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## 1. INTRODUCTION

This paper presents the first application of PCA to data taken from weather surveillance radar. The method constitutes a major improvement to the method of Harasti and List (1997) which used a non-standard,  $S^2$ -mode PCA on Doppler velocity ( $v_D$ ) data to establish hurricane properties in real time. These properties include the polar coordinates of the position of the circulation center,  $(R, \phi_c)$ , and the radius of maximum wind,  $\zeta_m$  (see Fig. 1, and Fig. 1 in paper 7B.5 of these proceedings). The current approach uses a standard,  $S^2$ -mode PCA on the same data, as well as different eigenvectors to establish the same properties.

## 2. THE PCA METHOD

The  $v_D$  data are taken from a PPI scan executed at a low elevation angle ( $\theta \approx 0.5^\circ$ ), then arranged by their range and azimuth coordinates  $(r, \phi)$  into an  $N \times M$  matrix designated by  $\mathbf{D}$ , where  $N$  is the number of range gates and  $M$  is the number of azimuth positions. Each row of  $\mathbf{D}$  represents one of the  $N$  VAD circles in the PPI scan. It is an arbitrary choice to place the range index along the rows of  $\mathbf{D}$ . The data are centered in the range and azimuth directions separately; this paper describes the procedure and uses of the range-centered  $\mathbf{D}$  matrix.

Centering with respect to range is accomplished by subtracting the average  $v_D$  of each column (radial ray) of  $\mathbf{D}$  from every datum of that column; let this centered matrix be denoted by  $\mathbf{D}_r$ . If there are missing  $v_D$  values in  $\mathbf{D}$  then these are set to the average of their particular column in  $\mathbf{D}$  before centering. Such a procedure is common practice in PCA. The variance from the mean of the data is then expressed by formulating the covariance matrix

$$\mathbf{S}_r = \mathbf{D}_r^T \mathbf{D}_r / (N-1) \quad (1)$$

where "T" designates the transpose operation.  $\mathbf{S}_r$  is an  $M \times M$  matrix. Let the ranked, non-zero eigenvalues and eigenvectors of  $\mathbf{S}_r$  be denoted by  $\mu_j$  and  $\mathbf{a}_j$ , respectively. These are determined via solutions to

$$\mathbf{S}_r \mathbf{a}_j = \mu_j \mathbf{a}_j \quad (2)$$

where  $\mathbf{a}_j = [\mathbf{a}_{1j}, \mathbf{a}_{2j}, \mathbf{a}_{3j}, \dots, \mathbf{a}_{Mj}]^T$  and  $j \leq \min(N-1, M)$ . The normalized principal components of  $\mathbf{a}_j$  are,

$$\mathbf{b}_j = \mathbf{D}_r \mathbf{a}_j / |\mathbf{D}_r \mathbf{a}_j| \quad (3)$$

where  $\mathbf{b}_j = [\mathbf{b}_{1j}, \mathbf{b}_{2j}, \mathbf{b}_{3j}, \dots, \mathbf{b}_{Nj}]^T$ . The roles of  $\mathbf{a}_j$  and  $\mathbf{b}_j$  would become interchanged if the matrix operations in

(1) were reversed, which can be done to save on processing time if the dimensions of (1) are smaller with the reversal. Thus, the  $\mathbf{b}_j$  may also be considered eigenvectors as well. The eigenvector  $\mathbf{a}_j$  has  $M$  coefficients; one for each azimuth coordinate,  $\phi$ . Similarly, the eigenvector  $\mathbf{b}_j$  has  $N$  coefficients; one for each range coordinate,  $r$ . The current method is only concerned about the physical interpretation of these eigenvectors. That is, the coefficients  $\mathbf{a}_{\phi j} = \mathbf{a}_j(\phi)$  and  $\mathbf{b}_{rj} = \mathbf{b}_j(r)$  are graphed and analyzed for particular cusps whose abscissas are related to the above-mentioned hurricane properties. Therefore, the scaling of  $\mathbf{a}_j$  and  $\mathbf{b}_j$  is arbitrary. The main advantage of this approach compared to others (Lee and Marks, 2000; Wood and Brown, 1992) is that a *two-dimensional* (PPI) extrema-locating problem is reduced to two, *one-dimensional* extrema-locating problems that are easier to deal with. Also, the PCA method is very robust against typical missing data configurations, and it is not sensitive to the type of wind field being analyzed (e.g. flat vs. Rankine wind profiles; see Fig. 1).

