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NONMODAL GROWTH IN SPHERICAL GEOMETRY

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1. INTRODUCTION

The existence of growth rates in excess of the normal mode growth rate is evidence of nonmodal growth (NG). A great number of studies have examined NG in the quasi-geostrophic (QG) framework, however few if any of them have dealt with the subject in the context of spherical geometry. In this study, spherical coordinate results are compared to Cartesian coordinate results from corresponding linear QG models. We also examine how NG varies with the length scale of the initial perturbation using several global parameters.

2. MODEL DESCRIPTION

The model's governing equation describes the conservation of quasi-geostrophic potential vorticity (QGPV); this is solved as an initial value problem. Diagnostic quantities such as the L2 norm (L2), kinetic energy (KE), available potential energy (APE), potential enstrophy (H), and Total Energy (TE) are calculated from the perturbation quantities at each time step. These perturbation patterns are overlaid upon a background state. The model is linearized about a background state consisting of a zonal flow, varying in the vertical and meridional, and a vertical temperature profile, which stays constant in the horizontal.

The model uses a single governing equation which expresses the conservation of QGPV. The governing equation is

$$\frac{\partial q}{\partial t} = \frac{\partial \overline{\psi}}{\partial \mu}\frac{\partial q}{\partial \lambda} - \frac{\partial \psi}{\partial \lambda}\frac{\partial \overline{q}}{\partial \mu} + \frac{\partial \psi}{\partial \lambda}\left(8\mu \frac{\partial}{\partial p}\left(\frac{p}{\sigma}\frac{\partial \overline{\psi}}{\partial p}\right)\right)$$

where

q : perturbation QGPV
$\overline{q}$: basic state QGPV
ψ : perturbation streamfunction
$\overline{\psi}$: basic state streamfunction
μ : sine of latitude
λ : longitude
p : pressure
σ : static stability parameter

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It should be noted that μ is not a constant and changes with latitude, and hence this is one major difference between this model and most Cartesian QG models

The derivation of the governing equation followed the steps outlined in Hollingsworth, et. al. (1976).

Eigenvalue and initial value versions of the model were developed. The model uses a parallelogrammic truncation scheme, keeping 15 zonal wavenumbers and 30 meridional nodes (P15/30 truncation). Eight model levels were kept. The basic state zonal wind used in the model was a 30 degree jet, described in Simmons and Hoskins (1976) and Frederiksen (1978).

The model was tested against previously published results from similar models. Despite minor differences in the handling of the vertical structures of the models, the normal mode growth rate results obtained from the QG model described here were in close agreement with those published by Frederiksen (1978). The fastest growing normal modes occurred at zonal wavenumbers 8 and 9, both of which showed growth rates very near 0.6/day.

3. RESULTS

An examination of time series of growth rates for L2, KE, APE, H, and TE can illustrate situations where NG is occurring. We used the model to investigate NG in these quantities. Two general types of initial conditions (IC) were used: 1. a "connected" IC, which has been found to develop large NG in Cartesian models; and 2. a "separated" IC, which is similar to observed atmospheric conditions prior to cyclogenesis. The model was run for ten days using various variations on the two IC's (by varying amount the amount of vertical tilt).

Our results in spherical coordinates (Fig. 1) are broadly similar to our prior findings in Cartesian geometry. The connected IC has much more NG, especially for very short wavenumbers. In all cases studied the structure is essentially the most unstable normal mode within 3 days. Varying the upstream shift (between upper and lower trough locations) affects some properties within the first 3 days. For the connected IC, peak L2 growth rate occurs quickly (within 6 hours) and is largest in magnitude for zonal wavenumber >20. For the separated IC, excess growth does not occur, even for very short wavelengths. In fact, there is significant negative growth rate near the start. In Cartesian geometry, this was attributed to a large initial projection onto

