P1.9 ESTIMATION OF POLARIZATION ERRORS FROM COVARIANCE MATRICES OF CSU-CHILL RADAR DATA

J.C. Hubbert,* V.N. Bringi

1. INTRODUCTION

The covariance matrix of weather targets can be measured by the CSU-CHILL radar which alternately transmits pulses of H (horizontal) and V (vertical) polarized energy and receives simultaneously both H and V polarized signals. The covariance matrix is a function of not only the scattering process but is also contaminated by polarization errors of the radar. If the covariance matrices are well calibrated it is possible to estimate these polarization errors from data. The expected value of the polarization errors is quite small and thus the copolar measurends will be negligibly affected. However, the crosspolar measurands can be greatly affected even by polarization errors of a few tenths of a degree (Hubbert et al., 1999). Thus knowledge of polarization errors is important to the interpretation of radar variables such as LDR (liner depolarization ratio) and $\rho_{hh,vh}$, the co-to-cross correlation coefficient.

Polarization errors have been covered in detail by McCormick (1981). That treatment was analytical and applied primarily to circular polarization basis radar. Here the polarization errors are integrated and represented by a single error term for each channel. The polarization errors are easily included in the scattering model of Hubbert et al. (1999) by pre- and post- multiplying $\mathbf{S}'$ by the error matrix $\boldsymbol{\Upsilon}$

$$\mathbf{S_e} = \boldsymbol{\Upsilon}^T \mathbf{S}' \boldsymbol{\Upsilon} \tag{1}$$

where

$$\boldsymbol{\Upsilon} = \begin{bmatrix} i_h & \varepsilon_v \\ \varepsilon_h & i_v \end{bmatrix} \tag{2}$$

with constraints $i_h^2 + |\varepsilon_h|^2 = i_v^2 + |\varepsilon_v|^2 = 1$ and i_h, i_v being real. The polarization errors of the H and V channels are represented by ε_h and ε_v, respectively. The polarization errors can also be equivalently represented with the geometric ellipse parameters of tilt angle, τ and ellipticity angle, ϵ. These variables are related by

$$\tan 2\tau = \frac{2\Re(\chi)}{1-|\chi|^2} \qquad \sin 2\epsilon = \frac{2\Im(\chi)}{1+|\chi|^2} \tag{3}$$

where χ is the polarization ratio. For H errors $\chi = \varepsilon_h / i_h$ and for V errors $\chi = i_v/\varepsilon_v$. As can be seen from the equations, if the ε_h (ε_v) is real then ϵ is zero and if ε_h (ε_v) is imaginary then τ is zero. If the errors are orthogonal, i.e., $\varepsilon_v = -\varepsilon_h^*$, then $\boldsymbol{\Upsilon}$ is unitary and Eq.(1) represents an orthogonal change of polarization basis transformation. However, in general polarization errors will not be orthogonal. Separating the polarization errors into their geometric components gives a convenient and intuitive way to analyze polarization errors.

The covariance matrix has the form,

$$\boldsymbol{\Sigma}(\chi) = \begin{bmatrix} P_{co}^A(\chi) & \sqrt{2}R_x^A(\chi) & R_{co}(\chi) \\ \sqrt{2}R_x^{A*}(\chi) & 2P_x(\chi) & \sqrt{2}R_x^B(\chi) \\ R_{co}^*(\chi) & \sqrt{2}R_x^{B*}(\chi) & P_{co}^B(\chi) \end{bmatrix} \tag{4}$$

