

# MECHANISM FOR THE GENERATION OF SECONDARY WAVES IN WAVE BREAKING REGIONS

Sharon L. Vadas,<sup>a</sup>  
Co-author: D.C. Fritts  
Colorado Research Associates<sup>b</sup>

## 1. Introduction

It is well known that wave breaking generates secondary waves. During wave breaking in the mesosphere, momentum is deposited, and a 3D, temporally and spatially-localized body force results. It is generally assumed that a body force leads only to the acceleration and establishment of a mean wind and balanced thermal structure. However, a significant amount of energy and momentum fluxes are carried away in the process by secondary waves and can be described by the simple, linear model of Vadas and Fritts (2001, hereafter VF). Because these secondary waves have large intrinsic phase speeds, large vertical wavelengths, and propagate in all directions away from the body force region, they can transport energy and more importantly momentum into the thermosphere. Because wave momentum fluxes per unit mass grow with height due to the atmosphere's decreasing density, the dissipation of these secondary waves in the thermosphere could have appreciable effects that have previously been ignored. Therefore, it is important to understand the role these waves play in the dynamics of our atmosphere.

Here, we only consider a zonal body force with zonal acceleration  $F_x(\mathbf{x})\mathcal{F}(t)$ , where  $F_x(\mathbf{x})$  contains the spatial attributes of the force and  $\mathcal{F} = (1/\sigma)(1 - \cos 2\pi t/\sigma)$  represents a single pulse of momentum that is deposited by the primary wave over the temporal duration  $\sigma$ . For a typical 3D body force created from wave breaking that occurs in a localized region correlated with specific phases of the primary wave, about one-half of the energy spins up the mean response while the other half goes into secondary waves. A simple formula for estimating the fraction of energy in secondary waves for forces with short durations characteristic of wave breaking is (VF)

$$\frac{E_{\text{GW}}}{\bar{E} + E_{\text{GW}}} \simeq \frac{\sigma_x^{-2}}{\sigma_x^{-2} + \sigma_y^{-2}} \quad (1.1)$$

<sup>a</sup>Author address: Colorado Research Associates, 3380 Mitchell Lane, Boulder, CO 80301; email: vasha@colorado-research.com

<sup>b</sup>Northwest Research Assoc. Division

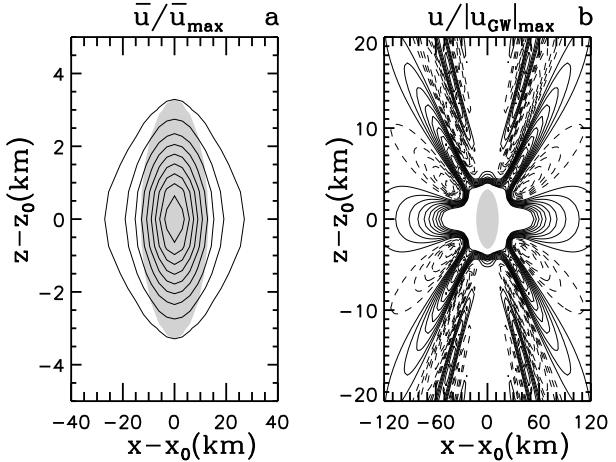


Figure 1: Response to a 3D monopole, body force with  $\sigma_x = \sigma_y = 6$  km,  $\sigma_z = 1.5$  km,  $u_0 = 20$  m s<sup>-1</sup>, and  $N = 0.02$  rad/s. a): Zonal mean wind, scaled by  $\bar{u}_{\text{max}} = 10$  m s<sup>-1</sup>. b): Zonal wind at  $t = 30$  min, scaled by the maximum gravity wave amplitude of 0.38 m s<sup>-1</sup>. The solid and dashed contours indicate 0.1 and -0.1 increments, respectively. The lightly shaded region indicates where the body force is greater than 10% of its maximum value.

for a Gaussian monopole body force, where  $\sigma_x$ ,  $\sigma_y$ , and  $\sigma_z$  are the half widths at half max of the force. The total spatial extents of the body force are approximately  $4.5\sigma_x$ ,  $4.5\sigma_y$ , and  $4.5\sigma_z$  in the  $x$ ,  $y$ , and  $z$  directions, respectively. [Here,  $F_x = u_0 \exp(-0.5 \sum_{i=1}^{i=3} [x_i - x_{i0}]^2 / \sigma_{x_i}^2)$  where  $\mathbf{x} = (x, y, z)$  and  $\mathbf{x}_0 = (x_0, y_0, z_0)$ . The amplitude of the body force is  $u_0$ .] For body forces that are determined by orography, say the Rocky Mountains, the zonal extent is much shorter than the meridional extent, so that nearly all of the energy resides in secondary waves.

