12.3 APPLICATION OF ENTROPY PRODUCTION EXTREMAL PRINCIPLE TO SIMPLIFIED CLIMATE MODELS

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1 INTRODUCTION

We study idealized models of the global climate to examine the hypothesis that the climate system adjusts itself to the state of maximum entropy production as first discussed by *Paltridge [1975].* We examine a hierarchy of simple box models of atmosphere, ocean, and the coupled system, and calculate analytical solutions for extremal entropy production. The box model solutions are compared to the output of general circulation models (GCM).

2 Theory Of Maximum Entropy Production Based On Box Models

Assuming the local thermodynamic equilibrium, we define the time rate of change of specific entropy, ds/dt,

$$\frac{ds}{dt} = \frac{1}{T}\frac{dQ}{dt} \qquad (1)$$

where dQ/dt is diabatic heating per unit mass. The global integral of Eq. (1) is the total entropy flux into the climate system, and is balanced by the internal production of entropy at steady state. We define the global entropy production, dS/dt, as;

$$\frac{dS}{dt} = -\oint_V \rho \frac{ds}{dt} dV \qquad (2)$$

where ρ [kg m^{-3}] is the density of the fluid.

2.1 2-Box Model Of Atmosphere And Ocean

Let us consider a single hemispheric box model of climate (FIG.1) which can be regarded both as the atmosphere or as the ocean. The model consists of equatorial region (with temperature Te [°C]) and polar region (with temperature Tp). We restore Te and Tp to Te* and Tp* with inverse time scale of k_T [s^{-1}]. In the atmosphere, Te* and Tp* are radiative-convective equilibrium profile and k_T is the radiative time scale. In the ocean, Te* and Tp* are air-temperature above the sea surface and k_T is the time scale of air-sea interaction. The global entropy production is;

$$\frac{dS}{dt} = X\left(\frac{1}{Tref + Tp} - \frac{1}{Tref + Te}\right) \qquad (3)$$

where Tref=273 [K]. Assuming that the system is in steady state and in global energy balance, the entropy production becomes a quadratic function of X and it has its maximum at;

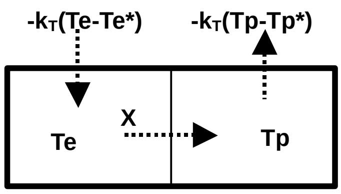
$$X = \frac{Ck_T \Delta T^*}{4} \qquad (4)$$

$$\Delta T = \frac{\Delta T^*}{2} \qquad (5)$$

where C [J K^{-1}] is the heat capacity of each box, ΔT=Te-Tp, and ΔT*=Te*-Tp*. In the atmosphere, within reasonable parameter range, X_A= 3.8 [PW] and ΔT_A=10 [K]. In the ocean, assuming that a half of the Earth's surface is covered by the ocean, X_O= 4.0 [PW] and ΔT_O=5 [K] which is consistent with gross measure of present climate.

FIG.1: 2-Box model

X is the meridional heat transport actross 30N or 30S.



2.2 Coupled Box Model

We couple the atmosphere and the ocean box models in the previous section and maximize the global entropy production. The entropy production of the coupled system is the sum of entropy production in the atmoshere, ocean, and air-sea interaction. The global entropy production is;

$$\frac{dS}{dt} = (X_A + X_O)\left(\frac{1}{Tref + Tap} - \frac{1}{Tref + Tae}\right) \qquad (6)$$

It is at its maximum when

$$X_A + X_O = \frac{k_T C_A \Delta T_A^*}{4} \qquad (7)$$

The extremal solution can determine the total meridional heat transport and ΔT_A which is determined

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by identical expression as Eq. (4). However, individual values for X_A and X_O and ΔT_O are not determined. Additional constraints, such as the parameterization of thermohaline circulation, are needed to fully determine the system.

3 Testing The Theory By Diagnosing Simplified GCM Experiments

We compare the extremal solutions of box models to the output of zonally averaged and 3D versions of MITGCM *[Marshall et al. 1997]* of the atmosphere and the ocean with idealized forcing.

3.1 Zonally Averaged Atmospheric GCM

The zonally averaged model employs the transformed Eularian mean formulation*[Wardle and Marshall 2000]* and simplified thermodynamics following *Held and Suarez[1994].* First, we run the model with various eddy diffusivities. Unlike the box model, X_A is calculated by detailed dynamics. We compare the spatial average of time-averaged temperature field with the box model. The diagnosis confirms the relationship between the heat transport and entropy production predicted by the box model (FIG.2). The maximum in the entropy production is near X_A=3.8 [PW] and is consistent with a reasonable value for the eddy diffusivity.

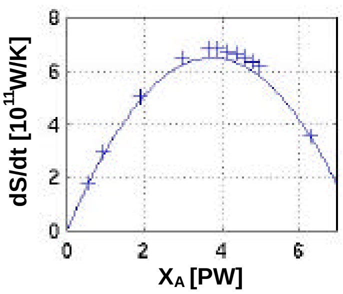

FIG.2 Atmospheric GCM diagnosis

Solid line: Box model
+ : GCM diagnosis
Each point represents different eddy diffusivities.

Secondly, we made model runs with various meridional temperature gradient in the forcing in order to test the relationship in Eq. (3) and (4). The GCM diagnosis seems to support the linear relation between the forcing (ΔT_A *) and diagnosed ΔT_A and X_A. We also examine the relationship between the restoration timescale k_T and heat transport.

3.2 Zonally Averaged Ocean GCM

We use a simplified ocean GCM to examine the entropy production in a hemispheric basin. The model employs parameterized zonal pressure gradient, which is loosely based on *Wright and Stocker [1991].* The model is forced by the restoration of SST to an idealized profile of air temperature. We make model runs with a range of diapicnal diffusivities. The extremal state is found near Kv = 2 10^{-4} [m^2s^{-1}] and is consistent with Eq. (4) and (5).

3.3 Extensions to An Eddy Resolving Model

We examine the entropy production in an eddy resolving atmospheric GCM which employs the same thermodynamic forcing as the zonally averaged model. We diagnose the model field for comparison with the analytical relationships calculated by variational method. The application of variational calculus suggests;

$$T = m\sqrt{T*} \quad (8)$$

$$X(f) = \int_0^{2p} dl \int_{pt}^{pb} dp \int_{f0}^{f} df \frac{c_P k_T}{g} \left(\frac{p_S}{p}\right)^k \left(m\sqrt{T*} - T*\right) \quad (9)$$

where μ is the Lagrange Multiplier. We show that diagnosed time-averaged temperature field from GCM has similar trend as Eq. (8) and that meridional heat transport predicted by Eq. (9) is comparable to the diagnosed heat transport of the GCM.

4 Summary and discussion

The extremal solution exhibits several remarkable properties. (1) the box model estimate of heat transport and pole-equator temperature difference are in a plausible agreement with the gross measure of present climate. (2) the box model provides analytical solutions which can be tested against more complicated models. (3) simplified GCMs seem to support extremal entropy production states within a reasonable parameter range. Global energy balance and extremal principle are not sufficient to fully determine the coupled system. Additional constraints could be used for closure condition.

5 Reference

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