8.7 A DYNAMIC AND SPATIAL SCALING APPROACH TO ADVECTION FORECASTING

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1. INTRODUCTION

Quantitative nowcasts of rainfall are frequently based on the advection of rain fields observed by weather radar. Major sources of error in this approach are the temporal development of the field during the forecast period, errors in the measured and forecast velocity of the field or rain cell, and the conversion of radar reflectivity into rainfall intensity.

Considerable theoretical argument and empirical evidence that rainfields can be modelled as self-similar or multifractal fields has been accumulated over the past 15 years. This implies that typically a rainfield is not organised as a collection of cells with a characteristic scale, but rather as a continuum or hierarchy of structures over scales from 100 m to 200 km at least. The lifetime of a turbulent structure has a power law dependence on the scale of the structure. Recent research has been conducted on methods to exploit this scaling behaviour in nowcasting applications. The promise of these new methods is their ability to model the rate at which the field is evolving as a function of scale in real-time in a parsimonious and robust manner. This information is then used to produce nowcasts where the small-scale detail is allowed to dissipate in a structured way as the forecast lead-time is increased.

This paper describes a Spectral prognosis (SPROG) advection based nowcasting system, the underlying model on which it is based and presents preliminary results from a recent field trial in Sydney as part of the Forecast Demonstration Project of the World Weather Research Programme of the W.M.O.

2. RAINFALL MODEL AND ALGORITHM

One way to generate a multifractal field is to use a bounded multiplicative cascade of random fields (Seed et al, 1999). The rain field is assumed to be the product of a hierarchy of correlated random fields where the correlation length of the field is the scale represented at that level and the variance of the field decreases as a power law with this scale.

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This approach can be used to generate nowcasts if the measured rain field can be used to infer the hierarchy of multiplicative fields.

Rainfall is related to radar reflectivity through a power law, so the multiplicative cascade in rainfall can be transformed into an additive cascade in radar reflectivity. The cascade of random fields is calculated from a measured field using notch filters in the frequency domain to successively filter out features in the image that fall with the range of scales represented by a level in the cascade.

The temporal development of each level in the cascade is modelled using an AR(2) process. The parameters for the hierarchy of AR(2) models are calculated at each time step by using the most recent estimates of the Lagrangian lag 1 and 2 auto correlations to solve the Yule-Walker equations, applying heuristic rules to maintain stationarity. This has the effect of making the model quite adaptive and generic, learning the characteristics of each rainfall event as it unfolds. The forecast fields are renormalised to maintain the observed conditional mean rain rate and raining area and are then converted from radar reflectivity into rainfall intensity.

The two basic assumptions in the method is that the probability distribution of a 2-D map (field) of instantaneous rainfall intensities is very close to a log Normal distribution, and that the field can be modelled by a multiplicative cascade such that

$$R_{i,j}(t) = \prod_{k=1}^{n} X_{k,i,j}(t)$$

where $R_{i,j}(t)$ is the rain rate at location i,j for time t, and $X_{k,i,j}(t)$ is a hierarchy of n fields, each field representing the variability of the rainfall over a certain range of spatial scales. In the case of radar reflectivity, the power law form of $Z = aR^b$ implies that we can construct a cascade such that

$$dBZ_{i,j}(t) = \sum_{k=1}^{n} \left[\mu_k(t) + \sigma_k(t) X_{k,i,j}(t) \right],$$

where $\mu_k(t)$ and $\sigma_k(t)$ is the mean and standard deviation of cascade level k at time t respectively, and then use a Z/R relation to convert the radar reflectivity into rain rate. The cascade of X fields are calculated by using a notch filter on the Fourier transform of the dBZ field such that level k represents the features of the field between $L_0 2^{-k}, L_0 2^{-k+1}$ where L_0 is the outer scale of the field, eg 256 km.

The temporal development of each level in the cascade is modelled using an auto-regressive lag 2 (AR(2)) model

$$\hat{X}_{k,i,j}(t+1) = \Phi_{k,1}(t) X_{k,i,j}(t) + \Phi_{k,2}(t) X_{k,i,j}(t-1)$$

Where $X_{k,i,j}(t-1), X_{k,i,j}(t)$ are the field values for cascade level k, position i, j, at times t-1 and t respectively. $\hat{X}_{k,i,j}(t+1)$ is the forecast at time t+1 for level k, position i,j based on the (time varying) model parameters $\Phi_{k,1}(t), \Phi_{k,2}(t)$. The mean and standard deviation of the field is assumed to be constant for the period of the forecast.

This model is constructed such that each level in the cascade approaches the field mean at that level as the lead time of the forecast is increased, which has the effect of producing a forecast which automatically becomes more smooth as the spatial scale of the unknown detail increases.

The basic algorithm is as follows

1. A pattern-matching algorithm is used to find the displacement between the current field and the field from the previous time step. This is accomplished by using a downhill simplex optimisation method to maximise the correlation between the two fields, assuming that the field advection can be characterised by a single linear displacement for the entire field. The running mean displacement over the past hour of data is used as the estimate since the estimated displacement for any particular time step has significant noise. Future developments in the SPROG will include more advanced field advection characterisation techniques that allow for shear in the field.