In Fig. 1, we show the mean (a) and wave (b) responses to an eastward zonal body force which is 27km wide and 7km deep. The mean zonal wind resembles the body force and is one-half the body force amplitude. In this case, 40% of the total energy is carried away by secondary waves. Fig. 2 shows the absolute value of the momentum fluxes,  $\sqrt{(uw)^2 + (vw)^2}$ , calculated at 10.5km (a) and 31.5km (b) above  $z_0$  from Fig. 1b. The waves radiate out in the shape of a cone, although there is no radiation northward and southward of the force. The light shading indicates the location of waves that propagate from  $\mathbf{x}_0$  with frequency  $\omega_c$ , where  $\omega_c$  is the characteristic source frequency formed

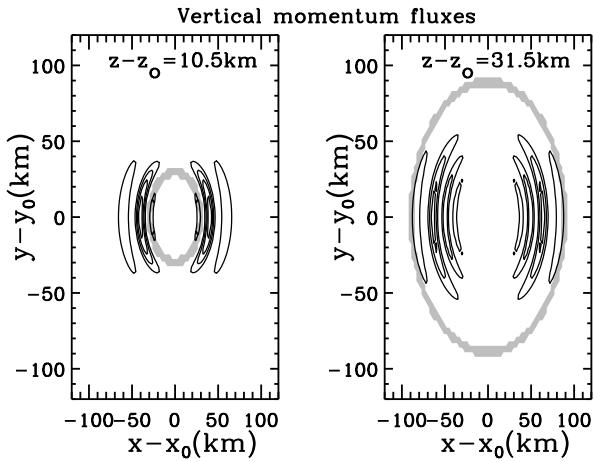


Figure 2: Combined zonal and meridional momentum fluxes calculated from Fig. 1b at a):  $z - z_0 = 10.5\text{km}$ , and a):  $z - z_0 = 31.5\text{km}$ . The contour intervals in (a) are  $0.005\text{m}^2\text{s}^{-2}$  and in (b) are  $0.002\text{m}^2\text{s}^{-2}$ . The light shading shows the location of waves with  $\omega = \omega_c$ .

from the spatial dimensions of the body force:

$$\omega_c = \left[ \frac{(\sigma_x^{-2} + \sigma_y^{-2})N^2 + \sigma_z^{-2}f^2}{\sigma_x^{-2} + \sigma_y^{-2} + \sigma_z^{-2}} \right]^{1/2}. \quad (1.2)$$

Here,  $N$  is buoyancy frequency,  $f = 2\Omega \sin \theta$ ,  $\Omega$  is Earth's rotation frequency, and  $\theta$  is latitude. One can generalize the secondary wave characteristics that result in general (Vadas et. al. 2001, hereafter VAF). As represented in the momentum flux spectra in variance content form, for deep (i.e.,  $\omega_c \gg f$ ) forces with  $\sigma_x \lesssim \sigma_y$ , the peak frequency is  $\omega \simeq (0.3-3)\omega_c$ , the peak zonal and meridional wavelengths are roughly twice the zonal width and twice the meridional width of the body force, respectively, and the peak vertical wavelength is  $\lambda_z \simeq 2-4$  times the vertical depth of the force. In Figs. 1-2, the secondary waves at the peaks of the spectra have horizontal phase speeds of  $c_x = \omega/k \simeq 50\text{ms}^{-1}$ ,  $\lambda_z \simeq 20\text{km}$ , and  $\omega \simeq (0.3 - 0.7)N$ . These however, are only the peak values. Because the Fourier transform of a Gaussian monopole has substantial spectral width, significant amounts of momentum flux reside in the off-peak values, which include larger phase speeds and larger vertical wavelengths. If these waves are created in the upper mesosphere, they can easily propagate into the thermosphere. With no mean winds, their

