P13.6 COMPARISON OF ANALYTICAL APPROXIMATIONS OF THE SHAPES OF THE CLOUD DROPSIZE DISTRIBUTIONS, UTILIZED FOR THE INTERPRETATION OF REMOTE SENSING DATA.

Oleg A. Krasnov * and Herman W.J. Russchenberg
International Research Centre for Telecommunications-transmission and Radar (IRCTR),
Faculty of Information Technology and Systems, Delft University of Technology

1.INTRODUCTION

The *dropsize distribution* (DSD) $N(D)$ is fundamental to the description the microphysics of hydrometeors, especially with respect to remote sensing applications. Traditionally, it is defined as the quantity $N(D)dD$ – the mean number of hydrometeor's particles with diameters between D and $D+dD$ present per unit volume of air. In this definition two different concepts are mixed – the spatial distribution of drops in a volume of air and the probability distribution of their sizes. Usually, when the DSD is analyzed, the hypotheses about spatial homogeneity and temporal stationarity of cloud microphysical processes are supposed to b valid, at least for analyzed scales. This simplifying (and questionable, see Kostinski and Jameson (2000)) assumption allows to concentrate attention on statistical behavior of drop sizes.

It is not difficult to see (Maisel, (1971), p.20) that the DSD has mathematical behavior similar to the *probability density function* (PDF). There is only one difference: the integral of a PDF equals 1, while that for a DSD gives the particle number concentration. This similarity means that it is possible to use powerful and well-developed statistical methods for the estimation of shape and parameters of the DSD. It suggests to use for the parameterization of cloud dropsize distribution, one of standard statistical PDF with the parameters depending of cloud type and surrounding meteorological conditions. Choosing an analytical form of PDF among so many other ones requires to take into account some of its peculiarities. The main natural feature of the DSD is that it's bounded from below by zero value. From experimental observations, it is also known that a DSD usually has a positive asymmetry. Both of these features also occur with gamma and log-normal distributions. As a result, they are widely used in cloud remote sensing researches and very often have been considered to be equivalent. In present work we try to estimate limits for which this equivalency is rightful.

2.THE MATHEMATICAL FORMULATIONS

We used the follow mathematical definitions: for gamma DSD:

$$N(D) = \frac{N_0}{D_{m,gam}^{\nu} \cdot \Gamma(\nu)} \cdot D^{\nu-1} \cdot e^{-\frac{D}{D_{m,gam}}},\ D \geq 0,$$

Corresponding author address: Oleg A. Krasnov, IRCTR-TUDelft, Mekelweg 4, 2628 CD Delft, The Netherlands. e-mail: o.krasnov@its.tudelft.nl

where $\Gamma(\nu)$ - gamma function, ν and $D_{m,gam}$ - shape and scale parameters of gamma distribution. And for log-normal DSD:

$$N(D) = \frac{N_0}{D \cdot \sigma_{\log} \cdot \sqrt{2 \cdot \pi}} \cdot e^{-\frac{\left[\ln(D/D_{m,log})\right]^2}{2 \cdot \sigma_{\log}^2}},\ D \geq 0,$$

where $\sigma_{\log}$ and $D_{m,log}$ - shape and scale parameters. In both equations, N_0 - total concentration of drops per unit of volume. It is simple to show that the n^{th} order non-central moment for gamma-distribution can be expressed in term of DSD parameters and first moment (mean size of drops) as:

$$m_n = D_{m,\,gam}^n \cdot \prod_{i=1}^{n} (\nu + i - 1) = = m_{1,\,\mathrm{gam}}^n \cdot \prod_{i=1}^{n-1} \left(1 + \frac{i}{\nu}\right),$$

and for log-normal distribution:

$$m_n = D_{m,\,log}^n \cdot exp\left(0.5 \cdot n^2 \cdot \sigma_{\log}^2\right) = = m_{1,\,\log}^n \cdot exp\left(0.5 \cdot (n-1) \cdot n \cdot \sigma_{\log}^2\right).$$

It is clear that mathematically these distributions and behavior of their moments are very different and it is necessary to estimate the influence of this difference when such distributions are used in remote sensing applications.

