RELATION OF RAIN RATES AT THE GROUND AND ALOFT — A MODEL STUDY

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1. INTRODUCTION

The radar reflectivity factor Z at a certain height level is commonly used in radar meteorology to estimate the rain rate at the ground. Aside from other involved assumptions this method only works if precipitation falls through purely horizontal air flow and fall speeds only depend on hydrometeor mass.

However, as noted by Battan (1976), Austin (1987), and Atlas et al. (1995) this is not the case in general. The precipitation rate (defined as the vertical hydrometeor mass flux density) is modified by convective up- and downdrafts within the precipitation core of deep convective clouds. Besides, the lower air density aloft leads to an enhanced precipitation rate for a given reflectivity factor compared to standard sea level conditions.

Our paper evaluates exemplary data obtained from mesoscale simulation case studies of single cumulonimbus clouds. The models contain balance equations for hydrometeor content ρq in which selfevidently effects of a vertical wind field w are considered. Also local variations of the hydrometeors' fall speed w_t due to changing air density ρ are included. Consequently, all information is available in the complete model volume to calculate both the vertical profile of rain rate $R(z)$ and the Z–R relation in the most general case.

2. ANALYTICAL APPROACH

The following normalized Γ–type spectrum is assumed for any hydrometeor class:

$$n(D) = N_0 \, \frac{\Gamma(4)}{\Gamma(\gamma+3)} \, \left(\frac{D}{D_0}\right)^{\gamma-1} \, e^{-D/D_0} \quad . \qquad (1)$$

Here, D denotes particle diameter, γ is a shape parameter, and N_0, D_0 are the spectral particle load and a measure of diameter, respectively. The parameter D_0 can easily be related to any specific measure of particle diameter, such as the volume median $D_v \approx (2.67+\gamma)\, D_0$. Note that both $n(D)$ and N_0 are given in units of m^{-4}, or conventionally in mm^{-1} m^{-3}. The normalization in Eq. (1) assures that the hydrometeor content ρq does not depend on the shape parameter γ:

$$\rho q = \pi \rho_h N_0 \, {D_0}^4 \quad , \qquad (2)$$

with ρ_h denoting individual hydrometeor density. For a complete discussion the reader is referred to Dotzek & Beheng (2000) and Dotzek & Beheng (2001).

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2.1 Radar reflectivity factor

The radar reflectivity factor Z for spherical particles under assumption of Rayleigh's approximation is given by

$$Z = \Gamma(4)(\gamma+5)(\gamma+4)(\gamma+3) N_0 \, {D_0}^7 \quad .$$

By eliminating D_0 using Eq. (2) we arrive at the relationship between Z and ρq:

$$Z = \frac{\Gamma(4)}{[\pi\rho_h]^{7/4}} \, (\gamma+5)(\gamma+4)(\gamma+3) \, {N_0}^{-3/4} \, (\rho q)^{7/4} \quad .$$

2.2 Rain rate

The precipitation rate R, or vertical hydrometeor mass flux density is given by

$$R = \frac{\pi}{6} \rho_h \int_0^\infty n(D) D^3 [w + w_t(D)] \, dD \quad .$$

Using bulk variables this becomes

$$R = -\left[w + \bar{w}_{t,00}(\rho q) \, \left(\frac{\rho_{00}}{\rho}\right)^{0.4} \right] \, \rho q \quad , \qquad (3)$$

being linked to vertical drafts, terminal fall velocity, air density, and hydrometeor content. Subscript 00 denotes variables at reference level.

3. MODELING CASE STUDIES

Two non–hydrostatic models were used to study storm cells: KAMM and MM5.

3.1 KAMM model

The 3D non–hydrostatic model KAMM (Adrian & Fiedler, 1991, **K**arlsruhe **A**tmospheric **M**esoscale **M**odel) was applied in a substantially revised and extended version. As described by Dotzek (1999) it presently includes a bulk–microphysical scheme for precipitation, cloud water, and cloud ice. As for the analytical approach, all hydrometeor fall speeds in the model are subject to a variation due to density stratification according to Eq. (3) with $\rho_{00} = 1.225$ kg m^{-3}.

A single shower cloud was initiated by a moist and warm air mass; the horizontal resolution of the model was 1 km and varied in the vertical from 10 m near the ground to ≈ 500 m at the model top (18 km AGL). The model data analyzed here were taken from the point of maximum cloud development: the cumulonimbus top exceeded 9 km AGL and the instantaneous rain rate at the ground was 420 mm h^{-1}.

For warm rain microphysics, the model uses the well–known terminal velocity formulation:

$$\bar{w}_{t,00}(\rho q) = -14.16 \left(\frac{\rho q}{\mathrm{kg\,m^{-3}}} \right)^{0.1364} \ \mathrm{m\,s^{-1}} \quad . \qquad (4)$$

The effects of mixed–phase precipitation were included in the model KAMM with the simple approach by Tartaglione et al. (1996): below the freezing level Eq. (4) according to the warm rain scheme is applied. For temperatures less than -35 °C a constant fall speed representative of snow or lump graupel is assumed:

$$\bar{w}_{t,00}(\rho q) = -2.5 \ \mathrm{m\,s^{-1}} \quad . \qquad (5)$$

For $[-35°\mathrm{C} < T < 0°\mathrm{C}]$ a linear interpolation between Eqs. (4) and (5) is performed.

