6.8 HORIZONTAL DISPERSION OF NEAR-INERTIAL OSCILLATIONS IN A TURBULENT MESOSCALE EDDY FIELD

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1. INTRODUCTION

The preceding studies (Kunze, 1995; D'Asaro, 1995; Klein and Treguier, 1995; Balmforth et al., 1998) have shown that a mesoscale flow characterized by a Rossby number smaller than one significantly affects the horizontal dispersion of the near-inertial oscillations (NIOs) on a slow time scale : $T = \mathcal{O}$ $[(Ro.f)^{-1}]$ with Ro the Rossby number and f the inertial frequency. The physics involved is the NIO spatial modulation (or refraction) by the mesoscale vorticity that leads to an NIO wavenumber *growing with time* as: $k = -\nabla Z.t$ with Z the vorticity and t the time (Kunze, 1995). The subsequent dispersion effects induce an NIO concentration in the $Z < 0$ regions and a NIO depletion in the $Z > 0$ regions. During a first period corresponding to t$< \mathcal{O}$ (T), the dispersion effects are still much weaker than the vorticity effects and the NIO field is close to the Z-field. This period is called the trapping regime. At a later time (t $> \mathcal{O}$ (T)) k has significantly increased, the dispersion effects are large, at least as large as the vorticity effects, and the NIOs are close to the streamfunction field. This later period is called the strong dispersion regime.

All the preceding studies have considered only isolated or monochromatic mesoscale structures (as a geostrophic jet) characterized by one typical length scale (or mostly one typical wavenumber) and the build-up of the spatial heterogeneity of the NIOs is related to the increase with time of their wavenumbers.

The question adressed in our study is : are the same mechanisms driving the build-up of the spatial heterogeneity of the NIOs when a more realistic mesoscale flow turbulent mesoscale flow is considered, i.e. a mesoscale flow involving a large number of mesoscale structures strongly interacting and characterized by a continuous wavenumber spectrum ?

Our results provide answers to this question that differ somewhat from the preceding results. They reveal new dispersion mechanisms that occur on a time scale shorter than the advective time scale of the mesoscale flow (Klein and Llewellyn Smith, 2001).

2. NUMERICAL RESULTS

In this study we have taken advantage of the NIO equations proposed by Young and Ben Jelloul (1997). Their main property is that the inertial period is filtered out and consequently they described only the evolution of the slower subinertial amplitude, A, given by :

$$u + iv = e^{-ift}.A$$

with $i^2 = -1$ and u and v the horizontal components of the NIO velocity. The NIO kinetic energy can be directly retrieved from A through :

$$u^2 + v^2 = |A|^2.$$

Such equations are easily incorporated in a 3-D quasi-geostrophic model used to simulate stratified quasi-geostrophic turbulence.

Starting with an equilibrated QG flow field dominated by the barotropic and the first baroclinic modes (see fig.1), obtained with the Hua and Haidvogel (1986)'s model, the NIO equation (governing the time evolution of A) is integrated with the evolving QG flow during 30 days. The influence of the eddy field on the horizontal dispersion of the NIOs is analyzed using a vertical normal mode expansion. The kinetic energy of the initial NIO field is *homogeneous* over the whole domain. The amplitude of this kinetic energy (equal to 1) is identical for all the NIO baroclinic modes considered in order to compare their respective behavior.

After 7 days the evolution of the QG flow is weak. The vorticity field (fig.2) is characterized by small-scale coherent vortices and strong vorticity fronts and a spectrum slope in $k^{-1.5}$. On the other hand the time evolution of the different NIO baroclinic modes is quite significant. Thus the kinetic energy of the 7^{th} NIO baroclinic mode is mostly concentrated in small-scale filamentary structures located in the negative vorticity regions close to the strong