## 3. MODEL RESULTS

The axisymmetric wind model shown in Fig. 1 was developed to test the PCA method. The tangential wind is assumed to have a flat, modified-Rankine, or Rankine profile in the  $\zeta$  direction. The radial wind is assumed to have a similar profile in the  $\zeta$  direction up to the inner-transition radius,  $\zeta_{Ti}$ , and beyond the outer-transition radius,  $\zeta_{To}$ , with a linear variation in between. The outflow and inflow vary linearly with altitude  $z$ . This is the first Doppler velocity model for hurricanes that accounts for typical vertical variations in the radial wind.

The PCA method was applied to simulated PPIs of the Doppler velocity corresponding to the model shown in Fig. 1 for typical values of  $\zeta_m$ ,  $\zeta_{Ti}$ ,  $\zeta_{To}$  and  $R$ .  $\phi_c = 180^\circ$  was used for convenience but without loss of generality. The model results, now discussed, were consistent up to  $\zeta_m/R \approx 0.5$ ; the azimuth-centered  $\mathbf{D}$  matrix approach is required beyond this limit.

Fig. 2 (left side) shows the Standard Azimuth Eigenvector (SAE) and the Standard Range Eigenvector (SRE) for the case of  $\zeta_m$ ,  $\zeta_{Ti}$ , and  $\zeta_{To}$  as shown in Fig. 1, using the modified-Rankine profile with  $R = 70 \text{ km}$ . For approximately  $R > 90 \text{ km}$ , the SAE and

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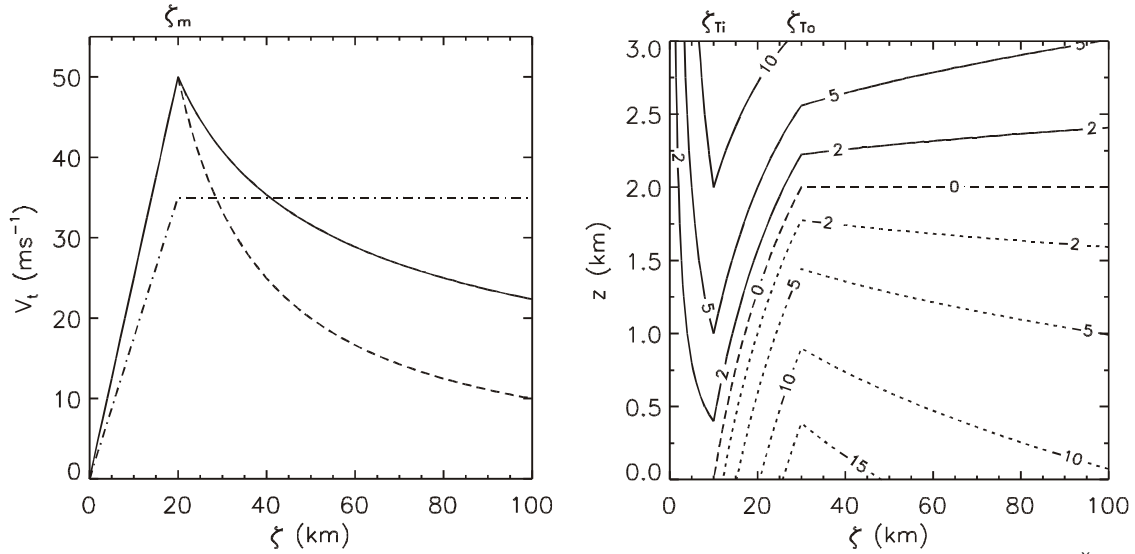


Fig. 1. . Left plot shows the axisymmetric tangential wind ( $V_t$ ) profiles in the radial direction ( $\zeta$ ) used in the model.  $V_t \propto \zeta^X$  where  $X = 1$  for  $\zeta < \zeta_m$ . For  $\zeta \geq \zeta_m$ ,  $X = 0$  (flat - dashed-dotted line),  $X = -0.5$  (modified-Rankine - solid line) or  $X = -1$  (Rankine - dashed line). Right plot shows the altitude ( $z$ ) - radius ( $\zeta$ ) contours of the axisymmetric radial wind model with the adjustable-radii of maximum outflow (solid lines) and inflow (dotted lines) indicated by  $\zeta_{Ti}$  and  $\zeta_{To}$ , respectively.

