# **7A.5-**THE NORMALIZED INTERCEPT PARAMETER $N_0^*$ TO DESCRIBE THE VARIABILITY OF THE PARTICLE SIZE DISTRIBUTION OF HYDROMETEORS, AND TO PARAMETERIZE THE RAIN AND CLOUD RELATIONS

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#### 1. INTRODUCTION

A rain drop spectra may be characterized by three elements mutually independent: the liquid water content LWC, the mean volume diameter  $D_m$  and the "intrinsic shape" of the drop size distribution (DSD). The "intrinsic shape" is defined as that of the spectrum after normalizing the diameters by  $D_m$ and the concentrations by  $N_0^* \propto LDC/D_m^4$ . Parameter  $N_0^*$  is also defined as the intercept parameter of the exponential DSD with same LWC and  $D_m$ . With spectra of ice particles the same normalization concept applies considering the "melted diameter" of each particle. This normalization is extensively applied to various microphysical data bases for rain an clouds.

Because the shape of the DSD (or PSD) is quite stable,  $N_0^*$  suffices to describe its variability. It is shown that the "normalized" cloud and rain relations, parameterized by  $N_0^*$ , are "universal" in the sense that they apply to any situation, whatever be the type of rain or cloud and the climate. Thus any remote sensing technique and associated algorithm (dual polarization radar, cloud radar and lidar combination) able to retrieve  $N_0^*$  will be able to perform cloud parameter retrieval not subject to DSD or PSD variability.

## **2-** THE NORMALIZED INTERCEPT PARAMETER OF THE PSD $N_0^*$

The physical characterization of any observed hydrometeor particle size distribution (PSD) raises three questions:

(i) What liquid water content LWC (or ice water content IWC, if solid particles) corresponds to this PSD?

(ii) What is the "mean" particle diameter?

(iii) What is the "intrinsic" shape of the PSD?

The liquid water content relates to the cloud droplet size distribution N(D) [D: equivalent droplet diameter] as:

$$LWC = \frac{\pi \rho_w}{6} \, {}_0^{\infty} N(D) D^3 dD \tag{1}$$

where  $\rho_w$  is the density of water. The expression of the ice water content *IWC* is more

complex since it depends on particle density and shape. We will use hereafter the formulation by Francis et al.(1998) who calculates the *IWC* from the microphysical observations as:

$$IWC = \frac{\pi \rho_w}{6} \, {}_0^{\infty} N(D_{eq}) D_{eq}^{-3} dD_{eq} \qquad (2)$$

where  $D_{eq}$  is the "equivalent melted diameter", and  $N(D_{eq})$  is the PSD in equivalent melted diameter.  $D_{eq}$  is empirically related to the cross sectional A of the ice particle observed by the 2D probe through:

 $D_{eq} = 1.097 A^{0.50}; \qquad A \le 0.0052 \text{ mm}^2 \qquad (3)$  $D_{eq} = 0.615 A^{0.39}; \qquad A > 0.0052 \text{ mm}^2$ 

As "mean particle size", we use in the following the "volume weighted mean diameter" (usually referred to as the "mean volume diameter" in the literature) defined as:

$$D_m = M_4 / M_3 \tag{6}$$

where  $M_4$  and  $M_3$  denote the fourth and third moment of the PSD in D if liquid droplets, or in  $D_{eq}$  if ice particles.

Thus we defined the normalization of the PSD from the general form:

$$N(D) = N_0^* F(D/D_m)$$
(7)

where  $N_0^*$  is the normalization parameter along concentration axis,  $D_m$  the normalization parameter along diameter axis and F(X) is the "normalized PSD" describing the "intrinsic" shape of the PSD (noting  $X = D/D_m$ ). For an ice particle spectrum D stands for  $D_{eq}$ .

Very simple mathematics (see Testud et al, 2000) shows that in order the "intrinsic shape" F(X) be independent of *LWC* and  $D_m$ ,  $N_0^*$  should be defined

as: 
$$N_0^* = \frac{LWC}{\pi \rho_w} \frac{4^4}{D_m^4}$$

A simple interpretation of  $N_0^*$  is that it is the intercept parameter of an exponential distribution with same *LWC* and  $D_m$  as the real one

#### 3- ILLUSTRATION OF THE NORMALIZATION TECHNIQUE

An illustration of the normalization technique is provided by Fig.1. The microphysical data comes

from an iced stratus observed during CLARE 98 (England, 1998). Note the stability of the "intrinsic shape" of the PSD as opposed to variability of the normalization parameters  $D_m$  and  $N_0^*$ . For reference the observed shape is compared with the exponential and wih a Gamma with  $\mu = 4$ . Clearly none of these theoretical shape may represent the actual one.

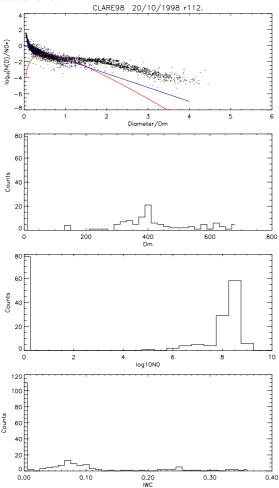


Fig.1: Top diagram: overlay of all normalized spectra (each integrated over 5s or 600m along track) obtained from PMS probes of the UKMO C130 aircraft for a 10 minute leg at -14°C. First middle diagram: histogram of  $D_m$ for the corresponding spectra. middle: Second corresponding histogram  $N_0^{*}$ . Bottom: of corresponding histogram of IWC.