2. The input radar field is disaggregated into the hierarchical cascade of fields where each level in the cascade represents the features at a particular scale. The first field contains features in the 128 – 256 km range, the second contains features in the 64 – 128 km range and so on down to twice the pixel resolution. This disaggregation is accomplished in Fourier space using a spectral notch filter.
3. The lag 1 and 2 Lagrangian correlation coefficients for the field are calculated for each level in the cascade by accounting for the field displacement.

4. These correlation coefficients are then smoothed through geometric averaging with past estimates and are used to solve the Yule-Walker equations for estimating the AR(2) model parameters for each level in the cascade. The correlation coefficients are checked to ensure that they are consistent with a stationary model and that the lag 2 correlation r_2 not less than $0.8r_1$. This is necessary due to the fact that artefacts from the spectral disaggregation cause underestimation in the correlation coefficients.

5. The lag 1 nowcast for t_1 is made using the t_0 and t_{-1} rain fields

6. The lag 2 nowcast for t_2 is made using the t_1, t_0 and so on.

Each forecast is renormalised to give a field with the same fraction of the field above some threshold (15 dbZ in this case) as the input measured field, and with the same mean conditional on being above the threshold. The conversion from radar reflectivity to rainfall intensity is based on a Z/R relationship, which is appropriate for the Sydney area:

3. EXAMPLES OF SPROG

Examples of the initial field and forecast fields are shown in Figures 1-3. Other products available include meteograms of precipitation rate and accumulated rainfall for specific venues.

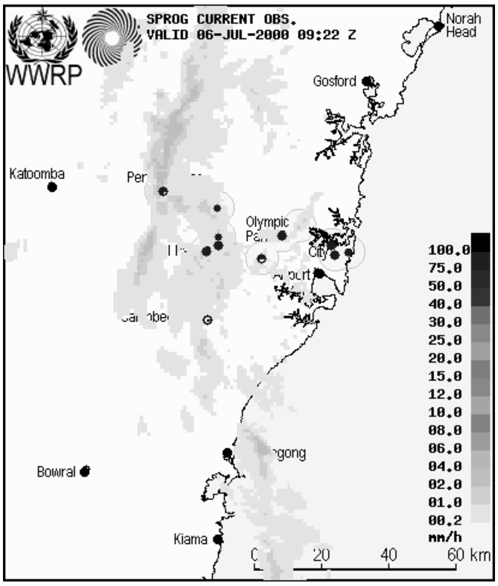

FIG. 1. An example of the measured rainfall over the Sydney area as displayed by SPROG.

4. SUMMARY

SPROG is an advection based nowcasting system that exploits the observation that rain fields commonly exhibit both spatial and dynamic scaling dependent properties i.e. the lifetime of a feature in the field is on the scale of the feature

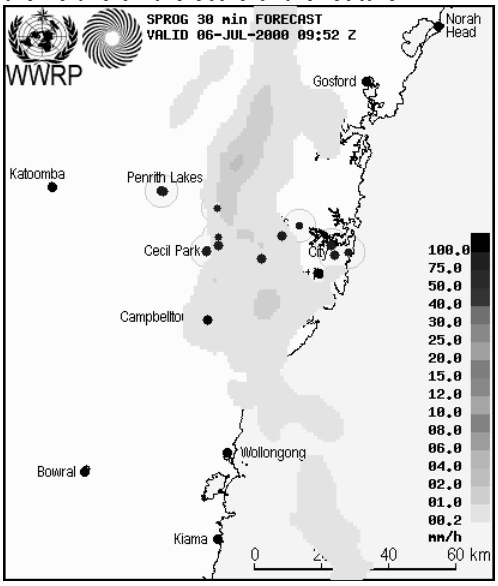

FIG. 2. A 30-minute forecast produced by SPROG. Note that the forecast field has less small-scale detail than the input radar field.

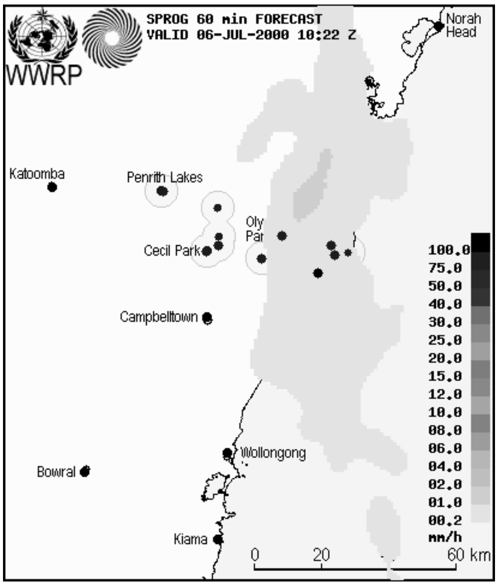

FIG. 3. A 60-minute forecast produced by SPROG

(large features evolve more slowly than small features) and that features at all scales between the outer and inner observed scales are present in the field. A Fourier notch filter is used to disaggregate the rain field into a multiplicative hierarchy or cascade of fields, where each level in the cascade represents features in the rain field at a particular scale. The Lagrangian temporal evolution of each level in the cascade is modelled using a simple autoregressive (lag two) model, which automatically causes the forecast field to become smooth as the structures at the various scales evolve through their life times

5. Reference

Seed, A.W., R. Srikanthan, and M. Menabde, 1999, A space and time model for design storm rainfall. J. Geophys. Res., Vol. 104 (D24) 31623-31630.