In a relevant range of $[0.1 \lesssim R \lesssim 420]$ $\mathrm{mm\,h^{-1}}$ the data from the two asymptotic cases of rain and snow or lump graupel collapse on a joint average relation

$$Z = [230 \pm 100]\ R^{1.51\pm0.09} \ , \quad R = [18 \pm 7]\ (\rho q)^{1.14\pm0.14} \ .$$

3.2 MM5 model

We also used a 3D non–hydrostatic PennState / NCAR MM5 (Dudhia, 1993). The bulk microphysics scheme introduced by Lin et al. (1983), Tao et al. (1989), and Tao et al. (1993) includes prognostic equations for cloud water, rain, snow, ice and graupel. As it is the case in the analytical approach and for the KAMM model, all terminal fall velocities for precipitation particles are subject to air density variations.

The system studied is a supercell storm observed in the northern Alpine foreland during the EULINOX project (Höller et al., 2000). As described by Fehr (2000), 50 vertical levels, 12 of them below 2 km AGL, were used with a horizontal resolution of 500 m. Convection was initialized by a temperature perturbation near the ground. The modeled cloud development corresponds in many respects to the observed supercell. Averaged surface rain rates reached values well above 200 $\mathrm{mm\,h^{-1}}$, while updraft velocities exceeded 50 $\mathrm{m\,s^{-1}}$. In such environments ordinary Z–R relations usually fail (Atlas et al., 1995).

4. DISCUSSION

Concerning Z–R relations, even for rapidly falling hydrometeors the model data are very noisy and have numerous outliers with high reflectivity factors at very small precipitation rates. These result from large hydrometeor contents within the main updraft core. For some of these data points the rain rate is reduced by a factor of 100 compared to the rain rate in air at rest. On the contrary, in the main downdraft within the rain shaft R is increased only by a factor of 2 for the KAMM case.

For more intense convective clouds with higher downdraft intensities like in the MM5 case study this range is expanded to higher enhancement factors. But even the moderate vertical drafts in the KAMM simulation suffice to introduce a standard deviation of ± 43 % in the prefactor of average Z–R relations. This noise cannot be attributed to the different asymptotic relations for rain and snow: the spread between the two curves is small for relevant rain rates. As noted by Atlas et al. (1995) and obvious from the present study, considering only a limited volume of a storm to evaluate $R(Z)$ can produce almost any functional form.

5. CONCLUSIONS

1. Vertical air motions as well as density variations affect the prefactor of standard Z–R relations by increasing both the average value and standard deviation,

2. vertical profiles of the precipitation rate R show considerable variations to the rain rate at the ground,

3. it is necessary to monitor the complete precipitation volume of a storm to obtain reliable Z–R statistics.

REFERENCES

Adrian, G., and F. Fiedler, 1991: Simulation of unstationary wind and temperature fields over complex terrain and comparison with observations. *Contr. Atmos. Phys.*, **64**, 27–48.

Atlas, D., P. Willis, and F. Marks, 1995: The effects of convective up- and downdrafts on reflectivity–rain rate relations and water budgets. *Proc. 27th Conf. on Radar Meteor.*, Vail, 19–22.

Austin, P. M., 1987: Relation between measured radar reflectivity and surface rainfall. *Mon. Wea. Rev.*, **115**, 1053–1070.

Battan, L. J., 1976: Vertical air motions and the Z–R relation. *J. Appl. Meteor.*, **15**, 1120–1121.

Dotzek, N., 1999: *Mesoskalige numerische Simulation von Wolken- und Niederschlagsprozessen über strukturiertem Gelände*. (In German). Dissertation, Univ. Karlsruhe, 127 pp. [Available at http://www.op.dlr.de/˜pa4p/]

Dotzek, N., and K. D. Beheng, 2000: The influence of deep convective motions on the variability of Z–R relations. *Proc. 13th Int. Conf. on Clouds and Precipitation*, Reno, 326–329.

Dotzek, N., and K. D. Beheng, 2001: The influence of deep convective motions on the variability of Z–R relations. Submitted to *Atmos. Res.*

Dudhia, J., 1993: A nonhydrostatic version of the Penn State–NCAR mesoscale model: Validation tests and simulation of an atlantic cyclone and cold front. *Mon. Wea. Rev.*, **121**, 1483–1513.

Fehr, T., 2000: *Mesoskalige Modellierung der Produktion und des dreidimensionalen Transports von Stickoxiden in Gewittern*. DLR Forsch. Ber. 2000–15, Cologne, 126 pp. [Available at http://www.op.dlr.de/˜pa10/PhDsite.html]

Höller, H., T. Fehr, C. Théry, J. Seltmann, and H. Huntrieser, 2000: Radar, lightning, airborne observations, and modelling of a supercell storm during EULINOX. *Phys. Chem. of the Earth: Part B*, **25**, 1281–1284.

Lin, Y.–L., R. D. Farley, and H. D. Orville, 1983: Bulk parameterization of the snow field in a cloud model. *J. Climate Appl. Meteor.*, **23**, 1065–1092.

Tao, W.–K., J. Simpson, and M. Mc Cumber, 1989: An ice–water saturation adjustment. *Mon. Wea. Rev.*, **117**, 231–235.

Tao, W.–K., J. Simpson, C.–H. Sui, B. Ferrier, S. Lang, J. Scala, M.–D. Chou, and K. Pickering, 1993: Heating, moisture, and water budgets of tropical and midlatitude squall lines: Comparisons and sensitivity to longwave radiation. *J. Atmos. Sci.*, **50**, 673–690.

Tartaglione, N., A. Buzzi, and M. Fantini, 1996: Supercell simulations with simple ice parameterization. *Meteor. Atmos. Phys.*, **58**, 139–149.