The real elements $P_{co}^A(\chi)$ and $P_{co}^B(\chi)$ along the diagonal are the copolar backscattered powers while P_x is the crosspolar power (eg. if χ corresponds to H polarization, then $P_{co}^A = P_{hh}$, $P_{co}^B = P_{vv}$ and $P_x = P_{vh}$ or P_{hv}). Also the three complex covariances would correspond to $R_{co} = R_{hh,vv}$, $R_x^A = R_{hh,vh}$ and $R_x^B = R_{hv,vv}$. Disregarding absolute power and phase, to construct a covariance matrix from data, two relative powers and two relative phases need to be calibrated. The relative powers calibrate $Z_{dr} = 10\log(P_{hh}/P_{vv})$ and $LDR = 10\log(P_{vh}/P_{hh})$. The LDR can be calibrated precisely with sun pointing data (termed *suncal*) that is routinely gathered by the radar. To calibrate Z_{dr} a scatter diagram of P_{vh} versus P_{hv} is constructed. By reciprocity these powers should be equal and any observed offset can be attributed differential gain bias (termed *xpow_bias*) of the radar system. It is then easily shown that $Z_{dr\_bias} = 2suncal - xpow\_bias$ (in dB scale). The system phase offset for $\Psi_{dp} = \arg\{R_{co}^*\}$ is easily found by plotting ray profiles of Ψ_{dp} and determining the phase that makes Ψ_{dp} begin at zero degrees. The co-to-cross phases, $\Psi_{hh,vh} = \arg\{R_x^A\}$ and $\Psi_{vv,hv} = \arg\{R_x^{B*}\}$, are much less understood. Since the correlation coefficient $\rho_{hh,vh} = |R_x^A|/(P_{co}^A P_x)^{0.5}$ is typically quite small (< 0.5 for most precipitation), the standard deviation of the $\Psi_{hh,vh}$ (and $\Psi_{vv,hv}$) is very high, typically in the tens of degrees. These phases are

*Department of Electrical Engineering, Colorado State University, Fort Collins, Colorado 80523, email: hubbert@engr.colostate.edu

also complicated functions of not only the mean canting angles of the backscatter medium (α) and propagation medium (θ) but also Z_{dr} and LDR. Next the scattering model as described in Hubbert et al. (1999) is used to examine the co-to-cross phases.

Fig. 1 shows the phases $\hat{\Psi}_{xh} = \Psi_{hh,vh} + \Psi_{dp}/2$ and $\hat{\Psi}_{xv} = \Psi_{vv,hv} - \Psi_{dp}/2$ as a function of principal plane Ψ_{dp} with the ratio α/θ as a parameter. The model shows two important behaviors: 1) $\hat{\Psi}_{xv}$ and $\hat{\Psi}_{xh}$ are nearly identical (some slight variations of $< 3°$ exists that are not shown in the plot) and 2) for positive (negative) θ, $\hat{\Psi}_{xv}$ and $\hat{\Psi}_{xh}$ are positive (negative). To find the system offset phases for these phases the fact that $\Psi_{vv,hv} - \Psi_{hh,vh} = \Psi_{dp}$ is employed. For the CSU-CHILL system it can be shown that the system offset phases for $\Psi_{vv,hv}$ and $\Psi_{hh,vh}$ are equal but opposite in sign. This common offset can be adjusted until range profiles of $\Psi_{vv,hv} - \Psi_{hh,vh}$ match Ψ_{dp}. This then completes the calibration of the covariance matrix.

To estimate polarization errors it is assumed the mean canting angle of the propagation medium will on average be zero. The individual backscatter resolution volumes may, however, have non-zero mean canting angles. It is known that if precipitation particles have a mean canting angle of zero and if they are symmetrically distributed around zero degrees, then theoretically $R_x^A = R_x^B = 0$ when operating in the H/V polarization basis. In practice these numbers are relatively small so that $\rho_{hh,vh}$ and $\rho_{vv,hv}$ are typically on the order of 0.1 to 0.2 for a rain medium. Since the polarization errors can be non-orthogonal, finding the tilt angle in the eigenpolarization basis will not reveal the true error terms. Thus the errors are determined by finding vaues of the error terms, τ_h, ϵ_h, τ_v, ϵ_v, that minimize $\rho_{hh,vh}$ and this is done via a simple search method.

