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P5.12 TO WHAT EXTENT ARE SEA LEVEL VARIATIONS DUE TO THE EXPANSION OR CONTRACTION OF THE WATER COLUMN?

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A coincidence between variations of observed and steric sea level (e.g. Pattullo, Munk, Revelle and Strong 1955)

$$\Delta\eta = \int_{-H_*}^{0} (\alpha_T \Delta T - \alpha_S \Delta S)\,\mathrm{d}z, \qquad (1)$$

where $z = -H_*$ represents the ocean bottom or a very deep level, may be interpreted in two different ways:

1. It expresses the expansion or contraction of the water column due to the *local* heat and fresh water fluxes through the ocean surface (e.g. Gill and Niiler 1973).
2. It shows evidence of surface intensified baroclinic motion in a deep enough ocean (e.g. Ripa 1997).

Heat and fresh water fluxes may *indirectly* contribute to **2** through the set up of baroclinic pressure gradients. The validity of both explanations is explored here, in wavenumber/frequency space, with a two-layer model in which salinity, temperature, and horizontal velocity are assumed to be depth independent in the top one. This type of model is not new. It was first developed, to the best knowledge of the author, by Dronkers (1969) in a one-layer rigid bottom set up, with the purpose of studying the tides in a coastal area. Afterwards it was used in a one-layer reduced gravity setting by Lavoie (1972) and Schopf and Cane (1983), in an atmospheric and oceanic problem, respectively. It has been used by many other authors ever since, but always with volume-conserving equations.

In (Ripa 1999) it is shown that neglecting vertical variations within an heterogeneous layer is a good approximation, even with finite horizontal variations of the density and velocity fields, as long as the perturbation horizontal scale is not much smaller than the resolved deformation radii. Here, the field variations will be considered infinitesimal –not finite– and therefore we ought to be in safe ground (within the same wavelength-scale restriction). One way to see these limitations imposed by this approximation is through the analysis of the free modes of the system. These are the usual Poincaré and Rossby waves (PW & RW), and a "force-compensating mode" (FCM), which maintains a vanishing velocity by balancing the pressure forces related to the heterogeneity of the top layer and the layers thickness (Ripa 1996). In (Ripa 1999) it is shown that the FCM is related to the RW in a higher vertical mode, and that it has $\omega = 0$ even with β effects, as if having a vanishing deformation radius. Consequently, the model fails at very short scales, of the order of the deformation radius of the (ill resolved) higher vertical mode.

Linearizing the model equations, it follows that the right hand side of (1) is made up of two parts: the integral within the top layer, $\eta_T + \eta_S$, with

$$\partial_t \eta_T = \frac{\alpha_T}{\rho_0 C_p} Q, \quad \partial_t \eta_S = \varepsilon_s (P - E),$$

plus the integral across the interface, $-\varepsilon\zeta$, where

$$\varepsilon_s = \alpha_S S_1, \quad \varepsilon = \alpha_T (T_1 - T_2) - \alpha_S (S_1 - S_2),$$

ζ is the interface elevation field, and $(T_j,\, S_j)$ are the mean temperature and salinity in the top ($j = 1$) and bottom ($j = 2$) layer. Typical oceanic values are $\varepsilon \approx 1 - 4 \times 10^{-3}$ and $\varepsilon_s \approx 2.5 \times 10^{-2}$. As an example, consider that Q has an amplitude of $200\,\mathrm{W\,m^{-2}}$ at $\omega = 2 \times 10^{-7}\,\mathrm{s}$ (one cycle per year); then η_T will have an amplitude of about 6 cm. On the other hand, if $P - E$ has an amplitude of $3 \times 10^{-8}\,\mathrm{m\,s^{-1}}$ (one meter/year) at the same frequency, then the amplitude of η_S will be about 4 mm, but that $\varepsilon_s^{-1}\eta_S$ will be about 16 cm.

The surface inputs of fresh water $(P - E)$ and heat Q produce directly the density changes represented by $(\eta_T + \eta_S)$ and indirectly, through the set up of pressure gradients, those represented in ζ.

