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WAVE–MEAN-FLOW INTERACTIONS IN A GRAVITY WAVE PACKET CRITICAL LAYER

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1. INTRODUCTION

The effect of gravity-wave–mean-flow interactions on the general circulation of the atmosphere is seen through numerous observed phenomena, such as the quasi-biennial oscillation of the equatorial stratosphere. Our understanding of such interactions has been greatly advanced in the past few decades by theoretical studies on wave instabilities in shear flows. A mathematical difficulty that arises in the linear theory of shear flows is that a singularity is generally present at the critical level where the mean flow velocity is equal to the phase speed of the waves. In the critical layer centered at such a level, wave breaking and reflection may occur, often leading to turbulence.

In the pioneering study of Booker and Bretherton (1967), upward propagating gravity waves produced by a sinusoidal forcing encounter a critical level where they are effectively absorbed. The vertical flux of horizontal momentum, which would be independent of height in the absence of a critical level (Eliassen and Palm, 1960), is reduced across the critical layer by a factor of $\exp\{-2\pi(\mathrm{Ri}_c - 1/4)\}$, where Ri_c is the initial Richardson number at the critical level. Brown and Stewartson (1982) extended this linear analysis to later times when the nonlinear effects become significant and concluded that, ultimately, the critical layer becomes a reflector of the incident waves.

Most theoretical studies of critical layers have been based on the assumption that the disturbance is periodic in the horizontal (zonal) direction. Although this may be appropriate for large scale disturbances in the stratosphere, Brunet and Haynes (1996) point out that a zonally periodic forcing is less realistic in the troposphere. This is particularly true for the case of gravity waves forced by topography, which would in reality be of finite extent only.

Bacmeister and Pierrehumbert (1988) studied gravity waves forced by flow over a horizontally localized obstacle. They observed that the absorbing state of the critical layer could continue even into the nonlinear regime. This was because, in their simulations, the discontinuity in the vertical momentum flux across the critical layer was balanced by a change in the horizontal momentum flux across the region above the obstacle, so that there was zero net flux into this region. This is the same scenario that was later suggested by Brunet and Haynes (1996) in their study of the related problem of the nonlinear critical layer for horizontally propagating Rossby wave packets. It was also observed in the nonlinear numerical simulations of the Rossby wave packet critical layer carried out by Campbell and Maslowe (2001). In this presentation, the nonlinear gravity wave packet critical layer is examined and the results contrasted to those obtained in the case of a monochromatic forcing.

2. RESULTS

The numerical experiments here are described in more detail in Campbell (2000). Finite differences and a pseudo-spectral method

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are employed to solve equations for the perturbation streamfunction, vorticity and density on a rectangular domain. The background flow varies with height only. x and z represent non-dimensional latitude and height respectively. A wave packet of the form $e^{-\mu^2 x^2} e^{ikx}$ ($\mu =$ a small constant) is forced at $z = 0$ and propagates up to the critical level located at $z = 5$. The upper boundary of the domain is at $z = 10$.

Experiments with monochromatic forcing ($\mu = 0$) are also carried out. In both cases, the disturbance is almost completely absorbed at the critical layer at early times; later, nonlinearity leads to overturning density and vorticity contours in the critical layer. According to the nonlinear critical layer theory of Brown and Stewartson (1982), one would expect this to occur when the non-dimensional time $t \sim O(\varepsilon^{-2/3})$, where ε is a non-dimensional parameter that gives a measure of the amplitude of the waves at the forced boundary relative to the mean flow. It is seen that the horizontal localization delays the onset of this nonlinear breakdown; in the figure shown below, the vorticity contours have not yet overturned although $\varepsilon = 0.05$ and $t = 30$. An outward flux of momentum is observed in the critical layer so that the horizontal extent of the region over which the packet interacts with the mean flow increases in time.

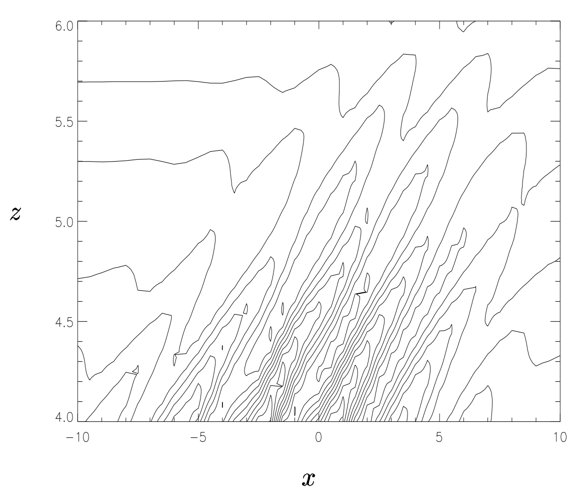

Figure 1: Vorticity contours in the critical layer

The effect of a horizontally localized forcing on the interactions in a Rossby wave critical layer has also been examined. Those results will be presented at the Wave Phenomena III conference in Edmonton next week.

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