4.11 THERMODYNAMIC RETRIEVAL EXPERIMENTS WITH A 2-D MODEL

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1. INTRODUCTION

The current generation of mobile platforms (e.g., Rasmussen et al. 1994) for observing convective storms and other small-scale phenomena provides detailed information on the wind field within the feature of interest, but it is difficult to obtain more than a few isolated measurements of pressure and temperature. The lack of thermodynamic data presents a challenge to the scientist who wishes to validate hypotheses of storm dynamics.

Traditional methods of thermodynamic retrieval (Gal-Chen 1978; Hane et al. 1981; Roux 1985) estimate the unknown fields by a direct solution of the equations of motion (and, in the case of the Roux method, the thermodynamic equation). Unfortunately, traditional retrievals yield unsatisfactory results when there are significant errors in the observations and/or when the temporal sampling is poor. When the time between observations is long relative to the characteristic time scale for evolution of the observed phenomenon, it is difficult to estimate the local time derivatives of velocity that are required in the retrieval.

Some of the limitations of traditional retrieval methods are avoided in data assimilation approaches (e.g., Sun et al. 1991; Sun and Crook 1997) for storm-scale retrievals. Sun et al. (1991) used a Boussinesq numerical model and its adjoint to produce dynamically consistent fields that agreed, in a least squares sense, with simulated radar observations of dry convection.

This paper is a description of a new and simple method, which also uses a numerical model, for retrieving temperature from observations of velocity. The advantage of this technique over traditional methods is that one may account for evolution that is nonlinear in time.

2. TWO-DIMENSIONAL MODEL

We are conducting experiments with a two-dimensional, dry, anelastic, frictionless, version of the COllaborative Model for Multiscale Atmospheric Simulation (COMMAS) (Wicker and Wilhelmson 1995). The model equations are as follows:

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - w\frac{\partial u}{\partial z} - c_p\bar{\theta}\frac{\partial \pi}{\partial x} \quad (1)$$

$$\frac{\partial w}{\partial t} = -u\frac{\partial w}{\partial x} - w\frac{\partial w}{\partial z} + B - c_p\bar{\theta}\frac{\partial \pi}{\partial z} \quad (2)$$

$$\frac{\partial \theta}{\partial t} = -u\frac{\partial \theta}{\partial x} - w\frac{\partial \theta}{\partial z} \quad (3)$$

where θ denotes the total potential temperature, overbars indicate base state quantities, buoyancy is defined as $B = g\left(\theta/\bar{\theta} - 1\right)$, and the other variables have their usual definitions. For the integration of the model equations, we use third-order Runge-Kutta time integration and fifth-order spatial differencing (Wicker and Skamarock 2001). Pressure is determined at each time step with a diagnostic equation obtained by taking the divergence of the equations of motion and incorporating anelastic mass continuity [$\nabla \cdot \left(\bar{\rho}\vec{u}\right) = 0$].

The numerical model is used both to produce simulated radar observations and to accomplish the retrieval of temperature. The retrieval method also requires a linearized version of the model:

$$\frac{\partial u'}{\partial t} = -\tilde{u}\frac{\partial u'}{\partial x} - \tilde{w}\frac{\partial u'}{\partial z} - u'\frac{\partial \tilde{u}}{\partial x} - w'\frac{\partial \tilde{u}}{\partial z} - c_p\bar{\theta}\frac{\partial \pi'}{\partial x} \quad (4)$$

$$\frac{\partial w'}{\partial t} = -\tilde{u}\frac{\partial w'}{\partial x} - \tilde{w}\frac{\partial w'}{\partial z} - u'\frac{\partial \tilde{w}}{\partial x} - w'\frac{\partial \tilde{w}}{\partial z} + g\left(\frac{\theta'}{\bar{\theta}}\right) - c_p\bar{\theta}\frac{\partial \pi'}{\partial z} \quad (5)$$