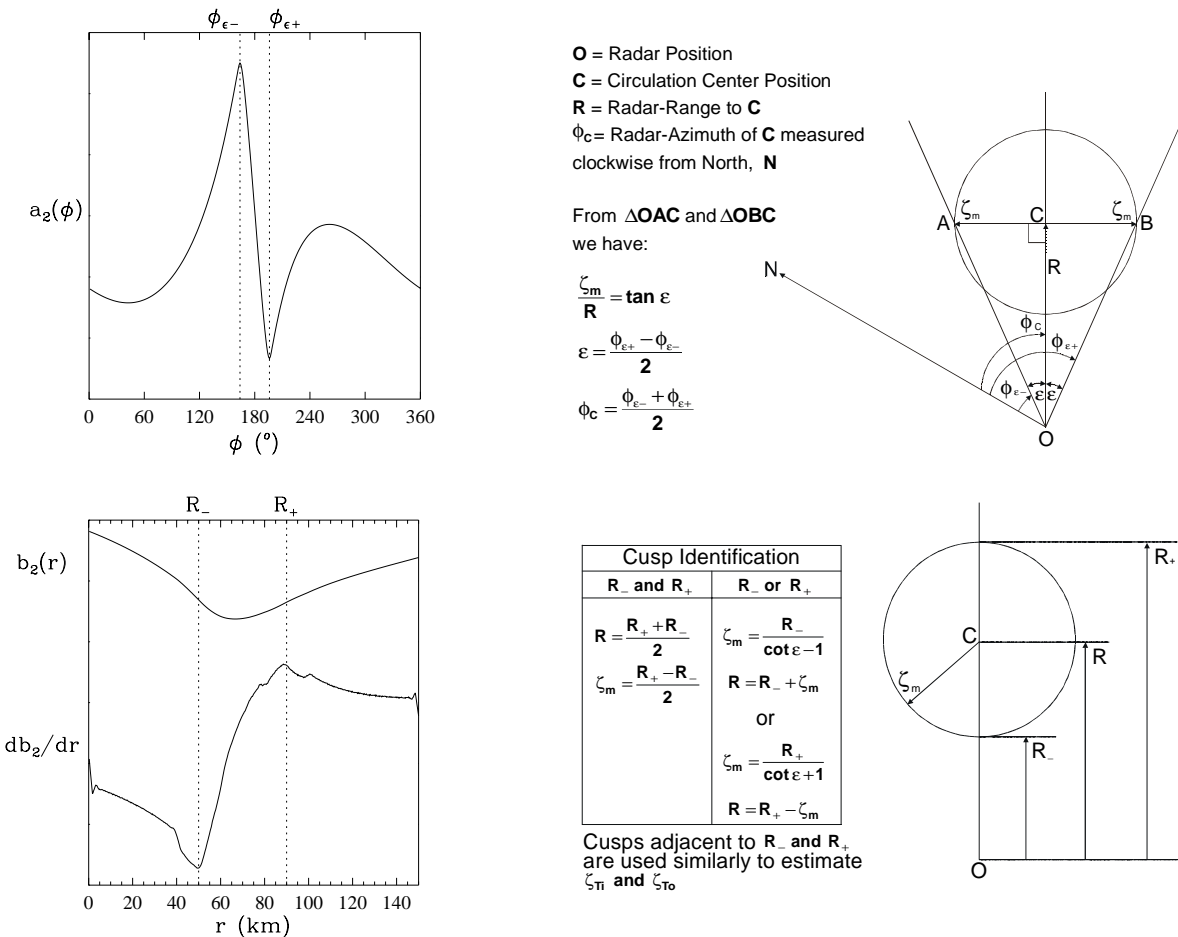


Fig. 2. Example plots of the eigenvector coefficients  $a_2$  versus  $\phi$  (top left), and  $b_2$  and its first derivative versus  $r$  (bottom left). Corresponding right diagrams describe the physical properties derived from these eigenvectors.  $\zeta_m$  is the radius of maximum tangential wind which is set to 20 km, and **C** is located 70 km south of the radar in these examples.

SRE appear as  $\mathbf{a}_1$  and  $\mathbf{b}_1$ , respectively; otherwise, they appear as  $\mathbf{a}_2$  and  $\mathbf{b}_2$ , respectively, as shown in Fig. 2. These are the most stable and easily identified eigenvectors in the  $\mathbf{S}_r$  eigenspace. The corresponding right diagrams in Fig. 2 describe the physical properties derived from these eigenvectors. The average of the abscissas  $\phi_{e-}$  and  $\phi_{e+}$  yield  $\phi_c$  from the SAE, whereas the average of the abscissas  $R_-$  and  $R_+$  yield  $R$  from the first derivative of the SRE. The first derivative is used to accentuate the subtle and hard-to-find inflection points in the SRE. Note that the first inflection in  $db_2/dr$  at, or just past, its maximum defines  $R_+$ . Half of the difference between the abscissas  $R_+$  and  $R_-$  yields  $\zeta_m$ . Also note the cusps and inflection points located 10 km on either side of  $R_-$  and  $R_+$  that are related to  $\zeta_{Ti}$  and  $\zeta_{To}$ . Of course, in real hurricanes, these radii vary in displacement from  $\zeta_m$ , and may be difficult to locate if the radial wind is weaker or has less-abrupt variations in the  $\zeta$  direction than those used in the model.