net effect on the thermosphere would be zero. However, with realistic winds and shears, the intrinsic wave properties of the eastward, westward, northward and southward components would evolve differently as they propagate upwards, and so they would dissipate at differing altitudes. This would create net forcing on the thermosphere. In this case, the integrated momentum fluxes of upgoing waves per body force volume at  $z_0$  is  $3\text{ m}^2\text{s}^{-2}$ . And the total amount of momentum flux in upgoing waves is roughly a percent of the change in momentum flux of the primary breaking wave (VAF). As these waves propagate into the thermosphere, their momentum fluxes per unit mass grow with altitude. For example, if they propagate 70km vertically before dissipating, the momentum fluxes increase by  $\sim \exp(70/7) \sim 2 \times 10^4$  (ignoring dissipation and viscosity effects along the way). Thus, this and other body forces could have a significant effect on the thermosphere where they dissipate.

For Figs. 1-2, we set  $f = 0$  in order to emphasize that rotation is not important for the wave responses to deep, 3D sources with  $\sigma_x \lesssim \sigma_y$ , because the creation of these secondary waves is *not* the geostrophic adjustment process. (For example, no waves are created from zonally-symmetric body forces at the equator.) In fact, changing the latitude of a deep, 3D body force negligibly alters the created momentum flux spectra when (VAF)

$$\sigma_x/\sigma_z \ll \sqrt{1 - \omega_c^2/N^2} N/f. \quad (1.3)$$

We also emphasize that even though the zonal body force in Figs. 1-2 is eastward, north-eastward and southeastward waves are created in equal amounts as northwestward and southwestward waves. This is a consequence of the non-conservation of fluid momentum (even though energy is conserved). And although we only examine zonal body forces here, because rotation is unimportant, we expect comparable meridional wave excitation from meridional body forces for similar spatial scales.

## 2. Importance of temporal variability

In Figs. 1-2, the body force duration is very short:  $\sigma = \tau_c/4 \simeq 4$  min, where  $\tau_c = 2\pi/\omega_c$  is the characteristic period formed from the spatial scales of the body force. Short duration forces occur during wave breaking because the temporal variability of the breaking process may occur on time scales that are a fraction of the period of the primary breaking wave,  $\tau_p$

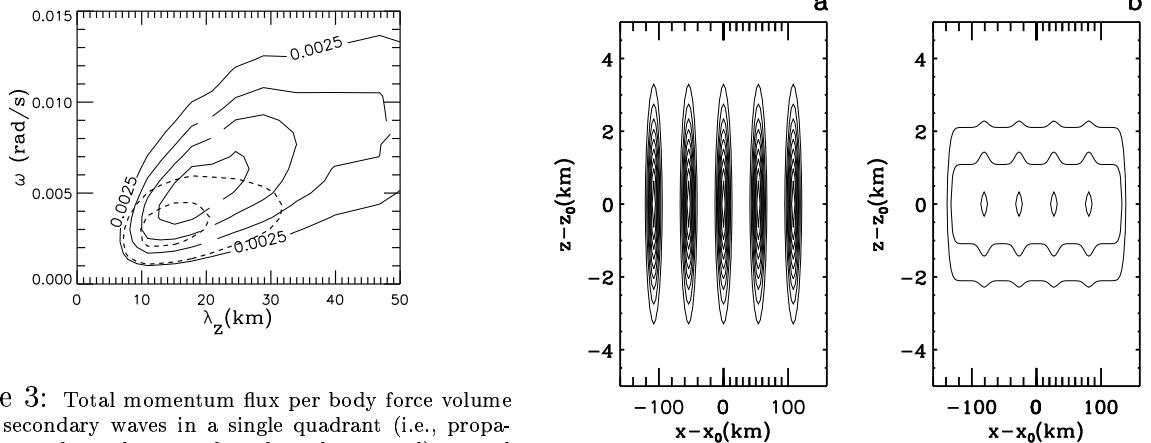


Figure 3: Total momentum flux per body force volume for the secondary waves in a single quadrant (i.e., propagating upward, northeastward, and northwestward) created from 3D monopole, zonal body forces with  $\sigma_x = \sigma_y = 10\text{km}$ ,  $\sigma_z = 2\text{ km}$ ,  $u_0 = 20\text{ m s}^{-1}$ , and  $N = 0.02\text{rad/s}$ . All contours are separated by  $0.0025\text{ m}^2\text{s}^{-2}$ . The solid lines are for  $\sigma = 0.3\tau_c$ , and the dash lines are for  $\sigma = \tau_c$ .