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By linearizing the evolution equations, the total sea level forced by those fluxes is calculated, say

$$\begin{aligned} E-P &\mapsto \eta_E, \\ Q &\mapsto \eta_Q, \end{aligned}$$

so that $\eta = \eta_E + \eta_Q$. Two conditions must be met in order to attribute sea level variations to the expansion or contraction of the water column. First, the actual sea level produced by the surface fluxes, η_E & η_Q, must be close to η_S & η_T. Second, this result must be obtained with a mass-conserving model (i.e. one that allows for the thermohaline expansion of seawater) but *not* with the usual volume-conserving model (with a non-divergent three-dimensional velocity field) (see Greatbatch 1994).

Both hypothesis can then be cast, for the simple model used here, in the form

$$\begin{aligned} &\mathrm{H1}: \eta \approx \eta_S + \eta_T && \text{only for } \lambda = 1 \\ &\mathrm{H2}: \eta \approx \eta_S + \eta_T - \varepsilon\zeta && \text{for any } \lambda \end{aligned} \tag{2}$$

where λ is a flag used to distinguish the usual volume-conserving models ($\lambda = 0$) from the mass-conserving ones ($\lambda = 1$). These hypotheses are tested for all values of the horizontal wavenumber $\mathbf{k}$ and frequency ω of the forcing.

The whole information on $\mathbf{k}$ and ω is contained in a single variable (see figure 1) with two quite different normalizations, κ_0 & κ_1, where $\kappa_0 = 1$ ($\kappa_1 = 1$) is the dispersion relation of barotropic (baroclinic) Poincaré waves and $\kappa_0 \gg \kappa_1$. More precisely

$$\begin{aligned} \kappa_0 &= \frac{k^2+l^2}{\omega^2-f^2} g\left(H_1+H_2\right), \\ \frac{\kappa_1}{\kappa_0} &= \varepsilon\gamma\left(1-\gamma\right) \le \tfrac{1}{4}\varepsilon, \end{aligned}$$

where $1-\gamma$ & γ are depth fraction of the top and bottom layers, i.e.

$$\gamma = \frac{H_2}{H_1+H_2}.$$

For $\omega^2 \ll f^2$ it is $\kappa \sim -\left(k^2+l^2\right) R^2$, where R is the corresponding Rossby radius of deformation. Both parts of the solution are found to be

$$\frac{\eta_E\left(\kappa\right)}{\eta_S} = \frac{\frac{1}{\varepsilon_s}+\lambda-\frac{1}{2}-\frac{1}{2}\gamma}{1-\kappa_0} + \frac{\frac{1}{2}\gamma+\gamma^2\frac{\varepsilon}{\varepsilon_s}}{1-\kappa_1} + \frac{1}{2} \tag{3a}$$

$$\frac{\eta_Q\left(\kappa\right)}{\eta_T} = \frac{\lambda-\frac{1}{2}-\frac{1}{2}\gamma}{1-\kappa_0} + \frac{\frac{1}{2}\gamma}{1-\kappa_1} + \frac{1}{2} \tag{3b}$$

$+\,O\left(\varepsilon\right)$ for sea level, and similar expressions for ζ and the velocity fields.

The first two parts are forced barotropic and baroclinic waves that vary continuously, as the frequency decreases from superinercial to subinercial, from Poincaré-like to geostrophic-like (Ripa and Zavala-Garay 1999). The factor $1-\kappa$ in the denominators is proportional to $\left(k-k_+\left(\omega\right)\right)\left(k-k_-\left(\omega\right)\right)$ where, for superinercial frequencies, $k_\pm\left(\omega\right)$ are the inverse of the dispersion relation of PW. In the forced problem there are two eingenvalues k for each ω, even though in the free waves problem there are three eigenvalues ω for each k (Philander 1978, Ripa and Zavala-Garay 1999). For subinertial frequencies, $k_\pm$ are complex. However, if we had included effects of Earth's curvature, there would be another resonance, with $k_\pm$ real, corresponding to the dispersion relation of the RW. The last part of the solution is the "force compensating mode".

Since $\kappa_1 \ll \kappa_0$, the variations of η_Q or η_E occur in quite different scales and consequently the properties of the solution are discussed in different regions. (These are regions in wave number/frequency space; their names reflect the size of $\mathbf{k}$ for fixed ω.)

Very long scales: $\kappa_0 \ll 1$
This solution is giving by (3) with $\kappa_0 = \kappa_1 = 0$, which gives

$$\begin{aligned} \eta &= \left(1/\varepsilon_s\right)\,\eta_S + \lambda\,\eta_T, \\ \zeta &= 0. \end{aligned}$$

No perturbation is transmitted to the interface elevation and the lower layer.