The remote sensing instruments are able to measure only integral parameters of drop size distributions: mean size m_1, effective radius $r_e = m_3/m_2$, second moment m_2 that is proportional to optical extinction, 3rd moment m_3 that is proportional to liquid water content (LWC), and 6th moment m_6 that is proportional to radar reflectivity. The shape of the DSD can be measured only with in-situ probes and mathematically can be described using an unlimited set of moments. When measured data is parametrized with analytical approximations of DSD (like gamma or log-normal PDF), only limited number of moments (usually two with order up to 3) are used for estimation of their shape and scale parameters. It is necessary to check the equivalence of other moments for these analytical approximations for different methods of their parameters estimation.

3. COMPARISON OF MOMENTS FOR GAMMA AND LOG-NORMAL DISTRIBUTIONS

3.1. The statistical approach for moments estimation

The gamma-distribution parameters can be estimated as follows:

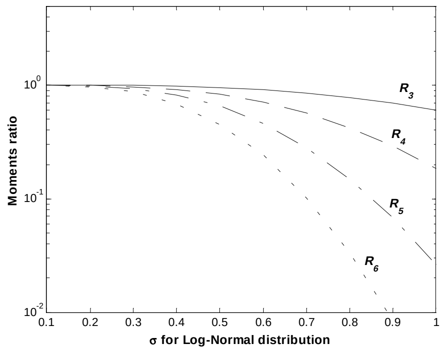

Fig.1. The gamma-to-log-norm moments ratios as functions of log-normal distribution shape parameter. For estimation of the gamma distribution parameters were used 1st and 2nd moments.

$$D_m = \frac{m_2 - m_1^2}{m_1} \quad , \qquad \nu = \frac{m_1^2}{m_2 - m_1^2} \quad ,$$

where m_1 - first (the mean) and m_2 - second moments. If substituted in these expressions, the moments of log-normal distribution, the result shows the relation between parameters of log-normal and gamma distributions:

$$\nu = (e^{\sigma_{\log}^2} - 1)^{-1} \; ,$$

$$D_{m,gam} = D_{n,log} \cdot e^{0.5 \cdot \sigma_{\log}^2} \, (e^{\sigma_{\log}^2} - 1) \; .$$

and the ratio of non-central statistical moments can be expressed as:

$$R_n(\sigma_{\log}) = \frac{m_{n,\,gam}}{m_{n,log}} = \frac{\prod_{i=1}^{n} \left[(2-i) + (i-1) \cdot e^{\sigma_{\log}^2} \right]}{exp[\, 0.5 \cdot (n-1) \cdot n \cdot \sigma_{\log}^2 \,]} \; .$$

The analysis of this expression show that only for n = 1 and n = 2 R_n = 1. The dependencies $R_n(\sigma)$ for n = 3..6 are shown in Fig.1. It can be seen that equality of first two moments of gamma and log-normal distributions doesn't mean equivalence of any other moments. Quite important for remote sensing applications is the 6th moment (this moment is directly proportional to radar reflectivity). The difference can amount to 20 dB and more.

3.2. The estimation of DSD parameters using 1st and 3rd moments

Miles et al., (2000) developed an another method for estimation of parameters for gamma and log-normal approximations of DSD. This method suggests a priori knowledge of concentration, mean (or effective) diameter of drops and liquid water content. In our context it means the estimation of PDF parameters using m_1 and m_3 moments. Using our designations, equations for this method can be rewritten: for gamma DSD

$$\nu = \frac{3 + \sqrt{1 + 8 \cdot Q}}{2 \cdot (Q - 1)} \quad , \qquad D_{m,gam} = \frac{m_1}{\nu} \quad ,$$