(and it is expected that  $\tau_c \sim \tau_p$ ). However, if one parameterizes the body force that results from wave breaking in the mesosphere as being constant in time (say, as a simplifying assumption in a global model), *no* secondary waves result (VF). The result of this parameterization then, is that only a mean response is created, even though the real process in the mesosphere would create the same mean response *and* significant amounts of secondary waves. Thus, it is important not to neglect the temporal variability of body forces. Fig. 3 shows the effect of lengthening the body force duration somewhat within the natural variability that likely occurs during wave breaking. The solid lines show the momentum fluxes for a body force that is fast, whereas the dash lines show the wave response for a body force which deposits the same amount of momentum but over a somewhat longer amount of time. Although the low-frequency, short  $\lambda_z$  portion of the spectra are the same, the high-frequency, large  $\lambda_z$  portion of the spectrum is removed for the longer duration forcing. This implies decreased momentum fluxes as well; the total, integrated momentum fluxes per body force volume per quadrant are  $1.2\text{m}^2\text{s}^{-2}$  and  $0.34\text{m}^2\text{s}^{-2}$  for  $\sigma = 0.3\tau_c$  and  $\sigma = \tau_c$ , respectively. We note that the spectra from very short duration forces with  $\sigma \ll \tau_c/3$  are virtually identical.

### 3. Importance of spatial variability

For deep, 3D body forces where the horizontal extents are equal,  $\omega_c \simeq \sqrt{2}N\sigma_z/\sigma_x$ ;

Figure 4: (a): Five monopole forces separated by their width. For each individual force,  $\sigma_x = \sigma_y = 6\text{ km}$ ,  $\sigma_z = 1.5\text{ km}$ ,  $u_0 = 20\text{ m s}^{-1}$ , and  $N = 0.02\text{rad/s}$ . (b): The same as in (a), but smoothed in y with a window  $\simeq 60\text{ km}$  wide.

the frequency of the wave response depends on the vertical to horizontal aspect ratio of the body force. And the wavelengths are determined by the spatial extents of the body force (see Sec. 1). Therefore if we horizontally average multiple body force regions (e.g., 3D global models with many degree horizontal resolution due to numerical/computer resource constraints), but retain much of the vertical resolution, then the vertical to horizontal aspect ratio decreases. This results in the creation of lower-frequency, longer horizontal-wavelength waves, rather than the high-frequency, short horizontal-wavelength waves that should be created from this body force. For example, suppose a primary wave with  $\lambda_x \simeq \lambda_y \simeq \text{few}100\text{ km}$  and  $\lambda_z \simeq 20\text{ km}$  breaks in the mesosphere and creates body forces in spatially-localized regions that are a hundred km across and ten km deep. As shown in VAF, as long as these body forces are separated by at least their diameter, the total spectra will be approximately the sum of the individual force spectra. Therefore, this force excites waves with high frequencies:  $\omega \sim N/10$ . If this force is smoothed over a few degrees latitudinally and longitudinally [e.g., by being incorporated into a global, 3D model such as TIME-GCM (Roble and Ridley, 1994)], the resulting secondary wave frequencies are inertial instead:  $\omega \simeq f$ , using Eq. (1.2). Thus, smoothing over

such large horizontal scales creates nearly inertial frequency waves instead of the high-frequency waves that would be created without the smoothing. Because inertia-gravity waves carry very little momentum flux, these waves likely have little impact on the thermosphere in these global models. However, the actual high-frequency, large momentum-flux waves may have a large role in the dynamics of the thermosphere. It would be extremely beneficial to include these high-frequency waves in global models in order to more realistically capture the net effect they have in the thermosphere.

