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## 1. INTRODUCTION

Since the work of Fujiwhara (1921), it has long been recognized that the paired cyclonic vortices in a quiescent environment rotate around each other resulting in the convergence or divergence in the final stage (e.g., Chang 1983; DeMaria and Chan 1984; Pokhil et al. 1990; Holland and Dietachmayer 1993; Pokhil and Polyakova 1994; Chan and Law 1995; Hart and Evans 1999).

In this study, we show by numerical experiments that separation distance between a paired vortices of typhoon scale oscillates with time in the course of mutual rotation due to Fujiwhara effect, rather than exhibiting a monotonic increase or decrease. The oscillation of separation distance is found for a wide range of model parameters that characterize the binary vortices of typhoon scale.

We will try to explain the oscillation in terms of the eddy-mean interaction, i.e., the interaction between the axisymmetric part and the perturbation in a plane polar coordinate system whose origin is the center of binary vortices.

## 2. EXPERIMENTAL DESIGN

## 2.1 Vorticity Equation and Boundary Condition

Nondivergent barotropic vorticity equation on a  $\beta$ -plane scaled by the radius  $a$  and the inverse of rotation rate  $\Omega^{-1}$  of the earth is written

$$\frac{\partial \zeta}{\partial t} = -\frac{\partial}{\partial x}(u\eta) - \frac{\partial}{\partial y}(v\eta) + \nu \nabla^2 \zeta, \quad (1)$$

where  $\nu$  is the dissipation rate. Definitions of the

variables are

$$\begin{aligned} \eta &= \zeta + f_0 + \beta(y - L_y/2) \\ f_0 &= 2\sin\theta_0, \quad \beta = 2\cos\theta_0 \\ \zeta &= \nabla^2 \psi, \quad \nabla^2 = \left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right\} \\ u &= -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}, \end{aligned}$$

where  $\theta_0$  is the reference latitude. The model domain  $L_x \times L_y$  is given ( $6000 \text{ km}/a \times 6000 \text{ km}/a$ ) with  $a=6371 \text{ km}$ . The boundary condition for  $x$ -direction is periodic while that for the  $y$ -direction is no normal-flow.

## 2.2 Spectral Representation

The orthogonal basis functions that are appropriate for the boundary conditions specified in Eq. (1) are introduced.

$$Q_{m,n} = \sin \frac{\pi ny}{L_y} \exp \frac{i2\pi mx}{L_x}, \quad i = \sqrt{-1}, \quad (2)$$

which are orthogonal so that

$$\begin{aligned} &\frac{1}{L_x L_y} \int_0^{L_y} \int_0^{L_x} Q_{m,n} Q_{m',n}^* dx dy \\ &= \begin{cases} \frac{1}{2} & (m, n) = (m', n') \neq (0, 0) \\ 0 & (m, n) = (m', n') \end{cases}, \end{aligned} \quad (3)$$

with the asterisk being the complex conjugate. We represent the variables in Eq. (1) using the basis functions with the truncation  $M=170$  and the transform grids of  $512 \times 512$ :

$$\zeta(x, y, t) = \sum_{m=-M}^M \sum_{n=0}^M \zeta_{m,n}(t) Q_{m,n}. \quad (4)$$

The spectral form of Eq. (1) is written as

$$\begin{aligned} \frac{d}{dt} \zeta_{m,n} &= -\frac{i2\pi m}{L_x} U_{m,n}^a \\ &- \frac{\pi}{L_y} \{ (n+1) V_{m,n-1}^a \\ &- (n-1) V_{m,n+1}^a \} + \nu d_{m,n}, \end{aligned} \quad (5)$$

where  $U_{m,n}^a$ ,  $V_{m,n}^a$  is the advection term,  $d_{m,n}$  comes from the Laplacian of the vorticity.

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### 2.3 Initial Condition

As an initial condition, a pair of vortices whose centers are aligned in zonal direction are given with the initial separation distance  $SD_0$ . The azimuthal velocity profile of each vortex is given by that of DeMaria and Chan (1984). The initial separation distance  $SD_0$  is varied from 470 km to 518 km with 4 km interval (Except for EXP IV, EXP V, and EXP IX).

### 2.4 Parameters of Numerical Experiments

The parameters of numerical experiments are listed in Table I (EXP I-X).

**Table I.** Parameters for numerical experiments. 'L' and 'R' denote the left and right vortex of the initial condition, respectively.  $V_m$  is the maximum speed,  $r_m$  is the radial distance at which the maximum speed  $V_m$  falls, and  $b$  is a parameter determining the decrease rate of the azimuthal wind with the radial distance.

