ON THE SELF-ORGANISATION OF BAROCLINIC WAVES INTO WAVE PACKETS

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Introduction

One-dimensional nonlinear wave systems in fluids often obey a dispersion relation of the form $\omega = \omega(k, \mathcal{A})$ that has a functional dependence similar to that shown schematically in Figure 1. Here $\mathcal{A}$ is typically some quadratic measure of wave amplitude a (e.g. $\mathcal{A} = a^2$). This is usually because the waves impart some $O(a^2)$ reversible change to the background flow, and this change to the flow in turn affects the dispersion properties of the waves.

If, as is typical, the background flow change is stabilising there will be a finite amplitude uniform wavetrain solution (i.e. with ω_i=0). By assuming the wave system is only weakly unstable, and can therefore be approximated by a Ginzburg-Landau expansion, Newell (1974) showed that this uniform wavetrain is unstable to sideband perturbation if and only if

$$Y = -\left(1 + \frac{\omega_{rkk}\ \omega_{r\mathcal{A}}}{\omega_{ikk}\ \omega_{i\mathcal{A}}}\right) > 0.$$

The growth of an unstable sideband can be thought of as the breakup of the uniform wavetrain into wave packets.

It can be further shown that the length scale of the most unstable sideband is given by

$$L_P = \frac{2\pi}{q_0}\left[\frac{D(D+1)}{(1+Y)(1+D) - \left[(1+Y)^2 + D\right]^{1/2}(1+D)^{1/2}}\right]^{1/2},$$

where q_0 is the width of the band of unstable wavenumbers in Figure 1, and $D = (\omega_{rkk}/\omega_{ikk})^2$. In nonlinear integrations of the Ginzburg-Landau equation wave packets are seen to emerge with length scales proportional to L_P.

In what follows we shall introduce a model of baroclinic waves that is entirely linear except for a simple representation of wave-mean flow interaction. Its behaviour can be compared directly with the nonlinear one-dimensional wave systems described above.

The Model

The model is adapted from the Phillips' two-layer model which describes quasi-geostrophic flow in a channel that is periodic in the x-direction with dimension L_x and bounded by sidewalls at $y = 0, L_y$. Details of the nondimensionalisation are given in Lee and Held (1993), and we follow the conventions established there. The equations we use are

$$\frac{\partial q_i}{\partial t} + J(\phi_i, Q_i) + J(\Phi_i, q_i) = d_i, \qquad i = 1, 2.$$

Here q_i and ϕ_i are the linearised perturbation potential vorticity (PV) and streamfunction in layer i, and Q_i and Φ_i are the background PV and streamfunction. Q_i and Φ_i include the β-effect, a uniform mean flow component, and a mean flow modification that is proportional to the local amplitude squared of the linear wave streamfunction ($\overline{\phi_1}^2$). Similarities exist between this model and the weakly nonlinear formulation of Esler (1997).

The form of the mean flow modification is varied between each experiment, with the different 'structure functions' used shown in Figure 2. These show the changes to Q_{iy} and Φ_{iy} in each layer as a function of latitude.

Results

Figure 3 shows snapshots of ϕ_{1x} after a long time in the four model experiments with the four different 'structure functions' A-D used in each case. Despite being apparently similar, each experiment generates very different wave packet behaviour, from a stable uniform wavetrain in (A) to isolated wave packets in (D). Estimating approximate values of Y for each structure function gives $Y = -0.693, 0.373, 1.823$, and 5.066 ($D = 2.384$), which is consistent with a stable uniform wavetrain in (A). The length scales of the wave packets in (B-D) can be shown to be approximately proportional to L_P.

Similar considerations can be shown to apply in more realistic turbulent flows, despite much more complex wave-mean flow interaction taking place.

References

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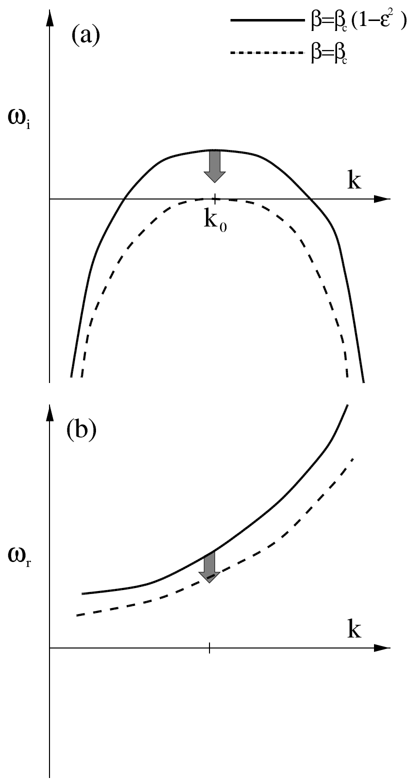

Figure 1: Schematic illustrating the complex frequency-wavenumber relationship for a typical weakly unstable, dispersive-diffusive, one-dimensional system. (a) Imaginary frequency ω_i. (b) Real frequency ω_r. The dotted curves illustrate the situation at marginal stability ($\beta = \beta_c$) and the solid curves the unstable situation when $\beta = \beta_c(1-\epsilon^2)$. The large arrows illustrate the direction in which the curves move with increasing wave amplitude, due to the effects of nonlinearity.