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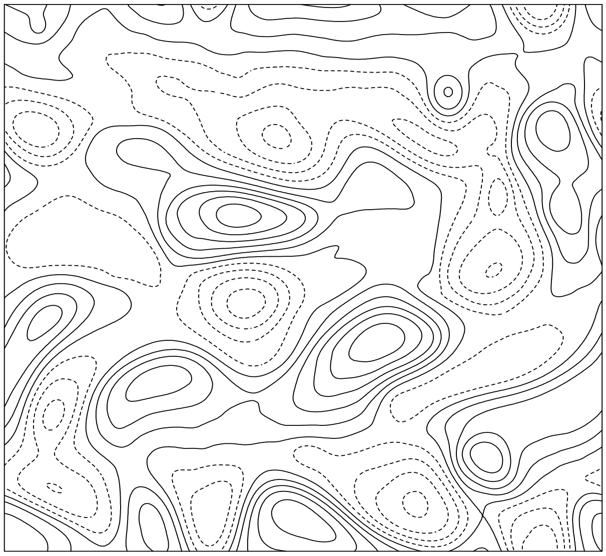

Figure 1: *Contour map of the initial streamfunction field at a depth of 100m. Dashed and continuous contours correspond respectively to negative and positive values relative to the mean.*

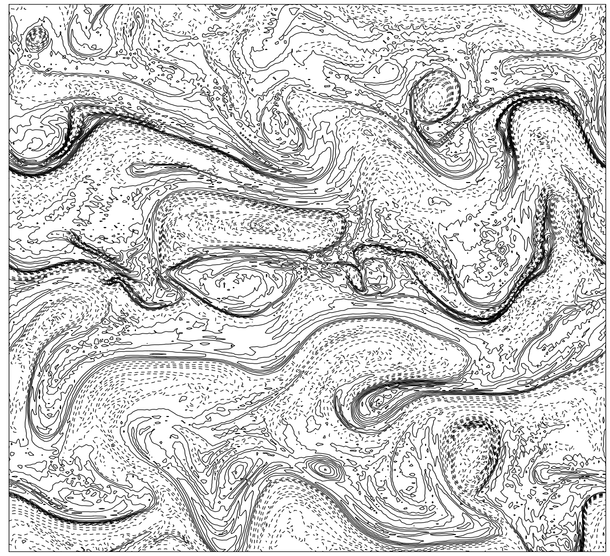

Figure 2: *Contour map of the vorticity field at a depth of 100m after 7 days. Dashed and continuous contours correspond respectively to negative and positive values relative to the mean.*

vorticity fronts (fig.3). Other regions are kinetic energy depleted. The scale of the structures where the NIO kinetic energy is concentrated varies from one baroclinic mode to the other. Statistical results are more interesting : they reveal the existence for each NIO mode of a specific wavenumber, k_s, where the spectrum slope is characterized by a *conspicuous discontinuity* (fig.4). Thus this wavenumber defines two regions : (i) a large-scale region ($k < k_s$) where all the NIO spectra have the same amplitude and a spectrum slope in $k^{-1.5}$, i.e. close to the vorticity spectrum slope, which is reminiscent of the trapping regime. (ii) a small-scale region ($k > k_s$) where all the NIO spectra have the same slope (in $k^{-5.5}$, i.e. close to the streamfunction spectrum slope, which is reminiscent of the strong dispersion regime) but with an amplitude that differs from one mode to the other. Furthermore the time evolution of these spectra shows that the amplitude in the large-scale region ($k < k_s$) grows with time whereas that in the small-scale region ($k > k_s$) is steady. As a result the specific wavenumber of each NIO mode *decreases* with time. At last the NIO kinetic energy spectra (fig.5) reveal that the kinetic energy is mostly concentrated in structures with scales close to k_s^{-1}.

3. ANALYTICAL ANALYSIS

Additional numerical simulations reveal that these features still persist when, in the NIO equation, the advection by the mesoscale flow is neglected and the refraction term (involving the vorticity of the mesoscale flow) is simplified by assuming that the initial large-scale NIO field dominates. This allows to use an analytical analysis to rationalize these statistical features and in particular the existence and time evolution of the specific wavenumber.