The main consequence of the effective evaporation is to raise or lower the surface by adding or subtracting water; its effect on water density is an $O\left(\varepsilon_s\right)$ less important: $\partial_t\eta_E = \left(1/\varepsilon_s\right)\partial_t\eta_S = -E_e$. Consequently, this response can be safely modelled with a volume-conserving model.

Heat flux, on the other hand, raises or lowers the sea surface by means of a true expansion or contraction of the upper layer. This effect can only be modelled with a mass-conserving model ($\lambda = 1$); a volume conserving model ($\lambda = 0$) gives an inacceptable incorrect result. Hypothesis 1 from the Introduction is satisfied here, just for the heat flux forced part of sea level.

Long scales: $\kappa_0 = O\left(1\right)$
At these scales it is, to dominant order is

$$\begin{aligned} \eta &= \frac{1/\varepsilon_s}{1-\kappa_0}\,\eta_S + \frac{\lambda-\left(\frac{1}{2}+\frac{1}{2}\gamma\right)\kappa_0}{1-\kappa_0}\,\eta_T, \\ \zeta &= \frac{\left(\gamma/\varepsilon_s\right)\kappa_0}{1-\kappa_0}\,\eta_S + \frac{\gamma\left(\lambda-\frac{1}{2}-\frac{1}{2}\gamma\right)\kappa_0}{1-\kappa_0}\,\eta_T. \end{aligned}$$

The structure of the response in wavenumber/frequency space is dominated by the forcing of the barotropic mode. As before, a mass-conserving model ($\lambda = 1$) is essential for the correct evaluation of η_Q, but hypothesis 1 is not necessarily satisfied. However, note that

$$\eta_Q - \eta_T = \tfrac{1}{2}\frac{(1-\gamma)\,\kappa_0}{1-\kappa_0}\,\eta_T + O(\varepsilon)\,,$$

and therefore hypothesis 1 is realized for an infinitesimally shallow upper layer, $\gamma \to 1$, if the forcing is sufficiently far from resonance $1-\kappa_0 \gg 1-\gamma$.

Medium scales: $\kappa_0^{-1} \sim \kappa_1 \sim \sqrt{\varepsilon\gamma(1-\gamma)} \ll 1$
The solution is giving by (3) with $\kappa_0 \to \infty$ and $\kappa_1 = 0$, namely

$$\begin{aligned}\eta &= \tfrac{1}{2}(\gamma+1)(\eta_S+\eta_T) + \gamma^2(\varepsilon/\varepsilon_s)\,\eta_S,\\ \zeta &= -(\gamma/\varepsilon_s)\,\eta_S - \gamma\left(\lambda - \tfrac{1}{2} - \tfrac{1}{2}\gamma\right)\eta_T.\end{aligned}$$

The coefficient $\frac{1}{2}(\gamma+1)$ comes from the contributions of the baroclinic and the force compensating modes. If $\gamma \to 1$ (i.e. a infinitesimally thin upper layer) it is $\eta \approx \eta_S + \eta_T$. Note that this effect is not due to the expansion or contraction of the upper layer because the same result is obtained with both a volume conserving ($\lambda = 0$) or a mass conserving ($\lambda = 1$) model. Rather is a confirmation of hypothesis 2, namely

$$\eta + \varepsilon\zeta = \eta_S + \eta_T + O(\gamma-1) + O(\varepsilon)\,.$$

Short scales: $\kappa_1 = O(1)$
At these scales it is

$$\begin{aligned}\eta &= \tfrac{1}{2}\frac{1+\gamma-\kappa_1}{1-\kappa_1}(\eta_S+\eta_T) + \frac{\gamma^2\varepsilon/\varepsilon_s}{1-\kappa_1}\eta_S,\\ \zeta &= -\tfrac{1}{2}\varepsilon^{-1}\frac{\kappa_1}{1-\kappa_1}(\eta_S+\eta_T) - \varepsilon_s^{-1}\frac{\gamma}{1-\kappa_1}\eta_S.\end{aligned}$$

The structure of the response in wavenumber/frequency space is dominated by the forcing of the baroclinic mode. If $\gamma \to 1$ it is

$$\eta + \varepsilon\zeta = \left(1 - \tfrac{1}{2}\frac{1-\gamma}{1-\kappa_1}\right)(\eta_S+\eta_T) + O(\varepsilon)$$

and therefore hypothesis 2 is satisfied if the forcing is sufficiently far from baroclinic resonance $1-\kappa_1 \ll 1-\gamma$.

