7A.4 STATISTICAL INTERPRETATION OF A GENERAL FRAMEWORK FOR THE ANALYSIS OF RAINDROP SIZE DISTRIBUTIONS AND THEIR PROPERTIES

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1. INTRODUCTION

Sempere Torres *et al.* (1994) have recently demonstrated that all previously proposed parameterizations for the raindrop size distribution (DSD) are special cases of a general formulation, which takes the form of a scaling law. In this formulation, the DSD depends both on the raindrop diameter (*D*) and on the value of a so-called reference variable, in most cases taken to be the rain rate (*R*). The generality of this formulation stems from the fact that it is no longer necessary to impose an *a priori* functional form for the raindrop size distribution. Moreover, it naturally leads to the ubiquitous power law relationships between rainfall integral parameters, notably that between the radar reflectivity factor (*Z*) and *R*.

Although the formulation has been successfully verified experimentally on a number of occasions (e.g. Sempere Torres *et al.*, 1998), a clear physical interpretation of its scaling exponents α and β and the general DSD function *g(x)* has been lacking until now. Our objective is to provide such an interpretation, to present methods to apply the general formulation in a meaningful way, and to give several examples of such applications.

2. SCALING LAW FORMALISM

According to the scaling law formalism, raindrop size distributions can be parameterized in terms of the scaling law (Sempere Torres *et al.*, 1994, 1998)

$$N(D,R) = R^{\alpha} g(D/R^{\beta}), \tag{1}$$

where *N(D,R)* (mm^{-1} m^{-3}) is the raindrop size distribution as a function of the (equivalent spherical) raindrop diameter *D* (mm) and the rain rate *R* (mm h^{-1}), α and β are (dimensionless) *scaling exponents*, and *g(x)* is the *general raindrop size distribution* as a function of the scaled raindrop diameter $x = D / R^{\beta}$. In agreement with common practice, *R* is used as the *reference variable* in Eq. (1), although any other bulk rainfall variable could serve as such (notably *Z*). According to this formulation, the values of α and β and the form and dimensions of *g(x)* depend on the *choice* of the reference variable, but do not bear any functional dependence on its *value*.

The importance of the scaling law formalism for radar meteorology stems from the fact that it allows an interpretation of the coefficients of *Z-R* relationships in terms of the values of the scaling exponents and the shape of the general DSD. Substituting Eq. (1) into the definition of *Z* in terms of the DSD leads to the power law

$$Z = aR^{b}, \tag{2}$$

with

$$a = \int_0^\infty x^6 g(x)dx, \tag{3}$$

and

$$b = \alpha + 7\beta \tag{4}$$

(Uijlenhoet, 1999). Hence, the physical interpretation of the scaling exponents and the general DSD we are looking for will directly lead to a physical interpretation of the coefficients of power law *Z-R* relationships.

In a similar manner, the scaling law formalism leads to power law relationships between any other pair of rainfall integral parameters. In particular, substitution of Eq. (1) into the definition of *R* in terms of the DSD (assuming a power law relationship with coefficients c and γ between the raindrop terminal fall speed *v* (m s^{-1}) and *D*) leads to the *self-consistency constraints*

$$\int_0^\infty x^{3+\gamma} g(x)dx = \frac{10^4}{6\pi c}, \tag{5}$$

and

$$\alpha = 1-(4+\gamma)\beta \tag{6}$$

(Uijlenhoet, 1999). Hence, *g(x)* must satisfy an integral equation (which reduces its degrees of freedom by one) and there is only one free scaling exponent. For the applied units, Atlas and Ulbrich (1977) proposed the widely used values *c* = 3.778 and γ = 0.67. Substitution of the latter into Eqs. (4) and (6) leads to $b = 1 + 2.33\beta$.

3. EXAMPLE: LAWS AND PARSONS' DATA

Uijlenhoet (1999) proposes various methodologies to estimate self-consistent values of the scaling exponents. It follows from Eq. (1) that, once α and β are known, *g(x)* can be identified graphically by plotting $N(D,R)/R^{\alpha}$ vs. D/R^{β}. With the aim of bridging the gap between the scaling law formalism and more traditional approaches

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for analyzing empirical DSDs (e.g. Smith, 1993), Uijlenhoet (1999) also presents a method to adjust self-consistent analytical parameterizations to the empirical *g(x)*.

Figure 1a,b provides two representations of the widely used family of mean DSDs presented by Laws and Parsons (1943) for different values of R. Figure 2a,b provides the corresponding empirical *g(x)* and two analytical parameterizations (exponential and gamma). The estimated values of the scaling exponents are α = 0.178 and β = 0.176. The corresponding *Z-R* coefficients, derived from Eqs. (3) and (4), are a = 351 (exponential fit) or a = 326 (gamma fit), both with b = 1.41, close to the standard NEXRAD *Z-R* relationship ($Z = 300R^{1.4}$).

