7A.3

ANALYTICAL SOLUTIONS TO SAMPLING FLUCTUATIONS IN RAINDROP SIZE DISTRIBUTION MEASUREMENTS

Remko Uijlenhoet[a,*], Daniel Sempere Torres[b], and Jean-Dominique Creutin[c]
[a] Princeton University, Princeton, NJ
[b] Universitat Politècnica de Catalunya, Barcelona, Spain
[c] LTHE, Grenoble, France and University of Iowa / IIHR, Iowa City, IA

1. INTRODUCTION

Fluctuations in disdrometer measurements of raindrop size distributions (DSDs) and derived rainfall properties are due 'both to statistical sampling errors and to real fine-scale physical variations which are not readily separable from the statistical ones' (Gertzman and Atlas, 1977). The terminology adopted here for these two types of fluctuations is "sampling fluctuations" and "natural variability", respectively.

It would be of great practical importance to be able to distinguish between both sources of variability. This is because the parameters of DSDs and the coefficients of *Z-R* relationships should represent the properties of the type of rainfall to which they pertain as much as possible and the properties of the raindrop sampling device from which they are derived as little as possible.

It is therefore necessary to investigate to what extent rainfall fluctuations observed with different types of instruments reflect the properties of the rainfall process itself and to what extent they are merely instrumental artifacts.

2. METHODOLOGY

We use a statistical model of the microstructure of rainfall to derive explicit expressions for the magnitude of the sampling fluctuations in rainfall properties derived from DSD measurements. The model is a so-called marked point process (e.g. Smith, 1993), where the points represent the drop centers and the marks their sizes (Fig. 1).

The simplest situation obviously is the case where only sampling fluctuations are present and no natural variability. As rare as this situation may be in practice, it is of more than merely academic interest. It provides a lower bound to the magnitude of the variability in a practical situation, where sampling fluctuations and natural variability exist side-by-side.

3. POISSON OR FRACTAL STATISTICS?

In the absence of natural variability, it is plausible to assume that (1) raindrops are uniformly distributed in space and (2) raindrop sizes are independent of each other and of the positions of the drops in space. This implies that the arrival process of drops at the disdrometer is a so-called homogeneous Poisson process and that the numbers of drops arriving at non-overlapping time intervals have independent Poisson distributions (e.g. Joss and Waldvogel, 1969). Figure 2 provides empirical evidence for the Poisson hypothesis in a stationary rainfall event, characterized by 35 minutes of uncorrelated fluctuations around a constant mean rain rate of 3.5 mm h^{-1} (Uijlenhoet, 1999).

As an alternative to the Poisson hypothesis and its extensions (e.g. Kostinski and Jameson, 1997), Lovejoy and Schertzer (1990) provide empirical evidence for a fractal description of rainfall. However, we show analytically that the fractal correlation dimension they obtain from a box-counting analysis of the spatial distribution of raindrop stains on blotting paper can be explained entirely as a boundary effect (Figs. 3 and 4). Hence, their test results are not significant enough to reject the Poisson hypothesis.

4. RESULTING SAMPLING DISTRIBUTIONS

Within the framework of the Poisson hypothesis, we show analytically that (and how) the sampling distribution of the estimator of any rainfall integral variable converges to a Gaussian distribution. Figures 5 and 6 show such sampling distributions for rain rate and for the maximum raindrop diameter. In addition to being useful in their own right, these results provide a theoretical confirmation and explanation of the simulation results of Smith *et al.* (1993).

5. OUTLOOK: TIME *AND* SPACE

We propose two extensions of the homogeneous Poisson process model of sampling fluctuations: (1) inclusion of natural variability (Fig. 7); (2) inclusion of spatial dimensions (Fig. 8). The latter is particularly important with regard to extending the classical theory of weather radar to include density fluctuations.

REFERENCES

Gertzman, H. R. and D. Atlas, 1977. *J. Geophys. Res.* **82**, 4955-4966.
Joss, J. and A. Waldvogel, 1969. *J. Atmos. Sci.* **26**, 566-569.
Kostinski, A. B. and A. R. Jameson, 1997. *J. Atmos. Sci.* **54**, 2174-2186.
Lovejoy, S. and D. Schertzer, 1990. *J. Appl. Meteor.* **29**, 1167-1170.
Smith, J. A., 1993. *J. Appl. Meteor.* **32**, 284-296.
Smith, P. L., Z. Liu, and J. Joss, 1993. *J. Appl. Meteor.* **32**, 1259-1269.
Uijlenhoet, R., 1999: *Parameterization of rainfall microstructure for radar meteorology and hydrology.* Doctoral dissertation, Wageningen University, the Netherlands, 279 pp.

