P4.8 MONTE CARLO SIMULATIONS OF RANDOMLY DISTRIBUTED FALLING RAINDROPS: PROPERTIES OF DROP SIZE SPECTRA AS MEASURED BY A VERTICALLY POINTING FM-CW DOPPLER RADAR

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1 INTRODUCTION

Vertically pointing Doppler-radars are so far the only instruments which can provide detailed information about the vertical profile of the velocity distribution of hydrometeors. For water drops a well defined terminal-velocity-size-relation exists and, therefore, the vertical profile of the drop size distribution is in principle accessible from the vertical profile of the velocity distribution.

Well known and often treated problems in this field are turbulent vertical air motions (e.g., Wakasugi et al., 1986), deviation of drops from a spherical shape, introduction of variance into the Doppler spectrum through the divergence of the radar beam in the presence of horizontal wind and wind shear (e.g., Sauvageot, 1992) and other "meteorological" sources of error.

In the past different vertically pointing radar setups have been applied to measure vertical profiles of Doppler velocity. Besides the widely used pulsed instruments an FM-CW–method invented by Strauch (1976) is frequently used. Here a frequency modulated microwave signal is transmitted continuously and is mixed with the backscattered signal. It can be shown that information about the range, the fallspeed and the number density of the scatterers are approximately accessible through calculation of the power spectrum of that mixed signal. This paper deals with the question of how accurate the drop size distribution can in principle be measured by an (ideal) FM-CW Doppler radar, disregarding the "meteorological" sources of error.

It turns out that a careful derivation of the theoretical power spectrum for a single scatterer differs from that originally given by Strauch (1976) in the sense that an additional term appears whose consideration leads to a large overestimation of the number of small raindrops in heavy rain (lots of scatterers with differing sizes and radial velocities). This finding is supported by Monte Carlo simulations of the FM-CW–measurement of drops having a predefined size spectrum which are randomly distributed in and falling through the radar volume.

These model simulations can generally be used to investigate other properties of the FM-CW technique, e.g. spectral leakage and echo statistics.

2 THE FM-CW–DOPPLER RADAR

Fig. 1 gives a sketch of the FM-CW method as proposed by Strauch (1976) to infer the radial velocity distribution profile of distributed scatterers. Essentially, the backscattered signal of distributed targets is instantaneously mixed with the transmitted signal, i.e., a signal with the difference phase is produced. The phase is the time integral over frequency f and contains information about the range and radial velocity of the scatteres because of the frequency shift between the transmitted and the received signal.

Often a linear frequency modulation pattern ("sawtooth") is applied because it greatly simplifies things, but

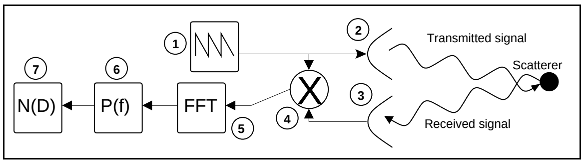

Fig. 1: *Sketch of the FM-CW method with Doppler processing: 1) frequency generator, 2) transmitting antenna, 3) receiving antenna, 4) mixer, 5) Fourier transformation, 6) power spectrum and 7) drop size distribution.*

in general other patterns can be used. In this paper, we take into account a linear downsweep mode (sweep = one sawtooth); fig. 2 shows the corresponding frequency time series of the continuously transmitted wave along with the received signals of a nonmoving and a moving target, both at the same range. T is the time for one sweep, f_0 the basic radar frequency.

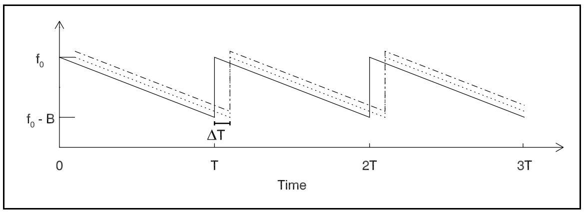

Fig. 2: *Sketch of the frequency of the transmitted (solid line) and received signal (dotted line: nonmoving target; dash-dotted line: moving target) as function of time for an FM-CW–Doppler Radar in downsweep mode.*

Consider a single scatterer moving towards the radar: When the backscattered signal arrives at the mixer, its frequency is higher than the transmitted signal at that moment because of both wave travelling time and the Doppler effect. Information about this difference is contained in the difference phase at the mixer output. A priori the two sources of difference are not distinguishable. Strauch (1976) showed that in the power spectrum of the mixed signal, taken over K ($\gg 1$) consecutive sweeps (not only one), the two components separate.