Very short scales: $\kappa_1 \gg 1$

This solution formally corresponds to (3) with $\kappa_0 \to \infty$ and $\kappa_1 \to \infty$, which leaves just the force-compensating mode,

$$\begin{aligned}\eta &= \tfrac{1}{2}(\eta_S+\eta_T)\\ \zeta &= \tfrac{1}{2}\varepsilon^{-1}(\eta_S+\eta_T)\end{aligned}$$

This means that hypothesis 2 is verified

$$\eta + \varepsilon\zeta \sim \eta_S + \eta_T.$$

However, as mentioned above, the validity of model, namely, of keeping the dynamical fields in the upper layer as depth independent probably breaks down at these short scales. Note, for instance, that with more vertical structure (e.g. one more layer) in the model setup, there would be terms proportional to $1/(1-\kappa_2)$ due to the second baroclinic mode, where κ_2/κ_1 is typically around 4.

The coincidence (1) between the actual sea level and the steric integral may or not be an *observational* fact. Its explanation, in particular, one of the two hypotheses stated in the first paragraph, is *model dependent*. With the simple model used here, the first hypothesis is realized only for the heat flux forcing and for very long scales ($\kappa_0 \ll 1$). It is also realized for long scales ($\kappa_0 \sim 1$) if the upper layer, where the heat is absorbed, is very shallow ($\gamma \sim 1$). The calculations made by (Gill and Niiler 1973), with a different model, roughly correspond to $-30 < \kappa_0 < 0$ and $\omega \ll f$; therefore their conclusion that "steric changes in sea-level [are] produced by expansion and contraction of the water column above the seasonal thermocline due to changing fluxes of heat and water across the surface" may depend upon the shallowness of the thermal forcing. Greatbatch (1994) proposed to evaluate sea level in rigid-lid volume-conserving numerical models by

$$\partial_t\eta + \nabla\cdot\left(\int_{bottom}^{surface}\mathbf{u}\,\mathrm{d}z\right) = \frac{\alpha_T}{\rho_0 C_p}\bar{Q}$$

where $\bar{Q}$ represents a global average. Again, the success of this recipe may depend upon the thickness of the heat absorbing layer.

Hypothesis 2, on the other hand, is related to the dominance of surface intensified baroclinic signals, which may be free waves or wind-forced motions, totally independent of the fresh water and heat surface fluxes (see Ripa 1997). As far as these buoyancy forcings are concerned, and for the present model, hypothesis 2 is realized for very short scales ($\kappa_1 \gg 1$). In addiction, it works for short ($\kappa_1 \sim 1$) and medium ($\kappa_0^{-1} \sim \kappa_1 \ll 1$) scales if the top layer is sufficiently shallow (so that the baroclinic signal is surface intensified). In any case, these effects can be modelled with the usual volume-conserving equations.

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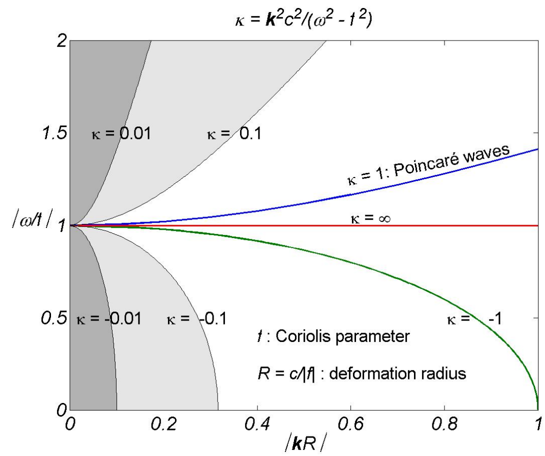

Figure 1: Definition of variable κ, used with different normalizations, defined by the choice of c^2. The heat input (output) Q through the ocean surface, with wavenumber $\mathbf{k}$ and frequency ω, produces a sea level η_Q, part of which, η_T, corresponds to the direct warming (cooling) of an upper layer, but most of η_Q is due to the pressure gradient generated by Q. However, in the shaded region ($\kappa \ll 1$) and for the κ corresponding to the barotropic mode, it is $\eta_Q \simeq \eta_T$. In order to obtain this result, a model must conserve mass instead of volume, i.e. the three-dimensional velocity field must have the possibility of a non-vanishing divergence.