Note that the effect of the scaling methodology is to collapse the eight different mean DSDs onto one single empirical *g(x)*. As such, all dependence of the DSDs on R (or any other rainfall integral parameter) is filtered out by this approach. Apparently, all effects of the spatio-temporal variability of rainfall integral parameters are entirely contained in the values of the scaling exponents.

4. INTERPRETATION OF THE SCALING LAW

Using a statistical formulation of the scaling law, Uijlenhoet (1999) shows that the scaling exponents can be expressed in terms of the variances of and the covariances between the parameters of the DSD. He finds that the values of these scaling exponents determine to what extent it is the fluctuations of the raindrop concentration (or the arrival rate) or the fluctuations of the characteristic raindrop size (or some combination thereof) which control the spatial and temporal variability of the DSD.

Figure 3a,b provides two graphical representations of the self-consistency constraint (Eq. (6)) for three values of γ reported in the literature (dashed line: 0.8; dash-dotted line: the previously mentioned 0.67; dotted line: 0.5). The cross at the point with coordinates (α,β) = (-0.27,0.27) corresponds to a situation of purely raindrop size-controlled rainfall, the plus at the point with coordinates (α,β) = (0,0.21) to Marshall and Palmer's (1948) exponential raindrop size distribution, and the circle at the point with coordinates (α,β) = (1,0) to purely raindrop concentration-controlled conditions. This figure shows that Laws and Parsons' (1943) data, for which (α,β) = (0.178,0.176), must originate from more concentration-controlled conditions than Marshall and Palmer's (1948) data. Uijlenhoet (1999) argues that pure size-control may occur during orographic conditions, pure number-control during equilibrium conditions, and different combinations of size-control and number-control during stratiform and convective conditions (with the former predominantly size-controlled and the latter predominantly number-controlled).

The interpretation of *g(x)* proposed by Uijlenhoet (1999) is straightforward. If R is chosen to be the reference variable, the general DSD simply represents an *equivalent* distribution for a rain rate of 1 mm h^{-1}. It remains a challenge, however, to establish connections between the shape of *g(x)* and the (micro)physical processes producing precipitation.

5. EXAMPLE: ISWS RAINDROP CAMERA DATA

Uijlenhoet *et al.* (2001) present a preliminary study towards relating the type of precipitation to the values of the scaling exponents and the shape of the general DSD. Their particular focus is the behavior of DSDs in extreme rainfall, an issue of considerable practical interest. Figure 4 presents the results of a global analysis of a year's worth of one-minute DSDs for Miami, Florida, a location for which the maximum DSD-derived rain rate exceeds 700 mm h^{-1}. A scaling analysis similar to that presented for Laws and Parsons' data (Section 3) has been applied to all spectra with rain rates exceeding 1, 10 and 100 mm h^{-1}, respectively.

As the rain rate threshold increases from 1 (or 10) to 100 mm h^{-1}, the value of the scaling exponent β decreases roughly from 0.1 to 0 (Figs. 4a,c,e). This indicates a change from a combination of size and number-controlled variability to a situation where the rainfall variability is purely number-controlled. In this situation all moments of the raindrop size distribution are approximately proportional to each other (Fig. 4e).

The shape of the general raindrop size distribution reflects a similar behavior (Figs. 4b,d,f). It also changes markedly while increasing the threshold from 1 (or 10) to 100 mm h^{-1}. Moreover, Fig. 4f seems to display a tendency toward multiple-peak behavior, consistent with the equilibrium hypothesis (e.g. List, 1988), although instrumental artifacts cannot be ruled out at this point.

6. SUMMARY

Our interpretation of the scaling law formulation is an attempt to establish a connection between the observed variability of the shape of DSDs at the ground and the microphysical processes aloft that shape them. We try to relate, in a systematic way, the values of the coefficients of the power laws used in radar meteorology to the type of rainfall (orographic, stratiform, convective, equilibrium) and the climatic setting.

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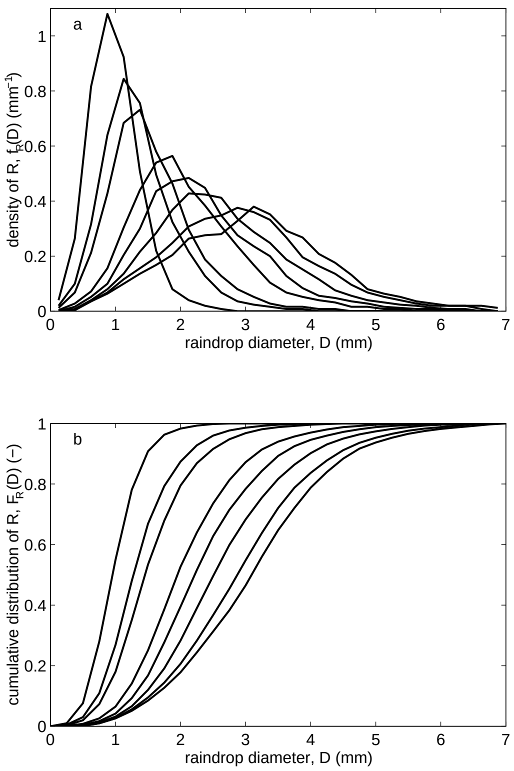

Figure 1. Drop size spectra for R = 0.254, 1.27, 2.54, 12.7, 25.4, 50.8, 101.6, and 152.4 mm h^{-1} (Laws and Parsons, 1943).

