AXIALLY SYMMETRIC CIRCULATIONS IN A MOIST ATMOSPHERE

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1. INTRODUCTION

The study of idealized axially symmetric circulations can provide some useful insights into the behavior of the tropical atmosphere. Held and Hou (1980) show that in the inviscid limit, a Hadley-like circulation is present whenever the horizontal gradient of radiative-convective equilibrium temperature is above a given threshold. Emanuel (1995) finds a similar criterium in a moist atmosphere. Lindzen and Hou (1988) and Plumb and Hou (1992) analyze the response to a heating centered off the equator. They find that the winter branch of the circulation is much stronger than the summer branch. The present paper investigates further the behavior of such a cross-equatorial Hadley cell in an idealized moist atmosphere, with special emphasis on the flow in the lower troposphere.

Consider a distribution of sea surface temperature (SST) with a maximum off the equator. If the SST perturbation is large enough, a cross-equatorial circulation is present. The upper branch of the circulation transports mass from the summer hemisphere (where the SST maximum is located) to the winter hemisphere. In an axially symmetric atmosphere, the return flow can only occur in two ways: (1) within a mixed layer, where surface friction and turbulence allow air masses to change angular momentum, or (2) within the free troposphere, in which case it follows surfaces of constant angular momentum or occurs within a region of homogenized angular momentum. In the strict sense, the inviscid limit discussed in Held and Hou (1980) implies that the thickness of the surface layer vanishes, so that the return flow must take place within a region of homogenized angular momentum within the free troposphere. However, in the atmosphere, convection maintains a mixed-layer near the surface. We consider here an alternative limit where viscosity vanishes in the free troposphere but remains large within the mixed-layer.

2. FLOW IN THE MIXED-LAYER

Assume that turbulence homogenizes the velocity within a mixed-layer (ML) of constant thickness ΔH, and that the turbulent momentum flux vanishes at the top of this layer. The momentum budget of the ML is obtained by integrating the momentum equation in the vertical between the surface and the top of the ML. For a steady circulation, the momentum budget in the zonal direction is

$$\Delta H \beta y v_b - \kappa u_b = 0, \qquad (1)$$

where u_b and v_b are the zonal and meridional winds respectively. The first term on the left-hand side is the Coriolis acceleration, and the second term is the surface friction. For simplicity, we limit ourselves to a Boussinesq flow on a β-plane and assume that the surface stress is proportional to the boundary layer wind, κ being the proportionality constant. Meridional advection is neglected in (1). The contributions of the pressure gradient and zonal advection vanish because of axisymmetry.

The momentum budget in the meridional direction is

$$-\Delta H \beta y u_b - \int_0^{\Delta H} \frac{1}{\rho_0}\frac{\partial p}{\partial y} \mathrm{d}z - \kappa v_b = 0, \qquad (2)$$

where $\partial p/\partial y$ is the meridional pressure gradient and ρ_0 is the reference density.

The wind in the free troposphere is assumed to be in geostrophic balance, so that $\rho_0^{-1}\partial p/\partial y(\Delta H) = -\beta y u_f$, where u_f is the zonal wind at the top of the ML and $\partial p/\partial y(\Delta H)$ is the meridional pressure gradient at the same level. The pressure gradient at any level is obtained by integrating the hydrostatic balance

$$\frac{1}{\rho_0}\frac{\partial p}{\partial y}(z) = -\beta y u_f - \int_z^{\Delta H} \frac{g}{T_0}\frac{\partial T_v}{\partial y} \mathrm{d}z'. \qquad (3)$$

Here, g is the gravitational acceleration, T_v is the virtual temperature in the boundary layer, and T_0 is the reference temperature.

If the virtual temperature gradient is constant through the ML, the momentum budget (2) becomes

$$-\Delta H \beta y (u_b - u_f) + \Delta H^2 \frac{g}{T_0}\frac{\partial T_v}{\partial y} - \kappa v_b = 0. \qquad (4)$$

The second term on the left-hand side is the contribution of the temperature gradient within the mixed layer, similar to that discussed by Lindzen and Nigam (1987). Let us first consider the flow in the absence of horizontal temperature gradients. The zonal wind in the boundary layer is then given by

$$v_b \approx \frac{\beta y \Delta H}{\kappa + \frac{g \beta y \Delta H^2}{\kappa}} u_f. \qquad (5)$$

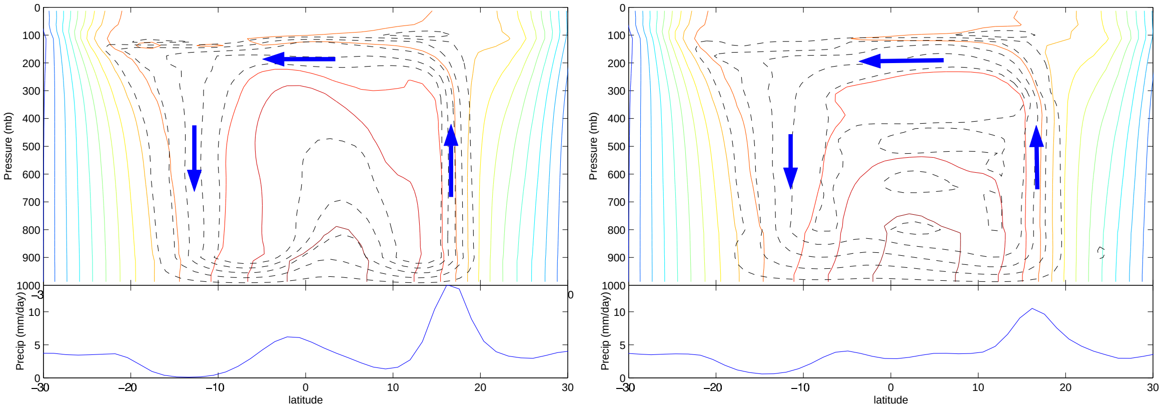

Figure 1: Angular momentum (solid line) and mass transport (dashed line) and precipitation (bottom plot). Contour intervals for angular mometum are 2% of its maximum at the Earth's surface. For the mass transport, they are $0.25\,10^{10}$kg s^{-1}. The arrows indicate the direction of the mean wind. Left pannel is for $\Delta P = 100$mb, right pannel is $\Delta p = 200$mb.