This analysis clearly reveals that, when the vorticity spectrum slope is less steep than k^{-4}, there exists a critical wavenumber whose expression is $k_c = \pi/(R_m\sqrt{\pi f_0 t})$ with R_m the Rossby radius of deformation associated the m^{th} NIO baroclinic mode. This critical wavenumber partitions the spectral space into two regions : (i) the large scale region ($k < k_c$) characterized by a spectrum slope equal to that of the vorticity spectrum and where the linear time increase of the NIO amplitude is the same for all the baroclinic modes. In this region the trapping regime dominates and the NIO structures are close the Z-field. (ii) the small scale region ($k > k_c$) characterized by a spectrum slope equal to that of the streamfunction spectrum and whose amplitude is steady but differs from one mode to the other. In this region, the strong dispersion regime dominates

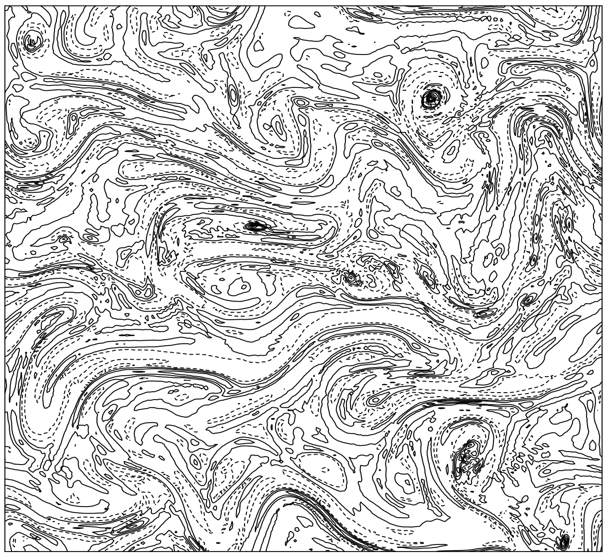

Figure 3: *Contour maps of the NIO speed ($\sqrt{u^2+v^2}$) associated with the 7th mode, 7 days after the initial time. Dashed and continuous contours correspond respectively to negative and positive values relative to the mean.*

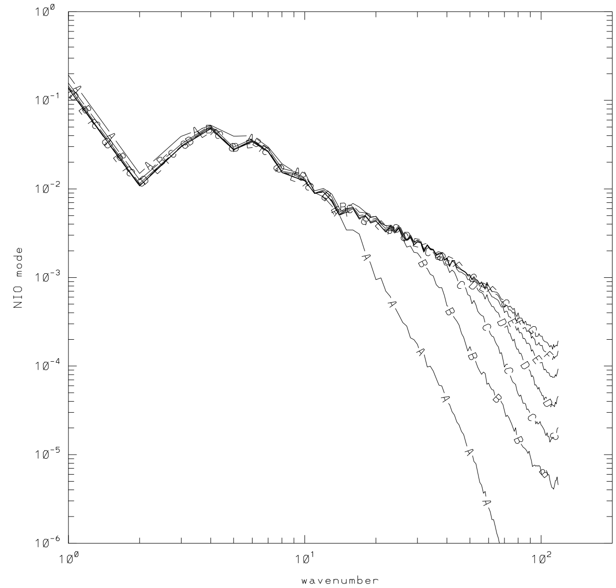

Figure 4: *Spectra of the NIO baroclinic modes, A_m for $m = 3,5,7,9,11,13,15$ (respectively A to G) after a simulation duration of 7 days.*

and the NIO structures are close to the streamfunction field.

The wavenumber k_c results from a balance between refraction and dispersion. This balance first occurs at the highest wavenumber. Thereafter k_c decreases with time, at a rate inversely proportional to the radius of deformation, R_m, of the baroclinic NIO mode considered. At last, the maximum efficiency of dispersion for scales close to k_c concentrates NIO kinetic energy at these scales. As a consequence, at any given time, higher NIO baroclinic mode energy can mostly be found in small-scale negative vorticity structures, such as filaments near sharp vorticity fronts, whereas lower NIO mode energy is concentrated within the core of mesoscale anticyclonic vortices. The value of this critical wavenumber has been found close to that of the specific wavenumber revealed by the numerical simulations :

