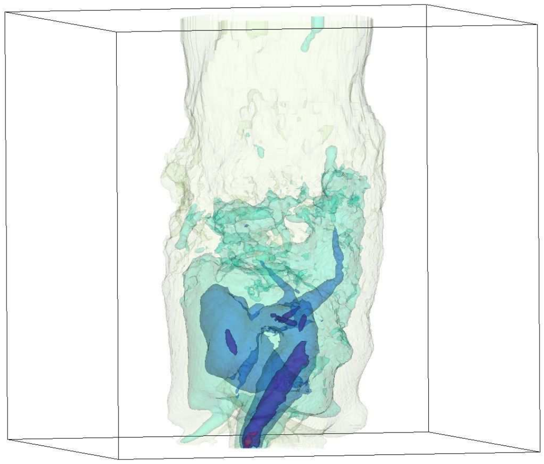

Figure 5: As in figs 3,4 but with the concentrated small-scale vortex driven by translation of the large-scale vortex over a rough surface to the right. The concentrated vortex is approximately stationary, located well off the central axis of the large-scale vortex and rotates cyclonically about its own axis. A weaker anticyclonic quasi-stationary small-scale vortex is located to the rear left of the figure.

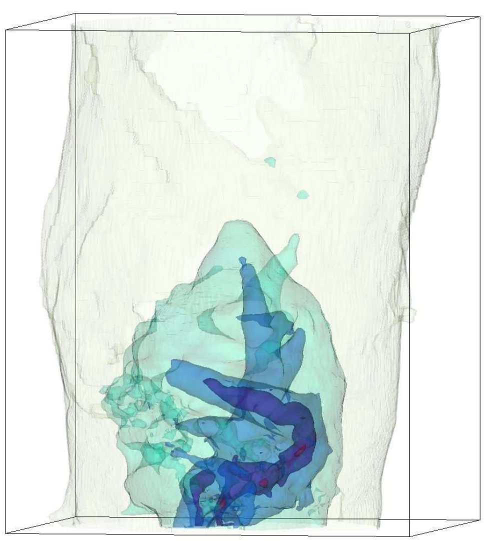

Figure 6: Snapshot from a time evolving asymmetric cornerflow collapse triggered by shutting off the low-swirl inflow into one quadrant of a translating large-scale low-swirl vortex. The large-scale vortex translation is to the right. The concentrated vortex evolves dramatically and is shown sometime after the period of peak intensification.

their analysis (e.g., as found with changes in swirl ratio, sensitivity to low-level inflow, transient effects and effects of turbulence); however, the budgets of mass continuity, axial momentum and angular momentum are all significantly simplified by the symmetry. Moreover, the mean flow is organized by the symmetry: all of the near-surface flow finds its way in to the vortex core, with the progressively higher layers of the inflow located radially outward in the core. Asymmetric vortices share all of the complications of the axisymmetric cases plus many in addition: tilted, twisted, and/or translating axes; no simple center line for computing angular momentum budgets and correspondingly significant torques from pressure gradients; core inflow from only a subset of the surface and thus no simple ordering of the origins of the mean core flow; and the self-induced motion of curved vortex lines.

To date we have made only modest progress in quantitatively studying asymmetric vortices using different approaches in different limits. For example, in the limit of a very high-swirl vortex with a thin surface inflow layer, the motion of different surface-layer parcels can be approximately mapped into a central force problem, allowing one to solve approximately for the near-surface trajectories and thereby understand some of the asymmetric behavior in that limit. There are also some additional constructs that one can make use of in the asymmetric case. For example in the symmetric case the linear momentum flux into the vortex is necessarily zero by symmetry (unlike the mass or angular momentum flux); however, in the presence of vortex translation and/or other sources of near-surface asymmetry, the total linear momentum flux into the vortex will be non-zero. Within an idealized model of a tilted vortex at the surface we have derived an analytic expression relating that linear momentum flux to the vortex tilt and strength, which is in at least qualitative agreement with simulation results.

4 CATEGORIZING TWO-SCALE VORTEX EMBEDDING

We have cataloged basic possibilities for how a concentrated small-scale vortex can be embedded within a larger-scale vortex circulation and determined some of the basic qualitative behavior that arises, in particular the direction of winding of the smaller-scale vortex within the larger, whether that structure can be quasi-steady or, if not, its direction of rotation. For simplicity we restrict attention here to quasi-steady boundary conditions that, for the large-scale vortex, are approximately axisymmetric. The key determining factors are: the corner-flow swirl ratio of the large-scale vortex; the location of the small vortex within the larger one (whether centrally located at the surface or well off axis); and whether the small-scale circulation is cyclonic or anticyclonic (taking the large-scale circulation as always cyclonic). Figs. 1,3,4 and 5 summarize the basic behaviors we have encountered in these scenarios. Some of the observations below are well known in a specific instance – that of secondary vortices within a high-swirl corner flow – but are expanded to more general cases here.