The polarization error estimation method is as follows. A range profile(s) of data is selected where there is significant increase in Ψ_{dp} which will accentuate the errors. For example, if significant tilt error is present (a few tenths of a degree is sufficient) then in general $\rho_{hh,vh}$ will increase significantly with increasing Ψ_{dp}. Calibrated covariance matrices are constructed at each sample point (0.15 km in the following case). For all the covariance matrices, error terms τ_h, τ_v, ϵ_h, ϵ_v, are varied and the minimum of the sum

$$\Omega = \sum_{i=1}^{n} \rho_{hh,vh}(i)(\tau_h,\ \tau_v,\ \epsilon_h, \epsilon_v) \qquad (5)$$

is found. The resulting tilt and ellipticity angles are considered polarization errors.

1. DATA ANALYSIS

Shown in Figs. 2-5 is a ray of data gathered during STEPS (Severe Thunderstorm Electrification and Precipitation Study) on 21 July 2000 through a heavy rain cell with reflectivities of 40 dBZ to 60 dBZ. Fig. 2 shows Ψ_{dp} increasing 100° over 50 km and a filtered version of $\Psi_{vv,hv} - \Psi_{hh,vh}$ which mimics Ψ_{dp} very well as expected. Fig. 3 shows a range profile of $\hat{\Psi}_{xh}$. If the mean propagation canting angle is zero then this phase should be 0° or 180° depending on the mean canting angle of the backscatter resolution volume. Since this phase is nearly always positive we surmise from Fig. 1 that θ must be positive. Thus if the actual mean canting angle of the propagation medium is zero, the polarization tilt error is likely to be positive. Fig. 3 is not a single ray anomaly but rather is seen in nearly all range profiles of CSU-CHILL data gathered during STEPS that possess significant amounts of increasing Ψ_{dp}. With 150 m gate sampling this allows for the construction of 333 covariance matrices which are used in the minimization procedure. The resulting polarization errors are $\tau_h = 0.5°$, $\epsilon_h = 0.1°$, $\tau_v = 90.5°$, $\epsilon_v = -0.4°$. Using these error terms a 3X3 transformation matrix can be constructed and the polarization errors can be removed from the data by pre and post multiplying the measured covariance matrices by this matrix (Huang et al., 2001). Range profiles of the corrected data can then be made. Fig. 4 shows filtered $\rho_{hh,vh}$ and corrected $\rho_{hh,vh}$ and as expected, corrected $\rho_{hh,vh}$ is significantly reduced to an average level that is consistant with rain. Similarly, Fig. 5 shows filtered LDR and corrected LDR. Again the the corrected LDR is reduced as it should be with the polarization errors removed.

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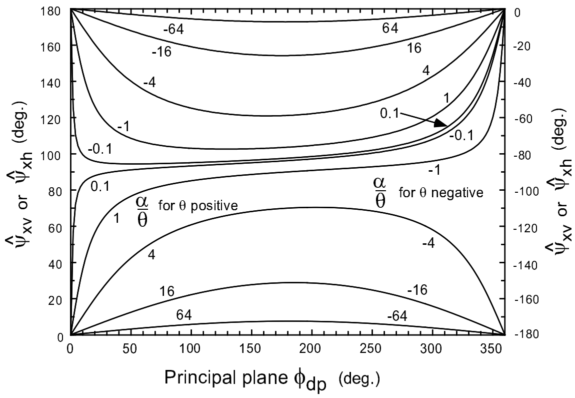

Figure 1: $\hat{\Psi}_{xh}$ or $\hat{\Psi}_{xv}$ as a function of principal plane Ψ_{dp} with the ratio of mean backscatter to mean propagation canting angle as a parameter.