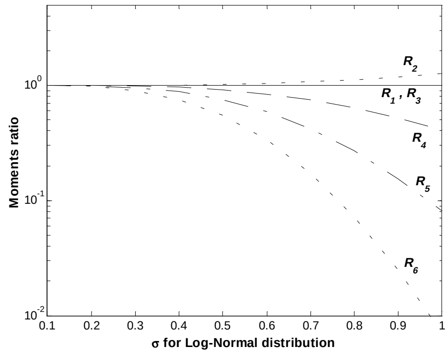

Fig.2. The gamma-to-log-norm moments ratios as functions of log-normal distribution shape parameter. For estimation of the gamma distribution parameters were used 1st and 3rd moments.

and for log-normal DSD:

$$\sigma_{\log} = \sqrt{\frac{1}{3} \cdot \ln Q} \; , \quad D_{m,log} = m_1 \cdot e^{-0.5 \cdot \sigma_{\log}^2} \; ,$$

where $Q = m_3 / m_1^3$. The ratio of non-central statistical moments estimated using this method can be written:

$$R_n(\sigma_{\log}) = \frac{m_{n,\,gam}}{m_{n,\,log}} = \frac{\prod_{i=1}^{n} \left[1 + \frac{i-1}{\nu(\sigma_{\log})} \right]}{exp[\, 0.5 \cdot (n-1) \cdot n \cdot \sigma_{\log}^2 \,]} \; .$$

The dependencies $R_n(\sigma)$ for n up to 6 are shown on the Fig.2. It can be seen that $R_1 \equiv R_3 \equiv 1$, but for other moments big variations of their ratios also take place.

3.3. Examination of real cloud data

Above was theoretically estimated, the non-equivalence of gamma and log-normal distributions from the viewpoint of difference between their high order moments. For the examination of quantitative importance of this effect for real DSD data, we analyzed the gamma and log-normal approximations of real DSD that was collected from different sources, calculated and published in Miles et al. (2000). For example, in Fig.3 is presented the histogram of the ratio between reflectivities, calculated for gamma and

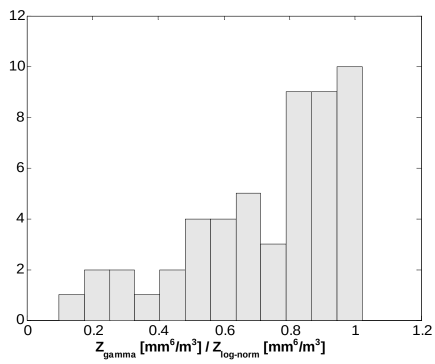

Fig.3. The histogram of the ratio between reflectivities, calculated for gamma and log-normal approximation of continental stratocumulus DSD.

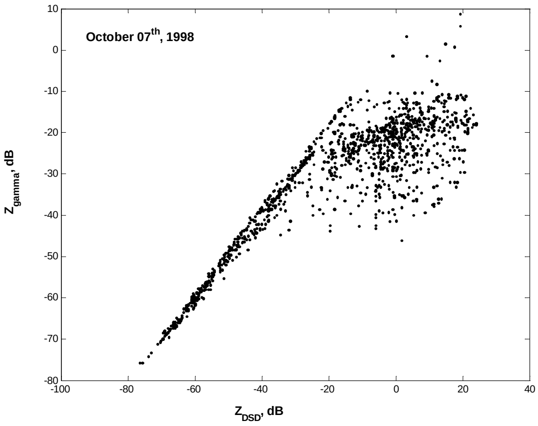

Fig.4. The relation between cloud's reflectivity, calculated using in-situ DSD and it's gamma approximation