#### 4. THE VALIDATION OF THE PCA METHOD

The PCA method has been validated using full resolution digital data of Typhoon Alex (1987) and Hurricanes Erin (1995) and Bret (1999). It was also validated operationally on Hurricane Debby (2000) at the NOAA/NWS/NHC using real-time radar imagery (WSR-88D Archive level IV) as a proxy for full resolution data. In all cases, the similarity of the eigenvectors to their theoretical counterparts was striking even in the presence of significant missing data. The PCA results for Hurricane Debby agreed well with aircraft reconnaissance information and provided the best circulation center fix for the GBVTD wind-retrieval method that was concurrently tested. (McAdie et al., 2001). Results obtained from several PPIs scans of Hurricane Erin were also in agreement with concurrent aircraft observations of the wind center corrected for the storm motion. These results will be presented at this conference. The PCA center results for Typhoon Alex and Hurricane Bret agreed within 2 km and 0.4 km, respectively, with the GBVTD-simplex method's (Lee and Marks, 2000) center results for the same storms.

Figs. 3 and 4 show the PCA method's results for Hurricane Bret on August 22, 1999, near 23:44 UTC. Fig. 2 of paper 7B.5 of these proceedings shows the  $\mathbf{v}_p$  data used from the WSR-88D radars at KBRO and KCRP. Except for the distortions due to the missing data shown in the PPI images, the overall shape of the eigenvectors agrees well with the model results. The KBRO and KCRP results also agree to within 2 km of each other as shown in the following:

	Circulation Center Position		
	Latitude (°)	Longitude (°)	$\zeta_m$ (km)
KBRO	26.858	-97.345	18.9
KCRP	26.876	-97.362	17.5

The results of these and future tests should vindicate the PCA method as a valuable tool for the diagnosis and subsequent forecasting of hurricane properties.

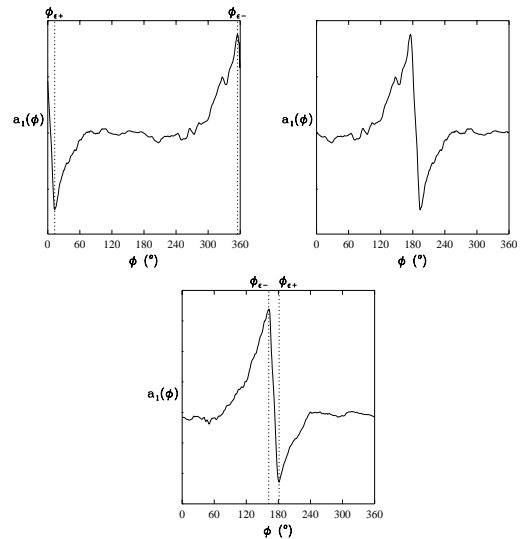


Fig. 3. The SAE derived from the PPI-Doppler-velocity data of Hurricane Bret observed by the WSR-88D radars at KBRO (top) and KCRP (bottom) on August 22, 1999, near 23:44 UTC. The top-right plot shows the SAE for KBRO shifted +180° in azimuth for comparison with the model results.

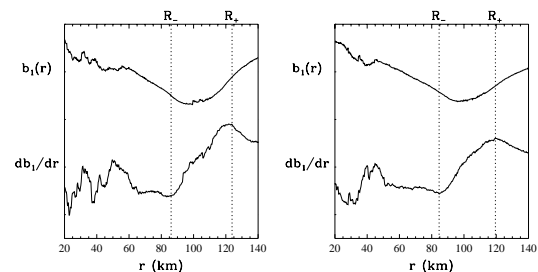


Fig. 4. The SRE and its first derivative derived from the PPI-Doppler-velocity data of Hurricane Bret observed by the WSR-88D radars at KBRO (left) and KCRP (right) on August 22, 1999, near 23:44 UTC.

#### 4. Acknowledgements

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#### 5. References

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