However, in the theoretical derivation of the power spectrum he neglected a "small" term which does not seem to be of importance in the presence of a single scatterer but can cause considerable errors when lots of scatterers with differing sizes and velocities are present (Blahak, 2000).

3 THEORETICAL POWER SPECTRUM OF A SINGLE DROP

In view of a proper interpretation of FM-CW–Doppler power spectra of distributed rain drops it is convenient to first look at the contribution of a single drop. A detailed derivation of the theoretical power spectrum of such a single moving target can be found in Blahak (2000). Here just a few words: If the drop does not move relative to the radar, the power spectrum P consists of sharp lines at frequencies of multiples of $1/T$. If its range r equals $n \times c/2B = n\Delta r$, $n \in \mathbb{N}$, c = speed of light

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and B = sawtooth amplitude, then exactly one line appears at $f = n/T$ (see also fig. 3, left graph), otherwise also weaker lines appear at the adjacent multiples of $1/T$ (spectral leakage). When the drop is moving with radial velocity v_r (positive towards the radar), the entire spectrum is shifted by the Doppler frequency $f_d = 2f_0v_r/c$ in positive direction; then other smaller lines appear at frequencies mirrored at the consecutive multiples of $1/T$, when the range is not exactly a multiple of Δr (spectral leakage coming from the negative side of the fourier spectrum F, which is an even complex function of frequency; $P = 2|F|^2$, when only $f > 0$ is considered). These are neglected in the original literature.

Therefore – when looking at the strongest line – if one expects only motions towards the radar (as in rain) and if the maximum Doppler frequency of the drops is smaller than $1/T$, the intervalls between n/T and $(n+1)/T$ are associated with a range cell with width Δr centered around $n\Delta r$. The shift of that line relative to "its" base gives the Doppler frequency f_d and therefore the velocity.

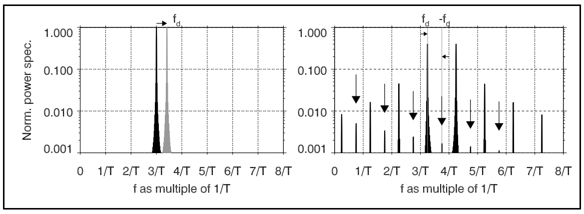

Fig. 3: *Normalized theoretical power spectra of example drops as a function of frequency (see text for explanation). T is the duration of one sweep.*

Fig. 3 shows graphical representations of that function for example drops. In the left graph, the normalized power spectrum of a drop exactly at $r = 3\Delta r$ is shown i) when $v_r = 0$ (black) and ii) when $v_r > 0$ (grey). In the right graph, the drop is at $r = 3.5\Delta r$ and $v_r > 0$. The vertical arrows indicate the lines which are neglected in the original literature, the small horizontal arrows suggest to the "mirroring". Note that the maximum power is only about 0.5 the maximum power of the line in the left graph.

4 DISTRIBUTED RAIN DROPS

To derive the drop size distribution in a certain range cell, usually the following formula is applied, provided the Rayleigh approximation is valid:

$$N[D(v_r)] \;=\; C\;\frac{r^2}{\Delta r}\;D(v_r)^{-6}\;\frac{\overline{p}[D(v_r)]}{dD/dv_r} \qquad (1)$$

where D is the drop diameter, C is a proportionality constant determinded by the radar system, $\overline{p}(D)$ is the measured ensemble average power spectrum and $r^2/\Delta r$ is the range correction valid if $r^2 \gg (\Delta r)^2$.

Essential assumptions herein are that $\overline{p}$ is the sum of the contribution of each single drop as presented in sec. 3 and that weaker contributions of drops at ranges between multiples of Δr are compensated by spectral leakage of drops with same D and v_r in adjacent range cells.