$$\frac{\partial \theta'}{\partial t} = -\tilde{u}\frac{\partial \theta'}{\partial x} - \tilde{w}\frac{\partial \theta'}{\partial z} - u'\frac{\partial \tilde{\theta}}{\partial x} - w'\frac{\partial \tilde{\theta}}{\partial z} \quad (6)$$

where $\tilde{u}, \tilde{w}, \text{and } \tilde{\theta}$ are the (total) model variables in the forward simulation, and $u', w', \text{and } \theta'$ are perturbations that are defined below. The utility of tangent linear models in data assimilation problems has been investigated previously by Wang et al. (1997), Pu et al. (1997), and Kalnay et al. (2000). The proposed technique here also involves integration of a linear model back in time.

The control simulation is of a cold bubble (minimum temperature perturbation of –16 K) descending and then spreading out at the surface in a neutrally stratified environment (Fig. 1). The initial velocity is zero. The θ field evolves rapidly as the outflow reaches the ground and spreads laterally (Figs. 1b and 1c). The magnitudes of u and w in the simulation (not shown) are as high as 30 m s^{-1} and 20 m s^{-1}, respectively, at 240 s. The rapid evolution of the flow presents a challenging test for retrieval methods.

3. RETRIEVAL EXPERIMENTS

The problem we pose for the retrieval experiments is as follows: Suppose we have complete, perfect sets of observations of the u component of motion at two times, e.g., at t_1=240 s and t_2=300 s. The goal is to determine θ at t_1 from the observations of wind. Since the problem is two-dimensional, the observations of u together with the continuity equation are used to synthesize the complete wind field. The problem is analogous to a three-dimensional case when dual-Doppler observations are available.

We use the following algorithm to retrieve θ:

1. Initialize the model at t_1 with a first guess of $\theta = \bar{\theta}$.

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2. Run the model forward to t_2, saving $\tilde{u}, \tilde{w}$, and $\tilde{\theta}$ at each time step for later use during step 4.
3. Define the forecast errors $u' = u_{ob}(t_2) - \tilde{u}(t_2)$ and $w' = w_{ob}(t_2) - \tilde{w}(t_2)$. Let $\theta' = 0$.
4. Integrate the linear model (4)-(6) back to t_1, to obtain $u'(t_1)$ and $w'(t_1)$.
5. Let $D_u = \dfrac{u'(t_1)}{t_2 - t_1}$ and $D_w = \dfrac{w'(t_1)}{t_2 - t_1}$. D_u and D_w are errors in the previous estimates of the time derivatives of velocity at t_1.
6. Retrieve π' and θ' from $\dfrac{\partial \pi'}{\partial x} = \dfrac{1}{c_p \overline{\theta}}\left(-D_u\right)$ and $\theta' = \dfrac{\overline{\theta}}{g}\left(c_p \overline{\theta} \dfrac{\partial \pi'}{\partial z} + D_w\right)$.
7. Initialize the model again, but with $\theta = \tilde{\theta}(t_1) + \theta'$. If the solution has not converged, return to step 2 and repeat the procedure.

This iterative technique provides a method for using errors in the forecast velocity at the final time to modify the θ field at the initial time. For the experiments, we determine u_{ob} and w_{ob} at both t_1 and t_2 with a variational method in which continuity is satisfied exactly and the observations of u are satisfied in a least squares sense (Shapiro and Mewes 1999). For other cases, it may be desirable to successively refine estimates of the wind field with other variational techniques that include additional constraints.

4. RESULTS

For the test with observations of u at 240 and 300 s, the retrieved perturbation temperature field after the first iteration (Fig. 2a) has already begun to resemble the actual temperature field (Fig. 1b). The mean absolute error in θ within the subdomain that is shown in the figures is 0.54 K after the first iteration. For comparison, we show the results of a traditional retrieval (Gal-Chen 1978; Hane et al. 1981) applied at the same time (Fig. 2d). Since significant nonlinear evolution in the cold pool occurs between 240 and 300 s (Figs. 1b and 1c), the finite-difference approximation of the velocity time derivatives over the 60-s interval introduces significant error into the retrieval. The mean absolute error in θ for the traditional method is 2.01 K.