The two basic ingredients for the concentrated vortex are (as always) convergence plus vorticity. The most intense, coherent vortices arise when one or both ingredients are provided or augmented by the larger-scale vortex itself. This largely determines the large-scale cornerflows of most interest and the possible locations for strong concentrated vortices within them. In a high-swirl cornerflow (fig. 7) the inner region of the updraft annulus near the surface provides significant shear in both the vertical and azimuthal velocity components as well as strong convergence. Strong secondary vortices can arise here without any further forcing. No other locations are conducive, on either count, to concentrated vortex formation or maintenance. On the other hand, given a low cornerflow swirl ratio together with a high swirl ratio for the vortex overall (i.e., low-swirl underneath high swirl) as in fig. 8 there are two distinct locations for possible concentrated near-surface vortices, but they each require additional forcing. The strongest convergence, which is driven by the large-scale low-swirl cornerflow dynamics, is located centrally, but significant shear is not encountered along that central streamline, at least until the flow has risen above into the large-scale vortex breakdown. There is weaker but still significant low-level convergence off the central axis; again there is no significant low-level shear but shear is available further aloft at the edge of the strong central updraft. Strong concentrated vortices can be found in either location if a suitable low level-level vorticity source is present (c.f. figs. 3,4,5).

In all cases the axes of the concentrated smaller-scale vortices tend to align with the vortex lines in the azimuthally averaged mean flow, and this determines the winding direction. In fig. 7 the radial gradients of swirl and vertical velocity are both positive at the location of the secondary vortices, leading to the anti-cyclonic winding in fig. 1. In fig. 8 at location "B" the radial gradient of swirl velocity is positive but of vertical velocity is negative leading to the cyclonic slant in fig. 5. In the vortex breakdown region in fig. 3 the gradients are as in fig. 1 leading again to anticyclonic winding, while in fig. 4 at the location of the concentrated vortex aloft the swirl gradient is reversed, leading to the cyclonic winding.

In each case strong embedded vortices tend to be shape preserving – either stationary if at all possible or rotating about the large-scale vortex center at a uniform angular velocity. This is nontrivial given that the mean flow in the relevant regions is not close to being in solid body rotation and that even isolated curved line vortices induce motions that alter the axis shape except for a few special geometries. The apparent motion of the axis is governed by its shape together with the advection velocity of core parcels. Only that velocity component normal to the axis contributes, i.e., velocity aligned with the axis everywhere leaves the structure stationary. This is the preferred configuration, e.g., if the winding of the mean vortex lines and streamlines is in the same sense then the concentrated vortex shape and location will tend to adjust so that its axis remains approximately stationary (e.g., as in fig. 5). On the other hand if the mean vortex lines and streamlines wind oppositely then stationarity is precluded; adjustments lead to a shape preserving rotation as the most

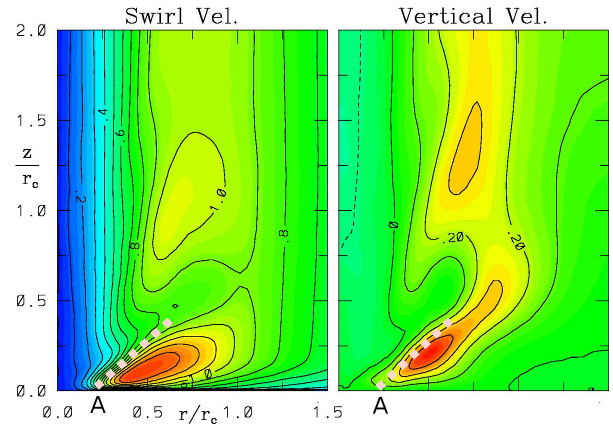

Figure 7: Mean (azimuthally averaged) contours of swirl velocity and vertical velocity in the radial-vertical plane for a high-swirl cornerflow. The region favorable for near-surface concentrated smaller-scale vortices is noted by the dashed line starting at "A".

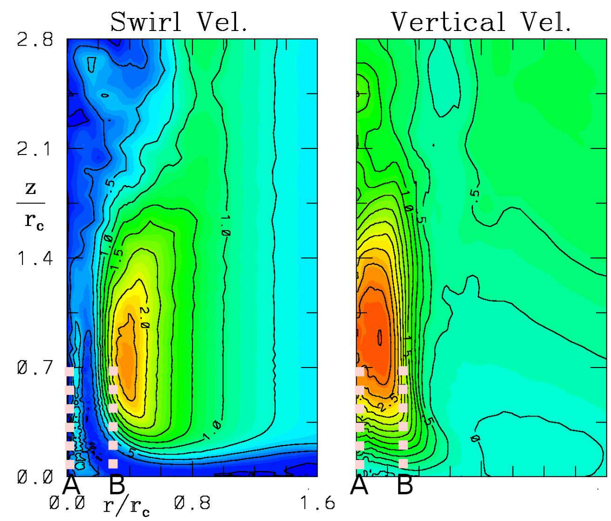

Figure 8: As in fig 7 but for a low-swirl cornerflow.

favored alternative.