* *Corresponding author address*: Dr. Remko Uijlenhoet, Department of Civil and Environmental Engineering, Princeton University, Princeton, New Jersey 08544; phone: (609)258-5429; email: ruijlenh@princeton.edu

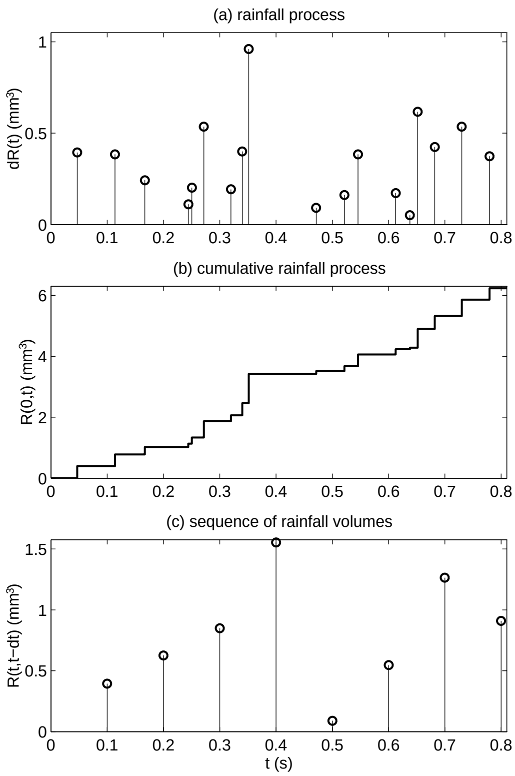

Figure 1. Three representations of the small-scale, discrete, stochastic temporal structure of rainfall at a surface: as a marked point process (a, b) and as a sequence of rainfall volumes (c).

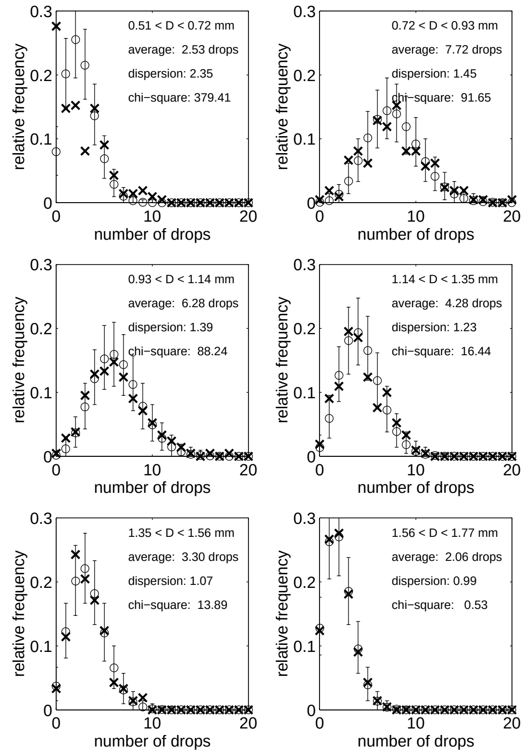

Figure 2. Empirical (crosses) and theoretical Poisson (circles) frequency functions of raindrop counts in 10-s intervals for 35 min of observations with 50-cm^2 optical disdrometer (Uijlenhoet,1999).