$$k_c \approx k_s.$$

Thus this analytical analysis well rationalizes the time behavior and the characteristics of the NIO spatial heterogeneity. The mechanisms involved in the build-up of this spatial heterogeneity concern the spatial modulation of the large-scale NIO field by the vorticity structures of different scales. This makes the growth in time of the NIO structures at a given scale to be proportional to the amplitude of vorticity structures of the same scale. This build-up is different from that invoked previously. The resulting NIO dispersion leads to the strong NIO energetic structures to have scales close to $\delta = R_m\sqrt{\pi f t}$ and the geometry of the NIO energetic structures should match the mesoscale Z-structures (for the low vertical modes) and the strong Z-fronts (for the higher vertical modes).

What is the impact of the neglected terms ?

- Taking into account the wave-wave interactions into the refraction term $(Z.A)$ leads to increase k_s and to decrease the spectrum amplitude for the small wavenumbers. These interactions become non-negligible after $t = 6$ days. They involve the mechanism invoked in the preceding studies :
  $dk/dt = -\nabla Z$.
- Effects of the advection terms become non-negligible after $t = 6$ days and lead to increase k_s. This is due to the cascade effects.

However the characteristics statistical features are still present which emphasizes the robustness of the analytical solution. Therefore, the build-up of the NIO spatial heterogeneity differs from that related by the preceding studies when a turbulent mesoscale flow is considered.

4. SUMMARY AND CONCLUSION

For a mesoscale flow characterized by an advective time scale of about 30 days and a vorticity spectrum

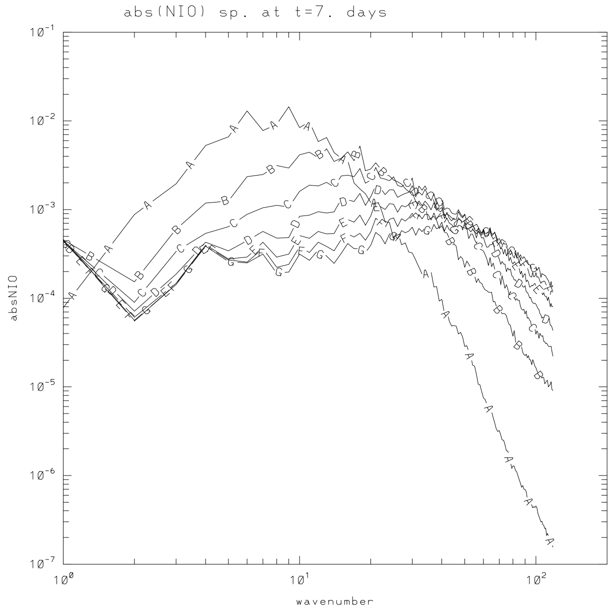

Figure 5: *Spectra of kinetic energy of the NIO baroclinic modes* $|A_m|^2$ *for* $m = 3,5,7,9,11,13,15$ *(respectively A to G) after a simulation duration of* 7 *days* .

slope less steep than k^{-4}, the build-up of the NIO spatial heterogeneity is the following :

During a short time period ($\approx$ 6 days): A given vertical mode, m, naturally selects a specific wavenumber, k_s, for which NIO energy concentration is maximum. The NIO field closely resembles the vorticity structures corresponding to $k \approx k_s$.

This specific wavenumber *decreases* with time as: $k_s^2 = \pi/(f.t.R_m^2)$.

For a longer time period (t > 6 days) : Mechanisms such as that involving $dk/dt = -\nabla Z$ become non-negligible. Cascade effects due to the advection by the mesoscale flow become non-negligible as well.

These mechanisms lead to *increase* k_s and to lessen the NIO spectrum slope.

For a much longer period (t > 30 days) : A saturation mechanism occurs with a resulting k_s close to vorticity spectrum peak.

The mechanisms revealed by the present study should be important for the horizontal dispersion of the near-inertial oscillations by a turbulent mesoscale eddy field when these oscillations are forced by a large scale wind stress whose time variability involves a time scale of a few days.

5. REFERENCES

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