P14.2 ANALYSIS OF CONDITIONAL BIAS IN RADAR RAINFALL PRODUCTS

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1. Introduction

The aim in radar rainfall (RR) estimation is to produce approximations of rainfall that are as close to the physical truth as possible. However, the question of how to rigorously formalize the requirement of "closeness to the truth" has no unambiguous answer. Ciach *et al.* (2000) discuss two RR estimation performance measures that contradict each other. One is the commonly-used mean square estimation error (*MSE*) and the other is a conditional bias.

The conditional bias is recognized in the statistical literature on the error-in-variable problem (Carroll *et al.* 1995) as an "attenuation" effect. It appears when predictors are corrupted with errors and the predictive relationship is built using least square regression. The resulting statistical predictions systematically underestimate higher range and overestimate lower range of the outcomes. Rosenfeld and Amitai (1998) discuss the conditional bias in the RR context and claim that their Window Probability Matching Method removes it. The studies by Ciach *et al.* (2000) and by Ciach and Krajewski (1999) do not confirm this assertion.

The conditional bias is just an example of a larger problem of comprehensive and conclusive evaluation of the remote sensing rainfall products. Following the verification techniques developed for the weather forecasts (Katz and Murphy 1997), one can distinguish several quality attributes that should be utilized in rationally based validation and intercomparison projects. Below, we only briefly outline this broader context and focus on the opposition of the conditional bias and the *MSE* criteria.

2. Verification of RR products

Quantitative assessment of RR estimates is a complex task. An efficient methodology should be able to describe the quality of given products in a few informative numbers. At present, the technique that seems the closest to this ideal can be based on an approach called "distribution-oriented" verification (Murphy 1997). It derives all the quality information from the verification distribution of RR and true rainfall:

$$(R_r, R_t) \sim f_{rt}(r_r, r_t), \quad (1)$$

where the symbol "~" denotes relation "distributed as", $f_{rt}(\cdot,\cdot)$ is a bivariate probability density function (*pdf*), R_r and R_t are the radar and true rainfall random variables, and r_r and r_t are their specific values, respectively. The basic prerequisite to use this framework is that, for a specified spatio-temporal domain, the distribution of these corresponding (concurrent and collocated) values is either known with sufficient accuracy, or can be retrieved from the observational data. The performance criteria are rigorously defined in terms of this constituting distribution. We now define several measures that seem to be the most important for the verification of RR products.

1) *Overall bias* measures systematic overestimation or underestimation of the RR products in comparison to the truth. Due to multiplicative character of most of this bias sources, it is best expressed as the following factor:

$$B_o = \mathbf{E}\{R_r\} / \mathbf{E}\{R_t\}, \quad (2)$$

where $\mathbf{E}\{\cdot\}$ is the operator of marginal expectation of the bivariate verification *pdf* (1).

2) *Association* is usually defined as a degree of statistical dependency between two random variables and its most popular measure is the correlation coefficient. More general measures are also known, but have not been applied to our area yet.

3) *Accuracy* quantifies the average degree of correspondence between individual pairs of R_r and R_t. The most popular measure of accuracy is:

$$MSE = \mathbf{E}\{(R_r - R_t)^2\}, \quad (3)$$

and minimizing of the *MSE* is a central part of the most common least-square regression techniques.

4) Conditional statistics are used to quantify behavior of one variable at fixed values of the other. The global measures of two types of the *conditional biases* can be defined:

$$CB_1 = \mathbf{E}_r\{ (\mathbf{E}_t\{R_t \,|\, R_r\} - R_r)^2 \}, \quad (4a)$$

$$CB_2 = \mathbf{E}_t\{ (\mathbf{E}_r\{R_r \,|\, R_t\} - R_t)^2 \}, \quad (4b)$$

generally called *type 1 conditional bias* and *type 2 conditional bias*, respectively. In the literature, the CB_1 is also called "reliability" or "calibration" (Murphy 1997), whereas the CB_2 has no such special name.

5) *Conditional variances* quantify the average scatter between R_r and R_t in situations when one of the variables is fixed. Similarly to the conditional biases, there are two types of these statistics:

$$CV_1 = \mathbf{E}_r\{ \mathbf{V}_t\{R_t \,|\, R_r{=}r_r\} \}, \quad (5a)$$

$$CV_2 = \mathbf{E}_t\{ \mathbf{V}_r\{R_r \,|\, R_t{=}r_t\} \}. \quad (5b)$$

where $\mathbf{V}\{\cdot|\cdot\}$ is the operator of conditional variance of the bivariate verification distribution (1).