Referring to this, a few remarks have to be made: The received $\vec{E}$-field is the sum of the backscattered fields of every single drop. Therefore, the complex fourier spectrum of the mixed signal is the sum of each single drop's fourier spectrum. But the power spectrum is not necessarily the sum of the power spectra associated with

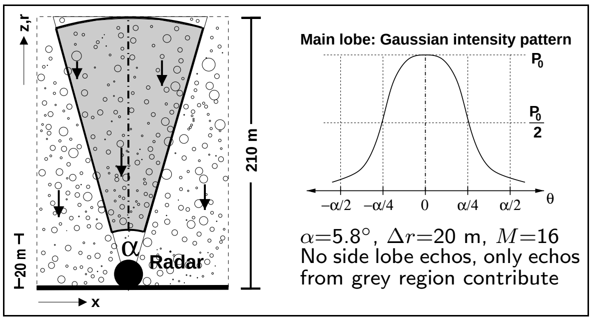

Fig. 4: *Sketch of the Monte Carlo model. M=16 is the number of range bins ("1": 10–30 m; ...; "10": 190–210 m; ...; "16": 310–330 m), α is two times the 3-dB–beamwidth.*

the single drops. Also the lines indicated by the vertical arrows in fig. 3 may play a role. Finally, echo statistics valid for ordinary pulsed radar systems (Marshall and Hitschfeld, 1953) have to be checked to be valid for FM-CW–Doppler systems, too.

5 MONTE CARLO SIMULATIONS OF FM-CW DROP SIZE DISTRIBUTION MEASUREMENTS

To check the assumptions outlined in section 4, a simple Monte Carlo model is used. The backscattered $\vec{E}$-field as sum of the contributions of randomly distributed falling spherical rain drops with an adjustable, spatially homogeneous size distribution is modeled (Rayleigh scattering). This signal is ideally mixed (i.e., without intermodulations) with the transmitted signal, an FFT is performed over $K = 64$ sweeps and the power spectrum is calculated. Then 200 consecutive power spectra are averaged incoherently to obtain the mean power spectrum $\overline{p}$. The drop size distribution is calculated with (1), in which the constant C was calibrated in a way that the rain rate R in the second range bin equals the "true" rain rate.

Fig. 4 shows the main characteristics of the model. Radar parameters are chosen in a way that the maximum unambiguous velocity v_{max} is 12 m s^{-1}. Note that the following results do not depend on the wavelength of the radar, as long as Rayleigh scattering can be assumed. Also the results can be generalized to arbitrary settings of the range resolution Δr.

First, an exponential distribution $N(D) = N_0\exp(-\lambda D)$ with N_0 = 8000 m^{-3}mm^{-1} and λ = 2.5 mm^{-1} ($R \approx 10$ mm h^{-1}) was chosen. To keep the computational amount within a manageable limit, no drops smaller than 0.7 mm have been modeled. In the interpretation, only drops with $D < 4.5$ mm have been considered because the radar volume contains a very small amount of larger drops, leading to statistical

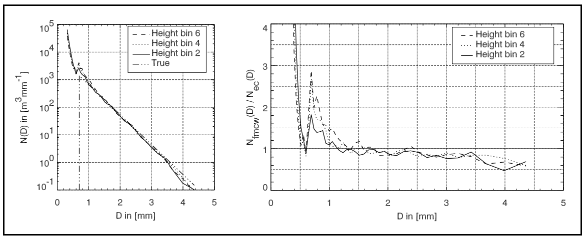

Fig. 5: **Left:** *Modeled FM-CW drop size spectra and predefined "true" exponential spectrum.* **Right:** *Ratio of the FM-CW drop size spectra and the exponential continuation of the "true" spectrum, $N_{ec}(D)$. No window functions applied.*

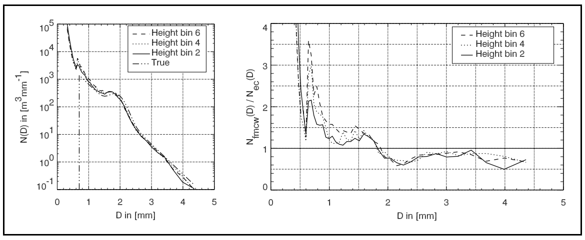

Fig. 6: *Same as fig. 5, but with an additional secondary mode in the "true" spectrum. No window functions applied.*

inhomogeneities in their spatial distribution which do not lead to useful results.