This can be interpreted as the relationship between the Ekman transport in the mixed layer and the geostrophic wind above it. An easterly wind in the free-troposphere is associated with a poleward pressure gradient and generates an equatorward flow in the ML. Conversely, a westerly wind produces a poleward flow in the ML.

The angular momentum in the free troposphere must be lower than that of the solid body rotation at the equator. It must also be larger than the angular momentum in the ascending regions in the subtropics. On a β-plane, this implies that the zonal wind in the free troposphere is bounded by

$$-0.5\beta(y_0^2 - y^2) \leq u_f \leq 0.5\beta y^2, \qquad (6)$$

where y_0 is the location of the subtropical ascent regions. For an equatorward flow, the maximum transport is

$$\Delta H |v_{b,ew}| \leq \frac{0.5(\beta y_0 \Delta H)^2}{\kappa} |y|. \qquad (7)$$

For the poleward flow, it is

$$\Delta H |v_{b,pw}| \leq \frac{0.5(\beta \Delta H)^2}{\kappa} |y|^3. \qquad (8)$$

In both cases, the mass transport vanishes at the equator. Furthermore, for the flow in the ML to be equtorward in the winter hemisphere and poleward in the summer hemisphere, the zonal wind at the top of the ML must change from easterly in the winter hemisphere to westerly in the summer hemisphere.

In the absence of temperature gradients in ML, we have the following situation. Subsiding air enters the mixed layer in the winter hemisphere. Low level easterlies drive a ML flow towards the equator. However, this flow cannot cross the equator in the ML. There is therefore a zone of low level convergence and enhanced precipitation on the winter side of the equator. The return flow then crosses the equator within the free-troposphere. Because of this ascent near the equator, it is possible to generate westerly winds in the summer side of the equator. Hence the return flow may subside on the summer side, although this is not necessary, and go all the way to the ascent regions in the summer hemisphere.

A virtual temperature gradient in the ML can potentially generate a cross-equatoward flow. If this temperature-driven flow is large enough, all of the return flow can occur in the ML. The mass transport generated by the temperature variations is proportional to ΔH^3. Hence, whether or not there is a zone of large-scale ascent and enhanced precipitation in the winter hemisphere depends strongly on the mixed-layer thickness.

3. NUMERICAL SIMULATIONS

The effect of the ML depth on the cross-equatorial Hadley circulation are investigated in a numerical model. The model used here is an axially symmetric GCM on a sphere, with an horizontal resolution of 1.4 degrees, and a vertical resolution of 25 mb. Convection is parameterized using the Emanuel and Zivkovic-Rothman (1999) convection scheme, without any convective transport of momentum. Horizontal diffusion is limited to a 4-th order Shapiro filter. Radiative cooling is represented by Newtonian cooling that relaxes the atmospheric temperatures toward a reference profile. Surface friction and heat fluxes are obtained from a bulk parameterization. This sea surface temperature distribution is chosen in order generate a cross-equatorial Hadley circulation. It is given by

$$\begin{aligned} \mathrm{SST}(\theta) &= 298.15\mathrm{K} \quad +2.5K cos(\frac{\pi}{2}\frac{\theta-\theta_0}{\Delta\theta})^2 \\ & \qquad\qquad \text{for} |\theta-\theta_0| \leq \Delta\theta \\ &= 298.15\mathrm{K} \quad \text{otherwise.} \end{aligned} \tag{9}$$

Here, θ is the latitude in degree, $\theta_0 = 15^0$ and $\Delta\theta = 20^0$.

To simulate a mixed-layer, vertical viscosity is large in a layer of thickness ΔP near the surface. Outside this layer, the vertical viscosity is limited to numerical diffusion. Figure 1 shows the cross equatorial circulation with $\Delta P = 100$mb and $\Delta P = 200$mb, corresponding approximatively to a boundary layer thickness of 1 and 2 km respectively. In the case $\Delta P = 100$mb, there is a strong return flow near the surface in the winter hemisphere that rises near the equator. This ascent is associated with enhanced precipitation in the equatorial region, even though the SST is barely above the mean value. There is also a subsidence region between 5^0 and 12^0 on the summer side, even though this corresponds to a region of warm SST. For $\Delta P = 200$ mb, there is a strong cross-equatorial flow in the viscous layer, and no region of enhanced precipitation in the winter hemisphere.

4. CONCLUSION

The mass transport within a mixed layer near the surface is strongly constrained by the zonal wind in the free troposphere. In the absence of temperature gradients in the ML, the meridional mass transport vanishes at the equator. In this case, the return flow of a cross-equatorial Hadley cell generates a region of of strong precipitation and large-scale ascent in the winter hemisphere, as observed in one of our simulations. This behavior of the return flow may depends on the temperature gradient within the ML. A scaling analysis shows that the temperature driven mass transport is highly sensitive to the depth of the mixed-layer. For a deep enough mixed-layer, a cross-equatorial temperature gradient may sustain a strong cross-equatorial flow. In this case, there is no convergence zone on the winter side of the equator. This implies that the large-scale distribution of precipitation depends strongly on the behavior of the planetary boundary layer.

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