The same normalization technique has been applied to rain: the TOGA-COARE microphysical data collected by the NCAR Electra, and various ground based disdrometers in Darwin, Zurich, or Trappes. The intrinsic shape of the DSD is found very stable for rain spectra, whatever the classification of the DSDs (stratiform or convective, intense or light precipitation) or the climatic zone [see Testud et al, 2001]. Which differentiates the various types of rain or the various climate is the statistics of the normalization parameters, as illustrated in Fig.2.

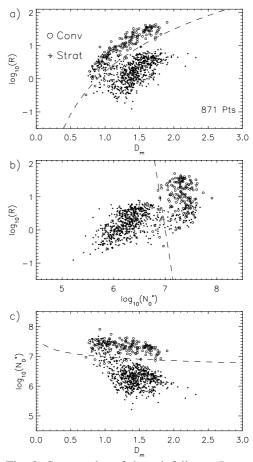


Fig. 2: Scatter plot of the rainfall rate *R* versus  $D_m$  (top), *R* versus  $N_0^*$  (middle), and  $N_0^*$  versus  $D_m$  (bottom) for all rain spectra of a TOGA-COARE flight (each spectra is integrated over 6 s or 720 m along track). Convective and stratiform spectra are distinguished.

In rain  $N_0^*$  varies over two decades, centered upon the Marshall and Palmer value of  $0.8 \times 10^7 \text{m}^{-4}$ . But stratiform and convective spectra adopt a distinct behavior as shown in Fig.2. Classification of the precipitation thus allows to reduce the scatter.

In ice clouds the statistics of  $N_0^*$  is more complex. It seems driven both by the temperature and by the importance of the aggregation processes (which tends to increase particle size and to reduce the concentration. In very cold and very thin cirrus clouds,  $N_0^*$  is of the order of  $5 \times 10^9$  to  $10^{10}$ m<sup>-4</sup>. In stratiform ice clouds producing precipitation,  $N_0^*$  is of the order of  $5 \times 10^9$  m<sup>-4</sup> at cloud top, but

decreases progressively downwards (probably due to the aggregation process) to reach about  $0.5 \times 10^7$  just above the melting layer.

### 4- RELATIONSHIPS BETWEEN INTEGRAL PARAMETERS OF THE PSD

Due to the large variability of  $N_0^*$ , any relationship between two integral parameter of the PSD is very much scattered [see *IWC-Z* rel. Illustrated in Fig.3]. However, it may be demonstrated theoretically (Testud et al., 2001) that after normalization by  $N_0^*$ , quasi "universal" relationships may be established, depending only weakly on the "intrinsic" shape of the PSD. Moreover the intrinsic shape is observed very stable. Fig.3 also illustrates the *IWC-Z* rel. after normalization of each of parameter by  $N_0^*$ .

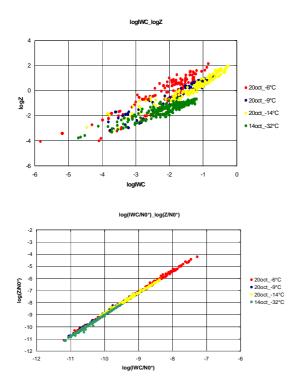


Fig.3: The relationships between IWC and Z, before (upper diagram) and after normalisation (lower diagram) by  $N_0^*$ , from the CLARE 98 microphysical data base.

#### 5- THE POSSIBLE USE OF OVER DETERMINATION WITH POLARIMETRIC RADARS

The fact that after normalization, the relationship between integral parameters becomes so accurate demonstrates that in order to derive accurate measurement of IWC or R from a remote sensing system, two parameters is a necessary and sufficient condition. For example a cloud radar and lidar system measures two parameters in nonprecipitating clouds, that may be used to accurately derive *IWC* and  $r_e$ . Similarly a polarimetric radar allow to measure R more accurately than a classical one. But the case of the polarimetric radar is of particular interest since more than two parameters may be derived: Z, the specific differential phase shift  $K_{DP}$ , and the differential reflectivity  $Z_{DR}$ . From what was said above, these three parameters are necessarily related one another, and this interrelation may be used to test the inverse model itself. Fig.4 shows how the radar itself, by testing the relation between  $K_{DP}/Z$  and  $Z_{DR}$ , may test an hypothesis of the inverse model, namely the oblateness law of raindrops.

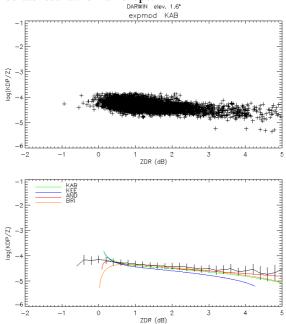


Fig.4: Top diagram: Scatter plot of  $K_{DP}/Z$  versus  $Z_{DR}$  derived from a data set from the Darwin C band polarimetric radar. Bottom: the dependence of the inverse model in respect to various raindrop oblateness laws (theoretical curves) and its comparison with that experimentally observed (average and standard deviation are derived from the top diagram.

#### REFERENCES

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- Testud J., S. Oury, P. Amayenc and R. Black, 2000: The concept of "normalized" distribution to describe raindrop spectra: a tool for cloud physics and cloud remote sensing, *Jour. of Applied Meteorology*, (in press).