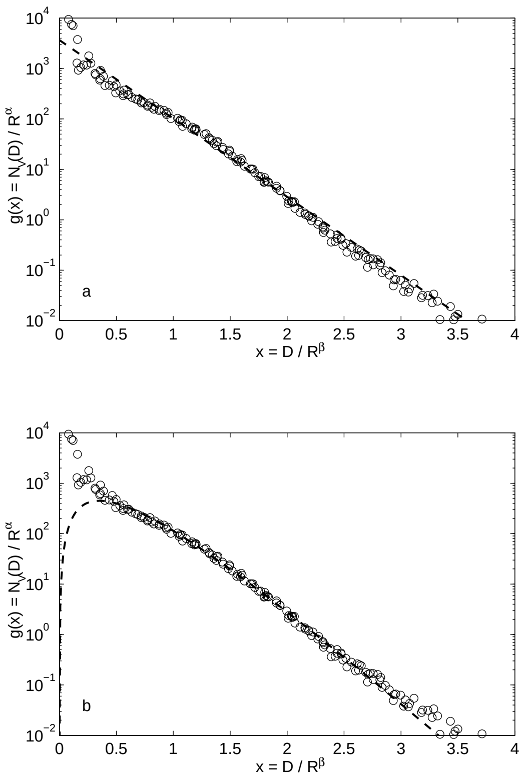

Figure 2. Exponential and gamma fits to general raindrop size distribution *g(x)* corresponding to Fig. 1 (Uijlenhoet, 1999).

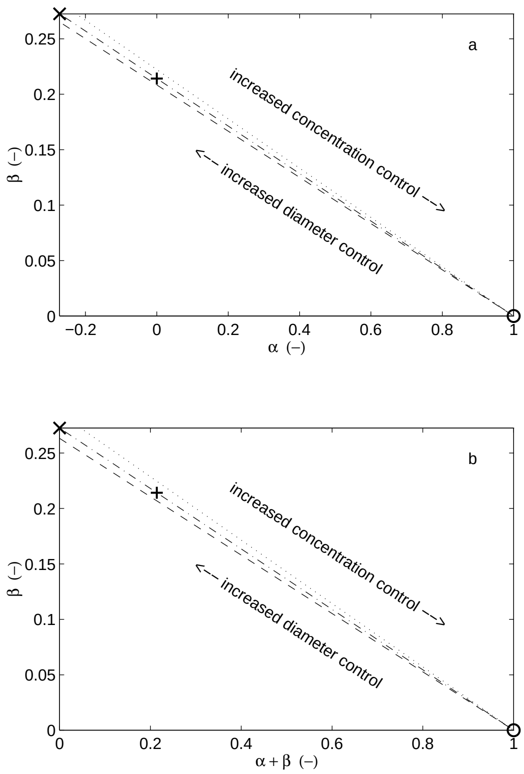

Figure 3. Statistical interpretation of self-consistency relationship (Eq. (6)) between scaling exponents α and β (Uijlenhoet, 1999).

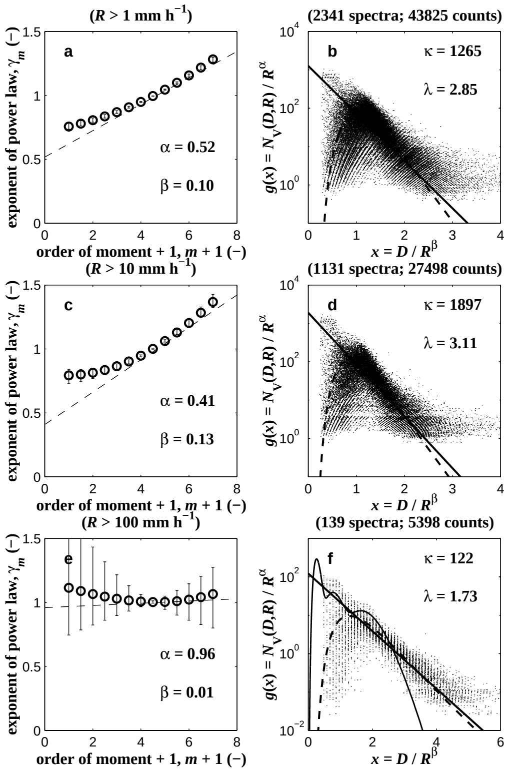

Figure 4. Global scaling analysis of Miami drop camera data for rain rate thresholds of 1, 10 and 100 mm h^{-1} (Uijlenhoet *et al.*, 2001).