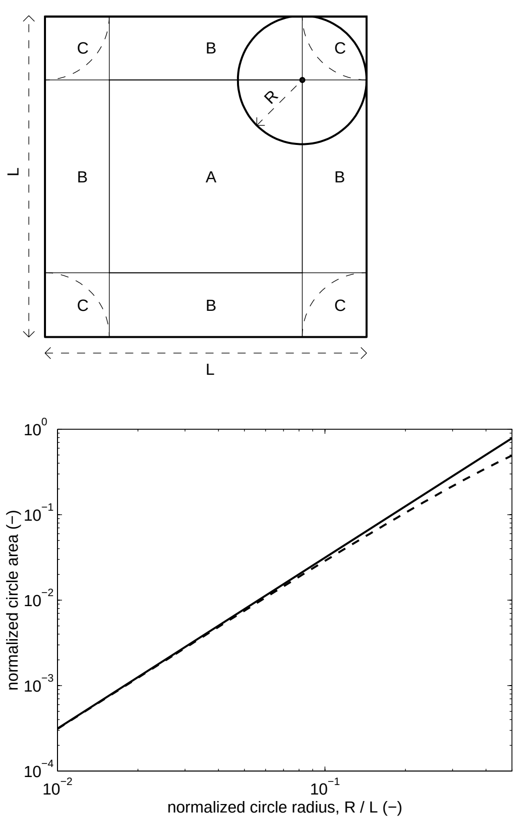

Figure 3. Schematic representation of raindrop stain on blotting paper (top) and expected surface area of circle falling inside blotting paper (bottom), without (dashed line) and with (solid line) correction for boundary effects.

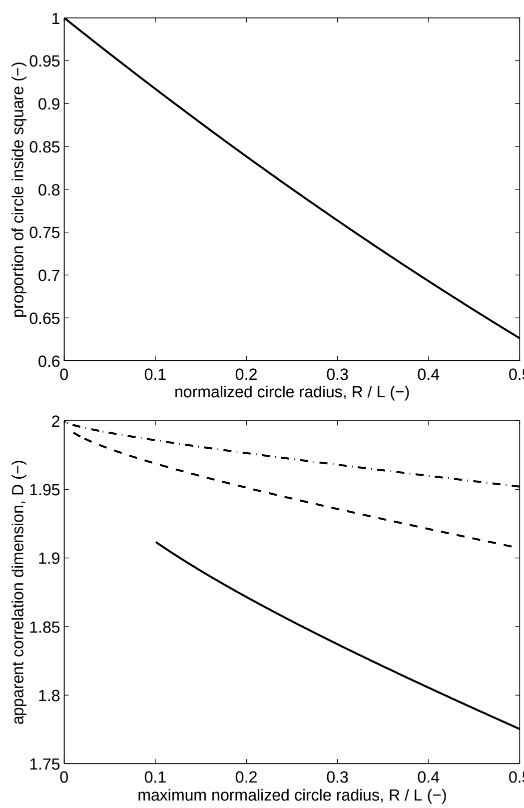

Figure 4. Expected proportion of circle falling inside blotting paper (top) and apparent fractal correlation dimension of uniformly distributed drops (bottom), for different values of minimum normalized radius.

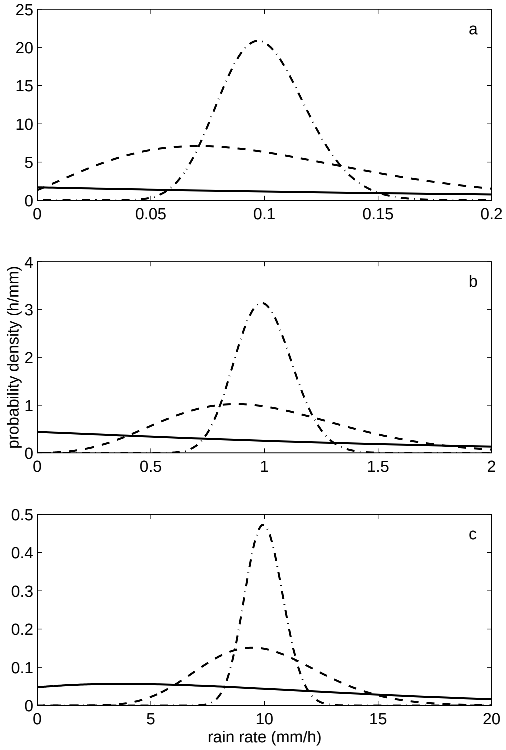

Figure 5. Sampling distributions of rain rate for 50-cm^2 disdrometer for different mean rain rates (0.1, 1, and 10 mm h^{-1}) and integration times (solid: 0.1 s; dashed: 1 s; dash-dotted: 10 s).