At later stages in the new iterative method, the retrieved θ field approaches that in the control simulation (Figs. 1b, 2b, and 2c). By iteration 25, the mean absolute error in θ is less than 0.1 K.

5. FUTURE WORK

The experiment just described involves a restricted set of circumstances – i.e., a perfect model and perfect observations. The emphasis was on retrieving θ when there is significant evolution in the fields between the times of the velocity observations. We are currently working on applying the retrieval in more complicated scenarios involving error in the observations, missing observations, and the presence of moisture.

ACKNOWLEDGMENTS

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REFERENCES

Gal-Chen, T., 1978: A method for the initialization of the anelastic equations: Implications for matching models with observations. *Mon. Wea. Rev*., **106**, 587-606.

Hane, C. E., R. B. Wilhelmson, and T. Gal-Chen, 1981: Retrieval of thermodynamic variables within deep convective clouds: Experiments in three dimensions. *Mon. Wea. Rev*., **109**, 564-576.

Kalnay, E., S. K. Park, Z.-X. Pu, and J. Gao, 2000: Application of the quasi-inverse method to data assimilation. *Mon. Wea. Rev*., **128**, 864-875.

Pu, Z.-X., E. Kalnay, J. Sela, and I. Szunyogh, 1997: Sensitivity of forecast errors to initial conditions with a quasi-inverse linear model. *Mon. Wea. Rev.*, **125**, 2479-2503.

Rasmussen, E. N., J. M. Straka, R. Davies-Jones, C. A. Doswell, F. H. Carr, M. D. Eilts, and D. R. MacGorman, 1994: Verification of the origins of rotation in tornadoes experiment: VORTEX. *Bull. Amer. Meteor. Soc*., **75**, 995-1006.

Roux, F., 1985: Retrieval of thermodynamic fields from multiple-Doppler radar data using the equations of motion and the thermodynamic equation. *Mon. Wea. Rev*., **113**, 2142-2157.

Shapiro, A., and J. J. Mewes, 1999: New formulations of dual-Doppler wind analysis. *J. Atmos. Oceanic Technol*., **16**, 782-792.

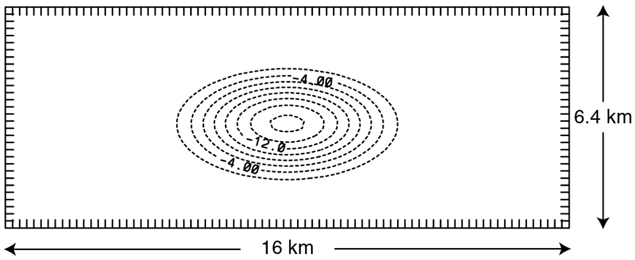
Sun, J., D. W. Flicker, and D. K. Lilly, 1991: Recovery of three-dimensional wind and temperature fields from simulated single-Doppler radar data. *J. Atmos. Sci*., **48**, 876-890.

Sun, J., and N. A. Crook, 1997: Dynamical and microphysical retrieval from Doppler radar observations using a cloud model and its adjoint. Part I: Model development and simulated data experiments. *J. Atmos. Sci*., **54**, 1642-1661.

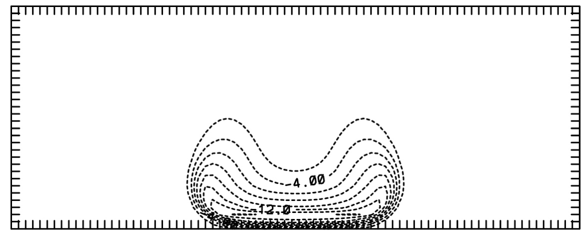
Wang, Z., K. K. Droegemeier, L. White, and I. M. Navon, 1997: Application of a new adjoint Newton algorithm to the 3D ARPS storm-scale model using simulated data. *Mon. Wea. Rev.*, **125**, 2460-2478.