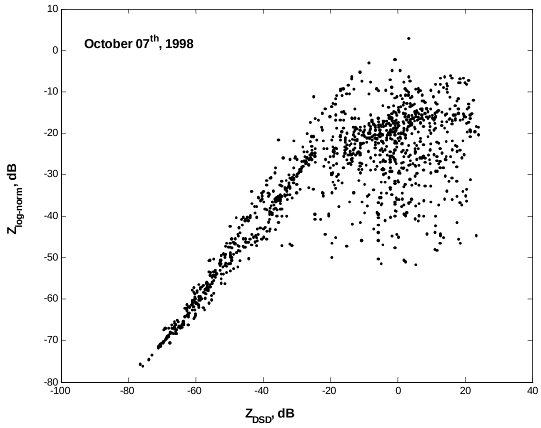

Fig.5. The relation between cloud's reflectivity, calculated using in-situ DSD and it's log-normal approximation

log-normal approximations of continental stratocumulus DSD. It can be seen that using the described approximations without checking the equality of high moments, it is possible to have errors in estimation of calculated reflectivity up to 10 dB.

4. THE APPLICABILITY OF ANALYTICAL APPROXIMATIONS FOR REAL CLOUD DSD

Quite important for remote sensing application is the question: which analytical distribution – gamma or log-normal is better for approximation of cloud DSD. We have shown that they are different, but which is preferable to use for description of real data? We have tried to answer this using water cloud in situ data obtained during the CLARE'98 campaign (ESA, 1999). For calibrated and merged FSSP and 2-DC spectrums, parameters of gamma and log-normal approximations of DSD were estimated, using the Miles et al., (2000) algorithm. For these approximations, the reflectivities Z_{gam} and Z_{log} were calculated. The results were presented on scatter diagrams "Z_{gam} versus Z_{DSD}" and "Z_{log} versus Z_{DSD}" (Fig. 4 and 5). It can be seen, that for a reflectivity up to –20 dBZ, gamma and log-normal distributions are in good agreement with real data (gamma distribution is a little better). For observed reflectivity more then –20 dB both approximation do not have good agreement with real data and very seriously underestimate the reflectivity. This effect can be explained as influence of drizzle particles that increase drops concentration in the tail area of distribution. Mathematically this effect can be described with bimodal distributions. All measured DSD that do not satisfy the criteria of distribution-unimodality (Kendall, 1994)

$$\frac{mean(x) - mode(x)}{std(x)} \le \sqrt{3} \ ,$$

are present in this area (Z_{DSD} > -20dB). Our observations of the shape of the distributions show that this criterion is not sufficient for the detection of bi- or multi- modal distributions, but most of all distributions with additional maximum(s) in the tail area have reflectivity more than –20 dB.

5. CONCLUSIONS

The representations of the cloud dropsize distributions by gamma and log-normal distributions are not equivalent. Although these approximations can have equal first moments, the 6^{th} moment that correspond with radar reflectivity, can show very big differences (up to ± 10 dB). It is necessary to be very careful with this approximations for DSD's which have reflectivity more than –20 dB. The shapes of such distributions can statistically only be described using complex representations with multi-parameters distributions (like Pearson's or Johnson's) or mixture of some distributions.

6. ACKNOWLEDGMENTS

The data used in this paper was collected during the CLARE'98 campaign carried out under the auspices of the European Space Agency (ESA, 1999). The FSSP and 2-DCP data sets were kindly provided by P.Francis from the U.K. Met. Office. This research received funding from SRON project EO – 035.

7. REFERENCES

ESA, 1999: CLARE'98 Cloud Lidar And Radar Experiment, International Workshop Proceedings. WPP - 170, ISSN 1022-6556, ESTEC, Noordwijk, the Netherlands, 239 pp.

Kendall, M., 1994: Kendall's advanced theory of statistics. Vol.1. Distribution theory. - London, Arnold, 676 pp.

Kostinski, A. B., and A. R. Jameson, 2000: On the Spatial Distribution of Cloud Particles, J. Atmos. Sci., 57, 7, pp.901-915.

Maisel, L., 1971: Probability, Statistics and Random Processes. - Simon and Schuster, NY, - 280 pp.

Miles, N. L., J. Verlinde, and E. E. Clothiaux, 2000: Cloud Droplet Size Distributions in Low - Level Stratiform Clouds. J. Atmos. Sci., 57, 295-311.