Mathematical Strategies for Stochastic Climate Modeling

The idea of simplified stochastic modeling for unresolved space-time scales in climate is over twenty years old and emerged from fundamental work of Hasselman [2] and Leith [4]. There has been a recent burst of activity in the atmosphere-ocean sciences community in utilizing stable linear Langevin stochastic models for the unresolved degrees of freedom in stochastic climate prediction [1], [3] with notable positive results, but also failure of this simplified stochastic model in some circumstances.

In this talk, authors will discuss recent theoretical results of systematically deriving effective equations for the slowly-evolving (climate) variables with corrections due to the unresolved variables [5], [6]. The key assumptions in this systematic theory are that the climate variables in a given nonlinear system necessarily evolve on longer time scale than the unresolved variables and that the nonlinear interaction among unresolved variables can be represented stochastically in a suitable simplified fashion. In this new mathematical approach once the climate variables are identified new closed nonlinear stochastic equations are derived for the climate variables alone on longer time scales. Several new phenomena occur through this systematic approach including the following:

A. Systematic nonlinear corrections to the climate dynamics due to the interaction with the unresolved variables.

B. The need for multiplicative stochastic noises besides additive noises for the climate variables; such noises and their structure are deduced in a systematic fashion from the theory.

C. Mathematical criteria and examples with unstable linear Langevin equations for the climate variables. Such examples with less stable stochastic models for the climate variables on a longer time scale indicate that interactions with the unresolved variables can diminish predictability in appropriate circumstances.

The authors will discuss the new phenomena mentioned above in the context of the barotropic quasi-geostrophic equations and, also, will give simple explicit examples of the triad interactions which illustrate the applicability of the theory. Also, an example will be presented where the effective nonlinear self-interactions and the noise in the equations for climate variables might be reduced due to fast wave effects. Comparison of the analytical predictions with the numerical simulations also will be presented for an idealized climate model.

References

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