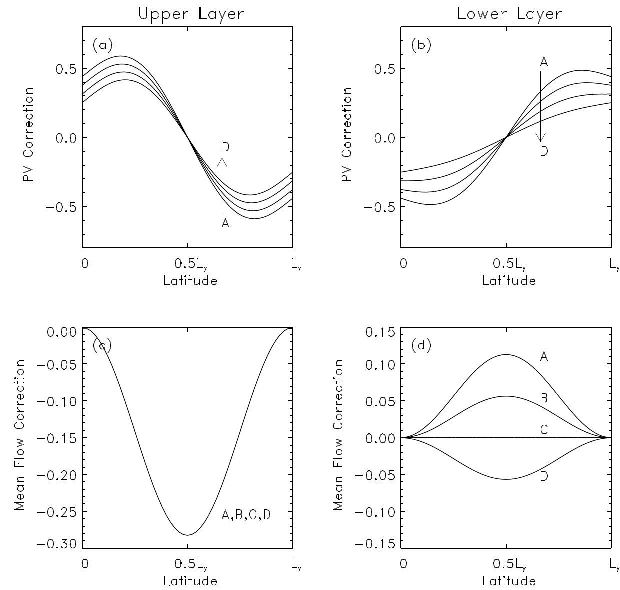

Figure 2: (a) Showing the latitudinal structure $\mathcal{Q}_1^R(y)$ of the PV change due to the waves in the upper layer, for all four of the different mean flow responses used in the numerical experiments, which are labelled A-D. (b) As (a) but for the lower layer $\mathcal{Q}_2^R(y)$. (c) Showing the upper layer zonal mean flow change induced by a uniform wavetrain, for mean flow responses A-D. (d) Showing the lower layer zonal mean flow change induced by a uniform wave-train, for mean flow responses A-D. The units are essentially arbitrary except for the purposes of comparison between the two layers.

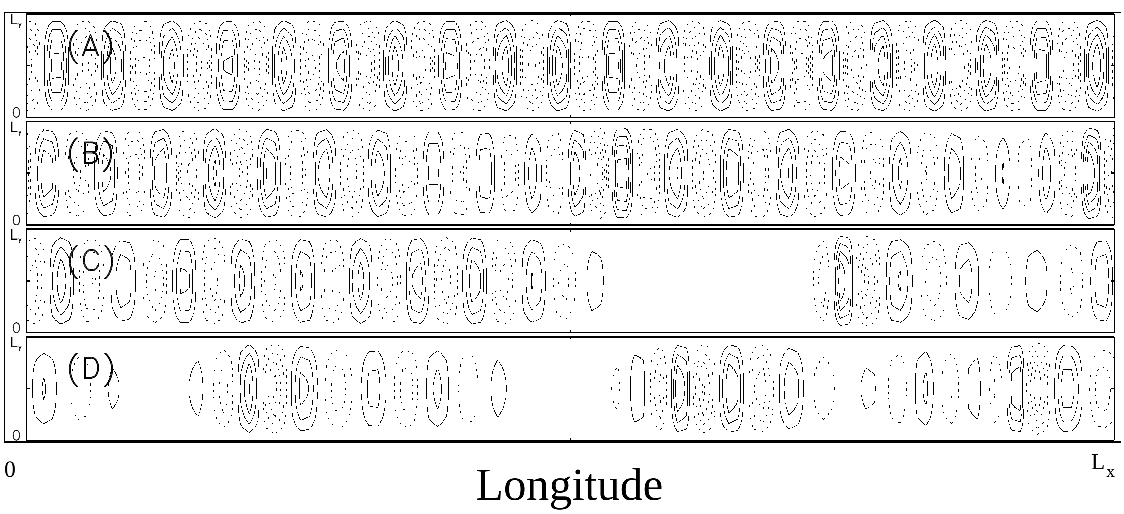

Figure 3: Contour plots showing snapshots at long times from the linear wave -mean flow interaction numerical experiments, for the four different, specified mean flow response structures A, B, C and D (see Figure 2). These correspond to estimated Y values $Y =$ -.693, 2.373, 1.ff23 and 5.066. Contour intervals are arbitrary (see discussion in text). β is fixed at 0.455 for each experiment ($\epsilon = 0.413$).