Wicker, L. J., and W. C. Skamarock, 2001: Time splitting methods for elastic models using forward time schemes. Submitted to *Mon. Wea. Rev*.

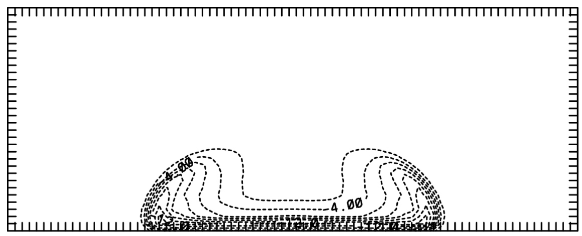
Wicker, L. J., and R. B. Wilhelmson, 1995: Simulation and analysis of tornado development and decay within a three-dimensional supercell thunderstorm. *J. Atmos. Sci*., **52**, 2675-2703.



a) T=0 s

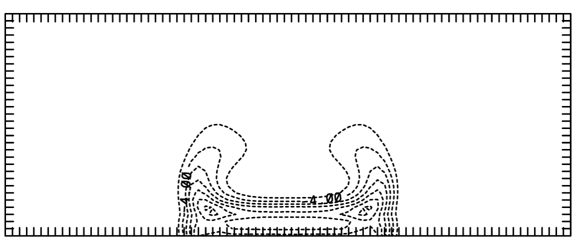


b) T=240 s

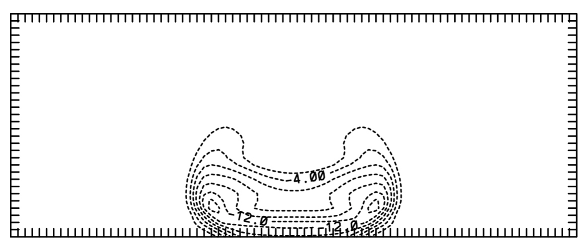


c) T=300 s

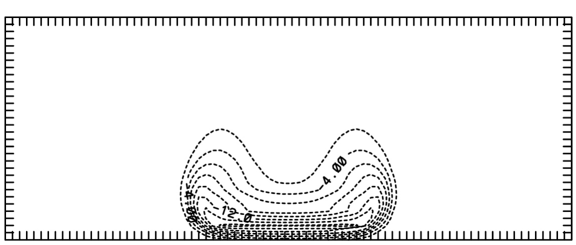
Figure 1. Perturbation temperature (contours at intervals of 2 K) in the control simulation. The domain for the simulation is 36 km wide and 6.4 km tall; only a portion of the domain is shown. The grid spacing is 200 m in both the horizontal and vertical. The model time step is 2.0 s.



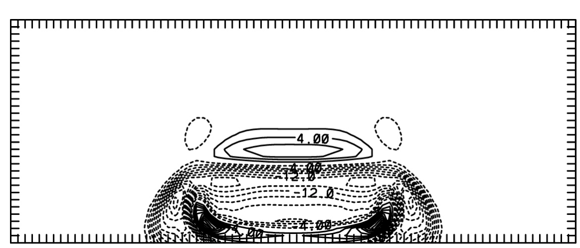
a) New method, after iteration 1 (MAE=0.54 K)



b) New method, after iteration 5 (MAE=0.25 K)



c) New method, after iteration 25 (MAE=0.09K)



d) Traditional method (Gal-Chen 1978; Hane et al. 1981) (MAE=2.01 K)

Figure 2. Retrieved perturbation temperature (contours at intervals of 2 K) at 240 s. The mean absolute error (MAE) in temperature within the subdomain is indicated in parentheses.