There are several interrelations between these seven performance measures. Some of them are still not fully understood and ask for more research. The CB_2 describes average differences between a true rainfall value and the conditional expectation of the RR conditioned on this true value. It is compared with the *MSE* in Section 4.

3. Mathematical framework

The conceptual model that we use below is discussed in detail in Ciach and Krajewski (1999) and in Ciach *et al.* (2000).

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The functional dependency between true rainrates (R_t) and radar reflectivities (Z_t) is a typical power law:

$$Z_t = A R_t^b, \quad (6)$$

and the measured reflectivities Z_m are corrupted with errors:

$$Z_m = Z_t E_z, \quad (7)$$

where E_z describes the measurement uncertainties. We define R_t and E_z as independent and lognormally distributed random variables with their standard deviations equal to s_r and s_e, respectively. The RR estimation is based on a conversion of the measured reflectivities Z_m into estimated rain-rates R_r:

$$R_r = a Z_m^{1/b}. \quad (8)$$

Substituting (6) and (7), we express the RR estimates as a function of the true rainfall and the error factor:

$$R_r = c R_t^{b/b} E_z^{1/b}. \quad (9)$$

If the multiplier a in (8) is adjusted so that the overall bias is removed ($B_o=1$), then the exponent ratio b/b in (9) governs the way the estimates R_r are related to different intensities R_t of the true rain-rates.

4. Comparison of *CB₂* and *MSE*

To compare the behavior of the CB_2 (4b) with the MSE (3), we express them as functions of the Z_m-R_r exponent and other parameters of our model. Using (9) and the definitions of the independent variables, we get:

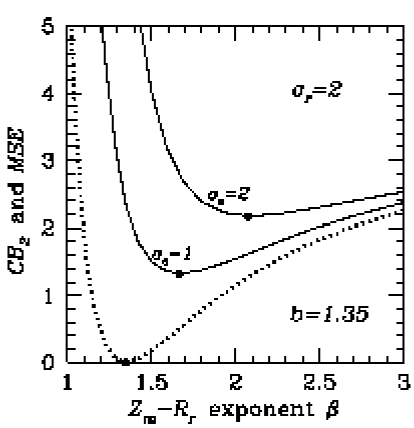
$$MSE(b) = D_r^{b^2/b^2} D_e^{1/b^2} - 2D_r^{b/b} + D_r, \quad (10a)$$

$$CB_2(b) = D_r^{b^2/b^2} - 2D_r^{b/b} + D_r, \quad (10b)$$

where $D_r=s_r^2+1$ and $D_e=s_e^2+1$. These functions are nonnegative and have single minima. For large values of b, they are both equal to s_r^2. In this case, the RR estimates are equal to the climatological average rain-rate and the estimation error is the same as the rain-rater variance. For small values of b, both functions go to infinity. However, the values of b minimizing the MSE and CB_2 differ significantly. The MSE has its minimum at:

$$b_{ms} = b\,[1 + b^{-2} \ln(D_e) / \ln(D_r)], \quad (11)$$

whereas $CB_2(b)=0$. The Figure below presents an example of the MSE and CB_2 in function of b for a system with $b=1.35$ and $s_r=2$.



Two cases of the radar reflectivity error E_z are shown: $s_e=1$ and $s_e=2$.

Note that the b_{ms} is always larger than b. For a given precipitation system defined by b and D_r, the difference between b_{ms} and b depends on the reflectivity measurement error. Thus, due to the inevitable radar reflectivity uncertainties, one cannot simultaneously optimize the MSE and CB_2 criteria. Minimization of the MSE (the most common Z_m-R_r optimization method) leads to substantial *conditional bias type 2*.

5. Conclusions

Expression (9) shows that non-zero CB_2 indicates that the RR estimates are not linearly related to the corresponding true rain-rates. For example, for $b>b$, the "attenuation" effect occurs resulting in underestimation of heavy rain-rates (compensated by overestimation in the range of weak rain-rates). Such a systematic misrepresentation of the true rainfall is certainly an undesirable effect and one would like to have it removed from the RR products. However, it can only be done at the cost of the increased MSE.

Choosing between the two estimation strategies discussed above (minimizing the MSE, or zeroing the CB_2) depends probably on a specific application of the RR products and has to be studied in an application-oriented setup. Our research tries to lead in this direction through better understanding and more rigorous definitions of the RR validation criteria and estimation procedures.

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6. References

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