When multiple body forces are smoothed horizontally, not only does the characteristic frequency of the smoothed body force shift to lower frequencies, but also the maximum force amplitude decreases. Because the secondary wave momentum fluxes are proportional to the force amplitude squared, smoothing results in a significant decrease in the total amount of secondary waves that are created from multiple body forces. In Fig. 4a, we show five unsmoothed, 2D, Gaussian, monopole body forces that are separated by their diameter zonally. Fig. 4b is the result of smoothing Fig. 4a zonally over a window approximately 60km wide. The resulting wave spectra are shown in Fig. 5. The solid line shows the momentum flux spectrum from Fig. 4a, while the dash line shows the momentum flux spectrum from Fig. 4b. Although the smoothed spectrum still peaks at high frequencies, the decrease of the force amplitude by a factor of three in this case creates secondary waves with nine times less momentum flux than the unsmoothed spectrum. This has obvious ramifications for the net forcing on the thermosphere. Therefore, it is necessary to retain both the temporal and spatial variability of the body force in order to estimate reasonably well not only the secondary wave frequencies and wavelengths, but also the total amount of secondary wave momentum fluxes.

#### 4. Conclusion and discussion

It is accepted that the breaking of small-scale, high-frequency gravity waves causes the reversal of the jets in the mesosphere by applying an average acceleration of  $\simeq 100\text{m/s/day}$  (Fritts and Vincent, 1987; Reid and Vincent, 1987; Garcia and Solomon, 1985). Because temporal and spatial variability must accompany these processes, in addition to spinning up a mean response, large amounts of secondary waves must also be created through the process discussed here. Because these secondary waves have large intrinsic phase speeds and large ver-

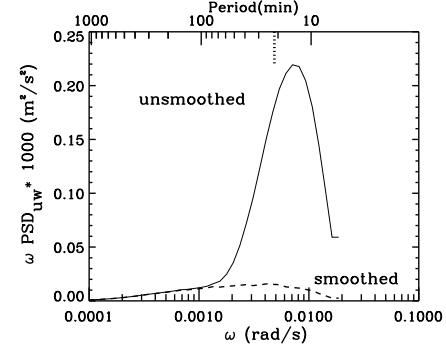


Figure 5: Power spectral density of the momentum flux. The solid line is from Fig. 4a, and the dash line is from Fig. 4b. The dotted line shows where  $\omega = \omega_c$ .

tical wavelengths, they can escape the large mean winds that persist in the upper mesosphere and lower thermosphere, and propagate well into the thermosphere. Upon dissipating there, they would influence the thermosphere to a presently unknown degree. Because the altitudes where these waves dissipate and the associated body forces depend sensitively on the temporal and spatial scales of the body forces, it is important to retain the intermittency information of the wave breaking events (which include the primary wave frequency and wavelengths) in order to estimate reasonably well the spectrum of secondary waves that are created from these events in the mesosphere.

*Acknowledgments.* We deeply thank M.J. Alexander for her important contributions to this work.

Fritts, D.C., and R.A. Vincent, 1987: Mesospheric momentum flux studies at Adelaide, Australia: Observations and a gravity wave/tidal interaction model, *J. Atmos. Sci.*, **44**, 605–619.

Garcia, R.R. and S. Solomon, 1985: The effect of breaking gravity waves on the dynamics and chemical composition of the mesosphere and lower thermosphere, *J. Geophys. Res.*, **90**, 3850–3868.

Reid, I.M. and R.A. Vincent, 1987: Measurements of mesospheric gravity wave momentum fluxes and mean flow accelerations at Adelaide, Australia, *J. Atmos. Terres. Phys.*, **49**, 443–460.

Roble, R.G. and E.C. Ridley, 1994: A thermosphere-ionosphere-mesosphere-electrodynamics general circulation model (TIME-GCM): equinox solar cycle minimum simulations (30-500km), *Geophys. Res. Lett.*, **21**, 417–420.

Vadas, S. L. and D. C. Fritts, 2001: “Gravity wave radiation and mean responses to local body forces in the atmosphere”, *JAS*, in press.

Vadas, S. L., M.J. Alexander, and D. C. Fritts, 2001: “Mechanism for the generation of secondary waves in wave breaking regions”, in preparation.