Fig. 5 shows the results for the 2nd, 4th and 6th height bin. Here no window functions have been applied to the mixed signal before the FFT. At $D > 1.2$ mm the agreement is good. But below, drop concentrations are systematically overestimated. The small peak at D=0.7 mm can be adressed to the lines marked by the vertical arrows in fig. 3: Large drops with $v_r \approx 9$ m s^{-1} produce significant contributions to $\overline{p}$ at frequencies corresponding to $v_r = v_{max}$ - 9 m s^{-1} = 3 m s^{-1} ($\Leftrightarrow D \approx 0.7$ m s^{-1}) when present in a sufficient number, since the contributions of the drops are proportional to D^6. The erroneously large amount of drops with $D < 0.7$ mm can be addressed to the fact that between the visible lines in fig. 3 there are very weak maxima ("noise"), which, for the drop collective, lead to a nonzero power spectrum in that size range.

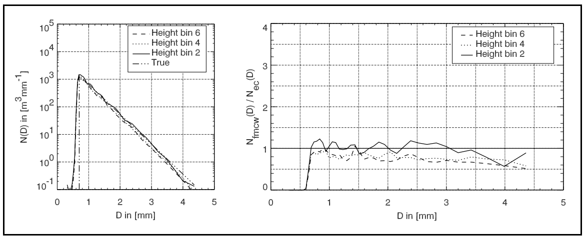

Fig. 7: *Same as fig. 5, but with window functions applied.*

Same effects can be seen in fig. 6. Additionally, here a secondary mode at $D \approx 2$ mm was predescribed to check the measurement of such modes. In the FM-CW–measurement, the mode appears slightly shifted and smeared, but qualitatively could be resolved very well.

The drop size distribution shown in fig. 7 is the same exponential as in fig. 5. The difference is that, before the FFT of the mixed signal, a combined window function (frequently used in FM-CW–radars) has been multiplied to the mixed signal: a square-cosine window for each single sweep and a hamming window over the $K = 64$ sweeps used for one FFT. This leads to a nearly complete elimination of the small-size-range overestimation, because spectral leakage and "noise" are weakened.

Fig. 8 shows the measurement of the spectrum of fig. 6 but with the same window function applied as in fig. 7. Again, the mode is slightly shifted and smeared,

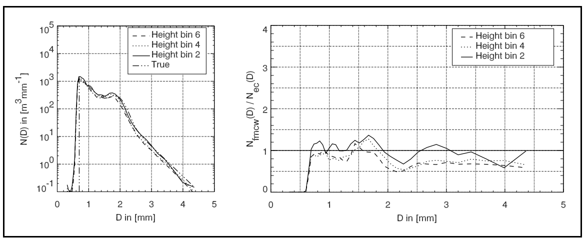

Fig. 8: *Same as fig. 6, but with window functions applied.*

but the spectrum could be resolved very well even in the small size range.

Note a decent underestimation of the drop number concentration in the size range of $D > 1.5$ mm above the level of calibration (height bin 2) which seems to increase with height, when the window function is applied (fig. 7, fig. 8). This could be due to the fact that the assumption for the range correction term in (1), $r^2 \gg (\Delta r)^2$, was not sufficiently satisfied for height bin 2, or, most likely, could be an artifact of spectral leakage in combination with the range-decreasing received powers.

6 SUMMARY AND CONCLUSIONS

Simple Monte Carlo simulations of the backscattered $\vec{E}$-Field of randomly distributed, falling rain drops have been used to study the effects of spectral leakage and the small lines (marked by vertical arrows in fig. 3) when measuring a drop collective with an FM-CW Doppler radar (Rayleigh scattering assumed). These small lines have been neglected in the original literature (Strauch, 1976). In the simulations, all other sources of error (turbulence, echos from side lobes, intermodulation by the mixer, etc.) have been disregarded. Note that turbulence could be easily included via the probability function of turbulence (PFT) since every single drop can be assigned a distinct v_r. The "calibration" of the constant C in (1) was done in a way that the rain rate R in the second height bin equaled the rain rate given by the predefinded drop spectrum.

It has been shown that the number of small drops may be considerably overestimated by an FM-CW Doppler radar in heavy rain. But this problem could be minimized through the use of specific window functions (section 5). A predescribed secondary mode at $D \approx 2$ mm could be resolved reasonably well. Echo statistics as described in Marshall and Hitschfeld (1953) are also applicable to FM-CW–power spectra, which is not presented in this paper but has been derived from the simulations. These results do not depend on a specific radar frequency or a specific range resolution.

The effect of the increasing underestimation of drop concentrations with height has to be checked by further simulations.

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