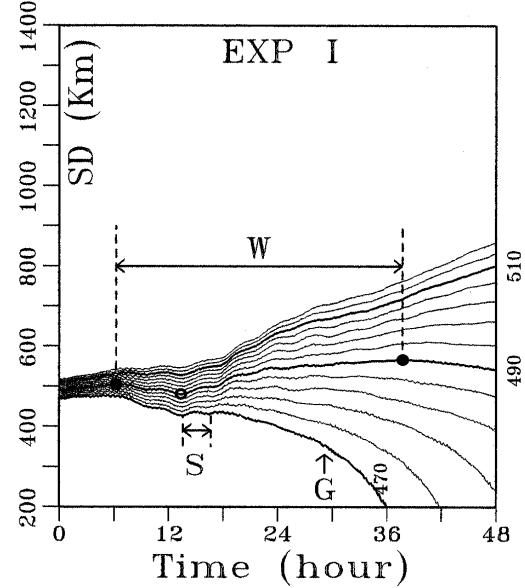
	$r_m$ (km)	$V_m$ (m/s) (L, R)	$b$	$\beta$
EXP I	100	(30,30)	0.75	None
EXP II	100	(40,30)	0.75	None
EXP III	100	(30,20)	0.75	None
EXP IV	100	(50,50)	0.75	None
EXP V	100	(50,50)	0.65	None
EXP VI	100	(30,30)	0.75	20°N
EXP VII	100	(40,30)	0.75	20°N
EXP VIII	100	(30,20)	0.75	20°N
EXP IX	100	(50,50)	0.75	20°N
EXP X	100	(30,30)	0.75	10°N

## 3. RESULTS

### 3.1 The Oscillation of Separation Distance

For all parameter range used here  $SD$  never exhibits a monotonic increase or decrease: an oscillation is embedded on the overall trend of monotonic increase or decrease even in the merging and repelling cases. Time variation of  $SD$

within 48 hours for EXP I are presented in Fig. 1.



**Figure 1.** Time variation of separation distance ( $SD$ ) for various initial separation distance ( $SD_0$ ).  $SD_0$  is varied from 470 to 518 km with 4 km interval. The solid (open) circle denotes the local maximum (minimum) associated with a long-period oscillation (denoted as 'W' for  $SD_0 = 490$  km). 'S' indicates the short-period oscillation, and 'G' points a noisy fluctuation of separation distance for  $SD_0 = 470$  km probably generated by the numerical method.

It is suggested for convenience that the interaction of vortices is categorized into three groups in terms of the time variation of  $SD$  during the initial three days: the merger (convergence), repulsion (divergence), and neutral case (maintaining a quasi-steady separation distance). The third case is where the vortices can not be classified into either the first case nor the second case at the time of 72 hour. This is somewhat subjective classification and the neutral case could be classified into either the merger or the repulsion in the final state. It is clear the amplitude of oscillation is largest for the neutral case, and the fluctuation for the divergence case is smaller than the merging case. For the neutral case in EXP V,

the greatest amplitude of the oscillation reach about 150 km, which corresponds to nearly half of the distance that a typhoon travels a day in average.

The oscillating feature varies in a systematic manner depending on  $SD_0$ . The period of oscillation in general increases with time and becomes longer for larger  $SD_0$ . The long-period of oscillation ( $W$ ) in  $SD$  is approximately one day or longer. An oscillating feature is not altered by the presence of the beta effect in the first couple of days, although the amplitude of oscillation is reduced to a certain extent.

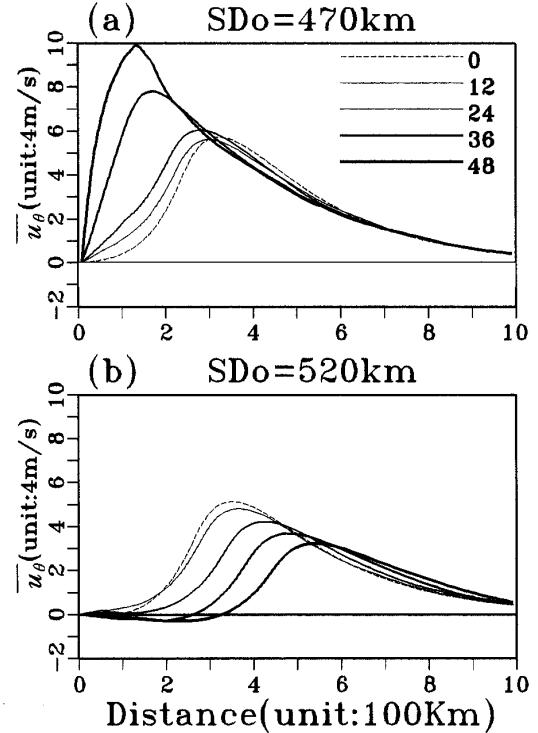
### 3.2 Interaction of Binary Vortices as the Eddy-Mean Interaction.

