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1. INTRODUCTION

A well-developed intense mesocyclone appears as a pair of maximal and minimal patterns on Doppler radar. The WSR-88D system includes a typical detection algorithm for mesocyclones and tornadoes, which examines azimuthal wind shear in a Doppler radial velocity field (Stumpf et al., 1998). However, target mesocyclones are always associated with natural smaller-scale perturbations and larger-scale ambient flows, which often contaminate the patterns of mesocylones. Therefore, it is difficult to precisely estimate the rotation and divergence intensities from the radial velocity observational data.

The purpose of this study is to develop a new algorithm for automated vortex detection of mesocyclones and tornadoes in a single Doppler radar. We suggest to use the two-dimensional continuous wavelet transforms as a preprocessor, to extract the particular spatial scale information from a wind field and to filter out the signal noise and larger-scale flows. This algorithm consists of two parts. The first part is to detect the locations of mesocyclones and the second part is to estimate their vorticity and divergence.

2. NUMERICAL METHOD

2.1 Wavelet Transforms

We apply a two-dimensional continuous wavelet transform to Doppler observational data instead of using original data to analyze vortex structures. A continuous wavelet transform is defined as follows.

$$W[f](a,\vec{b}) = \frac{1}{\sqrt{a}} \iint f(\vec{x}) \Psi(\frac{\vec{x}-\vec{b}}{a}) d\vec{x}$$

Here $f(\vec{x})$ is a data function, and $\Psi(\vec{x})$ a wavelet mother function. Here *a* and \vec{b} are dilation and translation parameters of the wavelet function which represent scale and position respectively. In our numerical experiments the normalized 2D Mexican hat function has been used as a mother wavelet. The reasons we chose the 2D Mexican hat wavelet are its continuity and axisymmetry. Furthermore, the function is easy to implement because it can be analytically described as a mathematical function. Other kinds of wavelets are also examined and similar results have been obtained (Liu 2000).

For our convenience, we numerically calculated the wavelet transforms in Cartesian coordinates with respect to the observation data, which are measured in polar coordinates. The integral can be calculated rapidly by the Fast Fourier transform (FFT) under the

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assumption that the radial variables are approximately considered as a constant number in the integral area.

2.2 Vortex Location Detection

A mesocyclone has been widely analyzed by a typical Rankine vortex model. The idea is to find an appropriate Rankine vortex, which approximates to an observational vortex. Therefore the location of the Rankine vortex can be seen as that of observational ones.

Let $u_r(x - \overline{x}, y - \overline{y})$ be radial elements of the Rankine velocity flow on radar field, where $(\overline{x}, \overline{y})$ is the vortex center. Also let $\hat{u}(x, y)$ be a Doppler radar observation function. The convolution function of an observational data \hat{u} and a Rankine vortex flow u_r is defined as follows.

$$f_{W}(\bar{x},\bar{y}) \equiv \iint_{\Omega} W[\hat{u}(x,y)] W[u_{r}(x-\bar{x},y-\bar{y})] dx dy$$

Here W[1] are indicated wavelet transforms. The wavelet transforms are applied to filter data into a specific frequency field. Both the observational data and the Rankine vortex field are filtered by wavelet transforms respectively. We calculate this convolution function and search its maximal points in order to detect positions of vortices in observations. A convolution function of an observation and the Rankine vortex will be significantly large on vortex locations of the observation data under the the atmospheric assumption that vortex approximates to the Rankine vortex. Compared with the WSR-88D mesocyclone detection algorithm, which searches for a pair of maximal and minimal points, our method is simpler and more practical to detect vortices because of using only one value: the maximum of the convolution.

2.3 Vorticity and Divergence Estimation

After a vortex location is given, the Sasaki variational method using the vortex model is applied to fit the observational data (Sasaki et al., 1989) to analyze the vortex structure. We define *J* as a integral of square of difference of observational data and vortex model data:

$$J = \iint_{\Omega} \{W[u_r] - W[\hat{u}]\}^2 \, dx \, dy.$$

The variables of *J* are divergence, vorticity and their core radii. The Rankine vortex model flow best approximates the observational flow when parameters are set at a local minimum. It is a linear and nonlinear mean square minimized variational

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problem, which can be solved numerically, for examples, by the Gauss elimination method and the quasi-Newton method.

3. RESULTS

3.1 Error Analysis

Three kinds of artificial data have been made to examine the efficiency of this detection method. The first one is the adding of a uniform wind flow to a Rankine vortex flow. The second one is a random number (noise) field added to the Rankine vortex flow. The third one is for a combined vortex flow of two different sized vortices. We calculate the vortex error in these three test data to investigate the influence of perturbation in detecting vortices. Our experiments show that the locations of vortices and their vorticity can be estimated and the combined vortex flow can be decomposed with negligible error. Note, that it is difficult to obtain similar results using the Sasaki variational method without filtering data by wavelet transforms.

3.2 Observational Experiments

We have applied our new algorithm to the Doppler radar observations. The data is observed from 18:50 May 10 (UTC) to 00:07 May 12 (UTC) 1992. According to the ground truth report, a total of 23 F1 to F4 tornadoes occurred in the eastern area of Norman radar site between 18:25 to 22:30 on May 11. After dealiasing and extrapolating processes, the observational data has been used to evaluate performance of this detection algorithm. The critical success index (CSI) has been evaluated and compared our method with the WSR-88D mesocyclone algorithm in detecting tornadoes. We define the thresholds of this new algorithm be the convolution function value, vorticity, divergence, and in addition the radii of rotation and divergence if necessary. We found that most of the CSI of all wavelet transform cases are better than the WSR-88D algorithm.

The second observational data experiment is to estimate the vertical structures of the mesocyclones. Observation data at several scanning elevations were used to calculate the vorticity and divergence of a storm in different heights. Our experiments show that convergence occurs at low-level, and divergence occurs at mid or high-level. On the other hand, vorticity tends to be larger at low- and midlevel. The intense vorticity appears firstly at low-level, descends to the ground, and then finally lifts to mid or high-level. The strong convergence is considered as an energy source, which amplifies vorticity, and is associated with tornado generation. At high-level divergence amplified, which implies the vorticity becomes weaker. Furthermore, the atmosphere of convergence to divergence with respect to the height implies updraft in the rotation storms (Figure 1).

4. SUMMARY

A new automated mesocyclone detection algorithm has been developed. This algorithm uses the radial velocity data, which observes tornadic storms by using a single Doppler radar, to detect mesocyclone or tornado positions and their vorticity. The vortex locations are calculated by measuring the maximal points of a convolution function of observational data and the Rankine vortex model. After a vortex location is obtained, the kinematical properties are estimated by the variational method. The two-dimensional continuous wavelet transforms filter data to a particular mesoscale field, so that vortex centers and intensities can be measured more precisely.

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Figure 1. The vertical vorticity and divergence distributions of a tornadic storm for three periods. The times, which are indicated under the graph, are the beginnings of the scans. A tornado of scale F0 is reported at 18:25, that is, in the middle period.