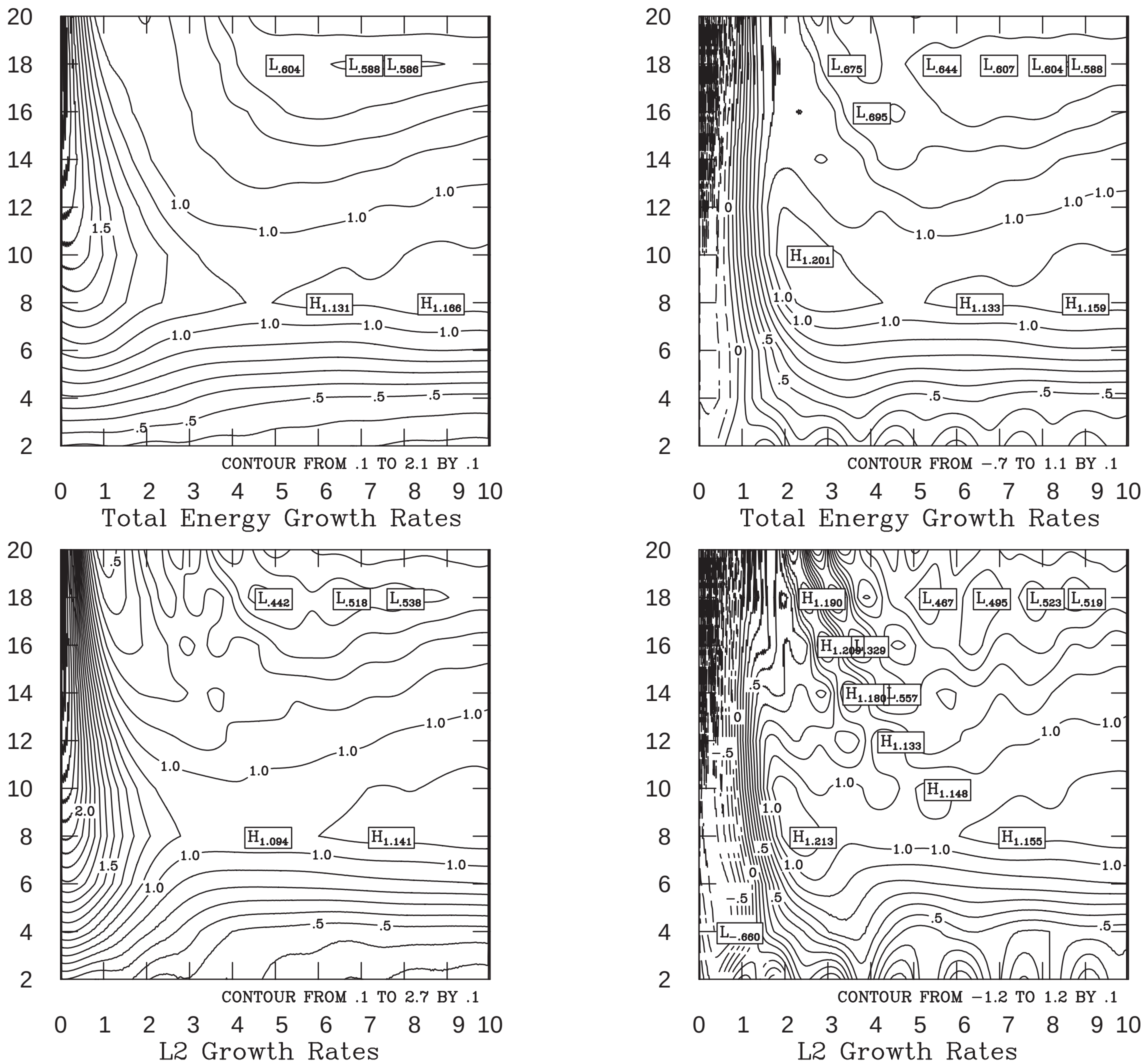

Fig. 1. Growth rates time series for connected trough IC (left column) and a separated trough IC (right column). Top row is TE, bottom row L2. Ordinate is zonal wavenumber; abscissa is time in days.

rapidly decaying normal modes. KE growth rates are precisely twice the L2 growth rates, but peak TE growth rates are much smaller than for L2. The reduction is caused by modest APE growth rates and APE is a larger fraction of TE than is KE at the start. APE develops its peak growth rates much later (12-48 hours after the start depending on the case). Peak growth rates of H occur at zonal wavenumbers between 10 and 20 and for both connected and separated IC. Prior work in Cartesian geometry proved that strong NG occurred in H for the separated IC due to differences between the boundary QGPV (BPV) in the initial state versus the asymptotic normal mode state. Subtle differences in initial structure made large changes in the amount of NG in BPV growth rate. Work in Cartesian geometry showed how the emergence of the more unstable normal modes could lead to horizontal tilts even though the basic flow has no horizontal shear. The tilts could be against or with the shear. Here, we have a strong jet and the tilts are with the shear.

4. REFERENCES

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