Note that in predicting the occurrence and behavior of smaller-scale vortices near the surface it is not sufficient to consider the interaction of ingredients locally. For example in the case of fig. 5 the large-scale low-swirl cornerflow provides a symmetric convergence at low levels. Acting purely locally on the vorticity sheet at the surface provided by the translation this would produce a symmetric pair of counter-rotating smaller-scale vortices; however, the interaction with the large-scale swirl flow well aloft ultimately strongly breaks that symmetry.

5 NEAR-SURFACE INTENSIFICATION

A key issue for tornado-like vortices in general is the level of near-surface intensification present and the physical mechanisms producing it. In concentrated vortices there is a physical feedback that naturally favors cylindrical symmetry: pressure gradients along the central axis associated with variations in vortex strength tend to drive axial velocities that oppose those strength variations. Near the surface, however, intensification of the vortex relative to conditions aloft is possible even for quasi-steady approximately axisymmetric vortices due to dynamics in the cornerflow (c.f. figs 7,8). Enhancement of radial inflow due to cyclostrophic imbalance in the surface layer leads to enhanced axial and swirl velocities in the cornerflow; the mean axial pressure gradient arising can then be balanced by the deceleration of the vertical flow. Generally this means of achieving the axial momentum balance (together with continuity) is not consistent with Bernoulli's equation,

$$p + \frac{1}{2}\rho v^2 = C_{\text{streamline}} \quad , \tag{1}$$

i.e. significant turbulent dissipation must be present, produced in the vortex by forcing changes over relatively short axial distances such as in a vortex breakdown (see e.g., Lewellen and Lewellen (2007)). This dynamic can be responsible for intensification in asymmetric vortices as well, either directly as in the symmetric case or indirectly as in fig. 5. In that case the intensification change within the the smaller concentrated vortex is enabled through its positioning within the vortex breakdown in the cornerflow of the larger-scale vortex which contains it (which also ties the intensity levels of the two scales together).

In some cases however, such as in figs 1,3,4 the intensification within the concentrated asymmetric vortex is of a type that does not have a counterpart for axisymmetric-in-the-mean vortices. In these cases a much more gradual pressure change along the axis is found, over axial distances long in comparison to the core radius. Both the secondary vortices in fig 1 and the spiral vortices in figs 3,4 weaken gradually as they extend aloft. How is this quasi-steady behavior consistent with the balance between axial momentum and pressure gradients in this case as well as with Bernoulli's equation (since turbulent dissipation is of at most secondary importance in this gradual change)? Put more generally, what is the nature of the "termination" of these vortices aloft?

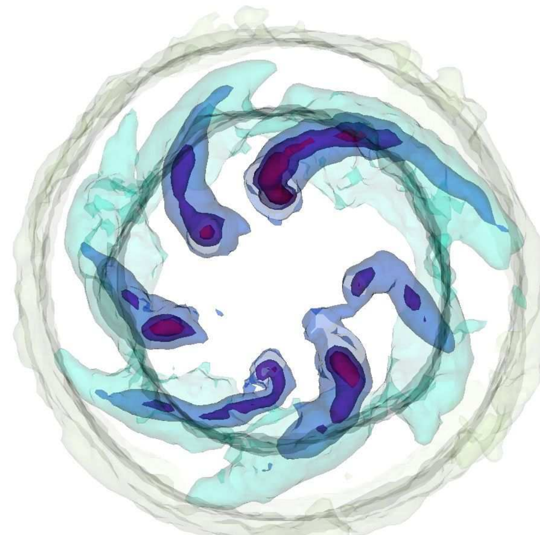

Figure 9: A high-swirl cornerflow, as in fig 1, viewed from directly above.

The answer is connected to two features not present in the axisymmetric case: the movement of the concentrated vortex within the larger-scale vortex and the increase in radial position of the former as it rises aloft within the latter (e.g., fig 9). The movement precludes the use of the simple Bernoulli equation as in (1), which is only applicable to steady flow. However, the stronger concentrated vortices in these cases are approximately steady in an appropriately rotating reference frame (though there are turbulent fluctuations and the structures do not live indefinitely). There is a version of the Bernoulli equation applicable to a steady flow within a rotating frame,*

$$p - \frac{1}{2}\Omega^2 R^2 + \frac{1}{2}\rho v^2 = C_{\text{streamline}} \quad , \tag{2}$$

where Ω is the angular velocity of rotation of the rotating frame and R is the radial position from the axis of rotation of the frame (i.e., the axis of the large-scale vortex for the cases considered here). Thus in the rotating frame pressure gradients along the axis of

*Note that the Coriolis "force", being perpendicular to the velocity, does not contribute to the Bernoulli equation.

the concentrated vortex can be balanced not only by changes in axial velocity (as in the axisymmetric case) but also by changes in R. Applying (2) has provided estimates on the pressure drop intensification within smaller-scale embedded vortices in different instances that is in general accord with that found in the simulations. This is of potential relevance in understanding the limits in some regimes to the strength of a tornado embedded within a mesocyclone or a secondary vortex embedded within a tornado.

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