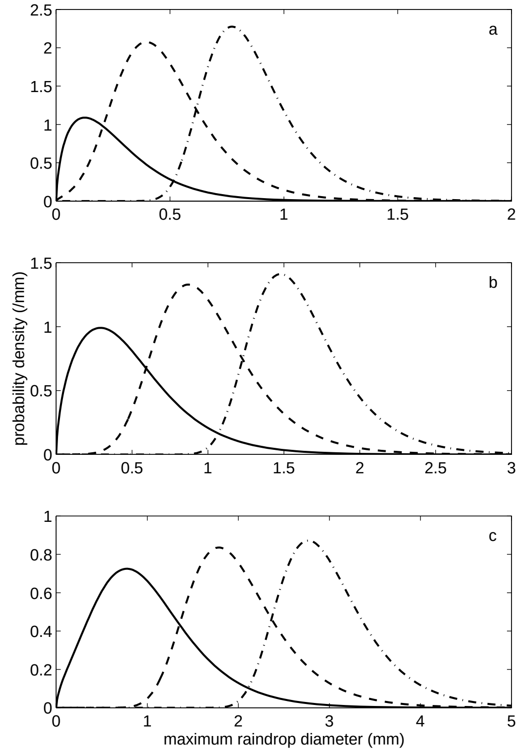

Figure 6. Sampling distributions of maximum drop diameter for 50-cm^2 disdrometer for different rain rates (0.1, 1, and 10 mm h^{-1}) and integration times (solid: 0.1 s; dashed: 1 s; dash-dotted: 10 s).

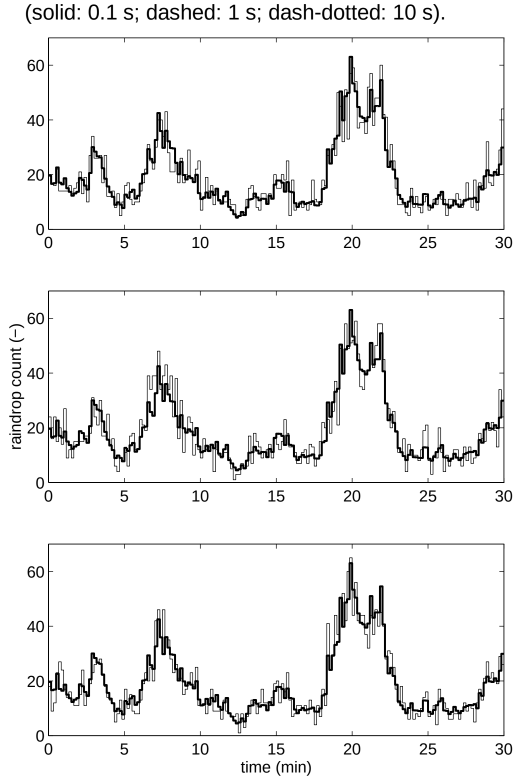

Figure 7. Three realizations of raindrop arrival process modeled as a doubly-stochastic Poisson process, including both natural variability (bold lines) and sampling fluctuations (thin lines).

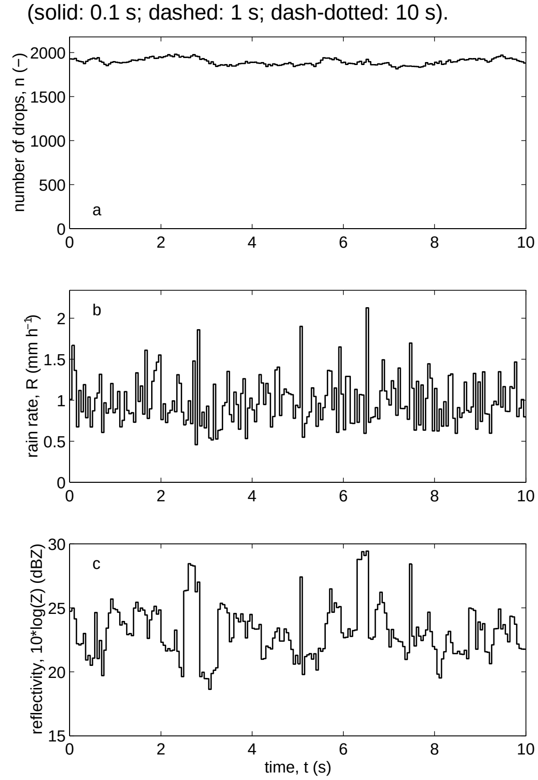

Figure 8. Simulation of temporal evolution of rainfall integral variables in 1-m^3 sample volume in statistically homogeneous rainfall (Uijlenhoet, 1999): number of drops (a); rain rate (b); reflectivity (c).