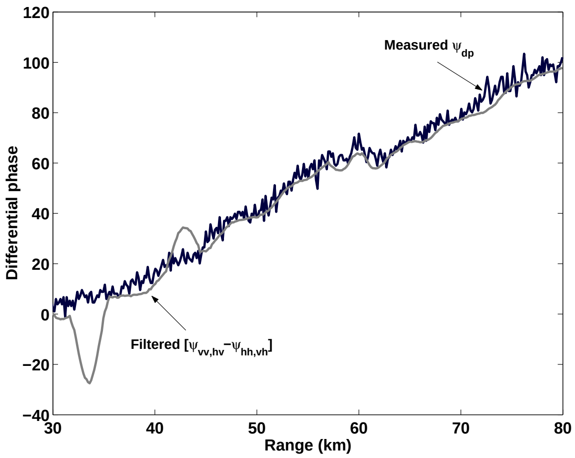

Figure 2: Range profile of Ψ_{dp} and $\Psi_{vv,vh} - \Psi_{hh,vh}$.

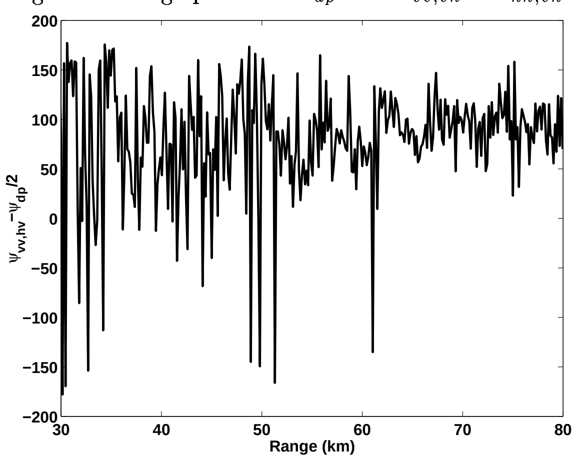

Figure 3: Range profile of $\Psi_{vv,hv} - \Psi_{dp}/2$.

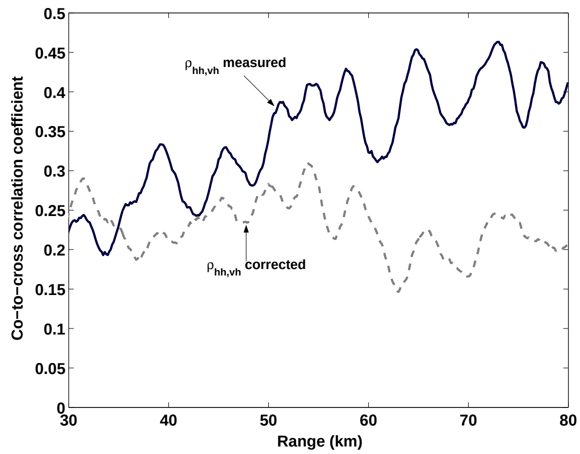

Figure 4: Range profile of filtered $\rho_{hh,vh}$ and polarization error corrected $\rho_{hh,vh}$.

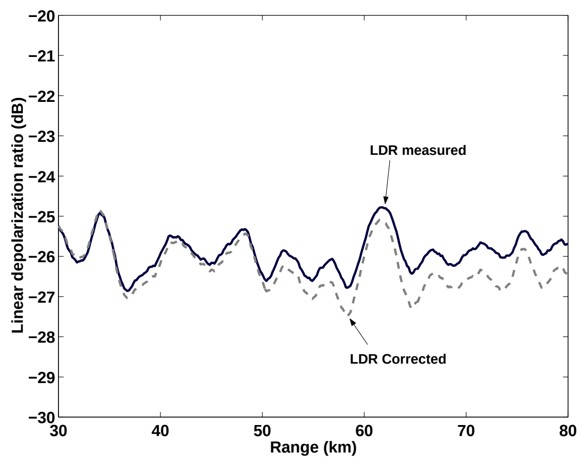

Figure 5: Range profile of filtered LDR and polarization error corrected LDR.