In this section we show that the oscillation of  $SD$  can be regarded as consequence of eddy-mean interaction(Möller and Montgomery 1999): The mean and eddy are defined as the azimuthal-mean and deviation from it, respectively, in a polar coordinate system of which the origin is taken as the midpoint of binary vortices.

Time variation of azimuthal-mean velocity ( $\overline{u_\theta}$ )  
 $\equiv \frac{1}{2\pi} \int_0^{2\pi} u_\theta d\theta$ ) for the cases  $SD_0=470$  and 520

km is shown in Fig. 2. Vortex merger (repulsion) is reflected on the intensification (weakening) of azimuthal-mean velocity: For the merged case ( $SD_0=470$  km) the azimuthal-mean velocity exhibits a rather sharp increase beyond day 1. For  $SD_0=520$  km the azimuthal-mean velocity near the center shows a negative sign inward of about 300 km from 48 h.

Time variation of  $\overline{u_\theta}$  is determined by the eddy momentum flux  $\overline{u_\theta' u_r'}$ , whose sign is related with the shape(phase tilt) of the eddy streamfunction. Shown in Fig. 3 are the eddy streamfunction for three cases  $SD_0=470$ , 490 and 520 km of EXP I, where the zero contour with thick line corresponds to the phase line. Differences in phase-line configuration near the



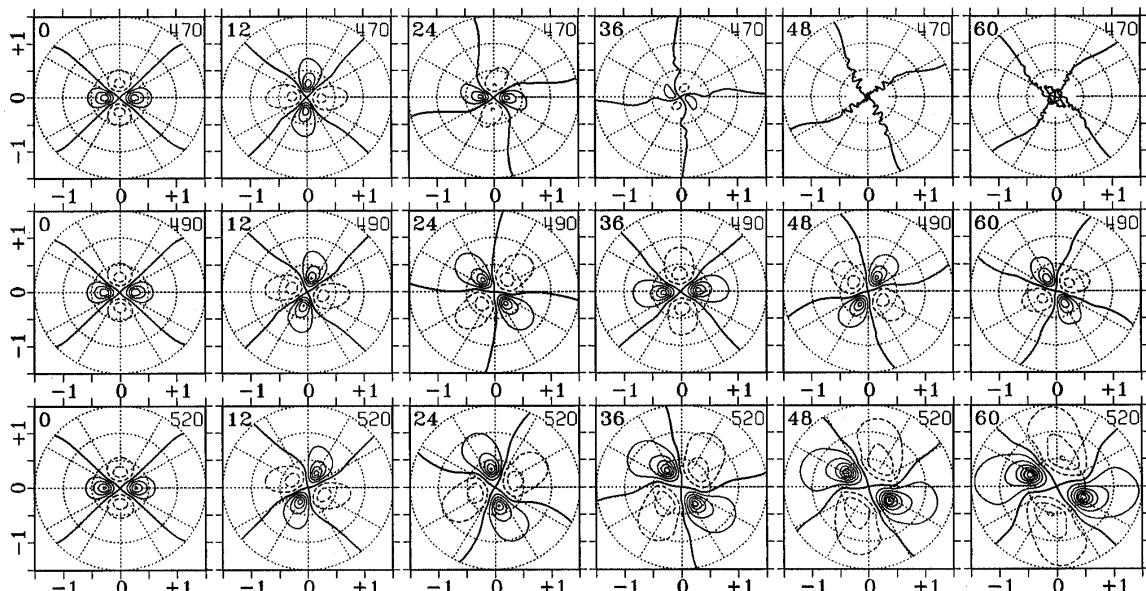
**Figure 2.** Time variation of azimuthal-mean flow in the polar coordinate system of which origin is the midpoint of binary vortices system. The profiles are drawn at every twelve hours.

origin are apparent: Inward-right tilt for the merging case, inward-left for the diverging case, and alternating with time for the neutral case. The oscillation is accompanied by the alternating phase-tilt of the anomaly (deviation from the axisymmetric part) with time, between the inward rightward tilt and the inward leftward tilt.

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**Figure 3.** Time variation of streamfunction deviated from the axisymmetric part for the case of  $SD_0 = 470, 490$  and  $520$  km of EXP I, respectively, with the contour interval of  $10^6 m^2/s$  and time interval of 12 hours. The numerals on the abscissa and ordinate represent the distance with unit of 1000 km.