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A THREE-DIMENSIONAL VARIATIONAL DATA ASSIMILATION SCHEME FOR A STORM-SCALE MODEL

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1. Introduction

Currently, the data analysis system (ADAS, Brewster 1996) of the Advanced Regional Prediction System (ARPS, Xue et al. 2001; Xue et al. 1995) of CAPS employs the Bratseth (1986) interpolation scheme based on successive corrections. The system has been successfully used in research and operational meso- and storm-scale simulations and forecasting, and is flexible in dealing with data of varying spatial densities. It is also computationally very efficient. A drawback of such schemes, including the until-recent-years rather popular optimal interpolation (OI) schemes (Bratseth scheme actually converges to OI), is that observations that differ from the analysis variables cannot be directly analyzed. Examples include the precipitable water from GPS, satellite radiances, radar radial velocity and reflectivity. Variational methods have the advantages of being able to directly use the observations in a cost function, and through the minimization of this function the desired analysis variables that give a best fit to the data, subjecting to background and other dynamical constraints, can be obtained.

While four-dimensional variational (4DVAR) data assimilation is generally considered superior, a 3D variational (3DVAR) assimilation system is the necessary first, and also computationally more efficient, step towards that goal. 3DVAR systems have been developed and operationally implemented for large-scale NWP at several operational centers in recent years (e.g., Parrish and Derber 1992; Courtier 1998) and progresses are also being made in developing systems for mesoscale models (e.g., Wu et al, 2001).

In this paper, an incremental 3DVAR system developed recently for the ARPS is described. In the system, the background error covariance matrix is modeled using a recursive filter (Hayden and Purser 1995) and the square root of the matrix is used for preconditioning. Some initial numerical experiments have been conducted based on this scheme and the results are compared with that from the ADAS.

2. 3DVAR formulation

The basic cost function *J*, may be written as the sum of two quadratic terms:

$$J(x) = \frac{1}{2}(x - x^b)^T B^{-1}(x - x^b) + \frac{1}{2}(H(x) - y^o)^T R^{-1}(H(x) - y^o). \quad (1)$$

The first term measures the departure of the analysis vector, x from the background x^b, which is weighted by the inverse of the background error covariance matrix B^{-1}; the second term measures the departure of the projection of analysis to the observation space, $H(x)$, from the observations themselves (y^o), which is weighted by the inverse of the combined observation and observation-operator error covariance matrix, R^{-1}. In our scheme, the background field can be provided by a single sounding, a previous ARPS forecast, or another operational forecast model. Observations currently tested include: single-level surface data (including Oklahoma Mesonet), multiple-level or upper-air observations (such as rawinsondes and wind profilers), as well as Doppler radar observations.

The analysis is to find model state x^a for which J is at a minimum. At the minimum, the derivative of J vanishes. The Hessian of $J(x)$ is:

$$\nabla^2 J(x) = B^{-1} + H^T R^{-1} H\ . \quad (2)$$

If $\nabla^2 J(x)$ is positive definite, then there is a unique x^a that minimizes the cost function $J(x)$.

By defining $v = \sqrt{B^{-1}}(x - x^b) = C^{-1}\delta x$, (3)

and letting,

$$H(x) = H(x^b + \delta x) \simeq H(x^b) + \mathbf{H}\ \delta x\ , \quad (4)$$

we obtain a new representation of the incremental cost function:

$$J_{inc} = \frac{1}{2} v^T\ v + \frac{1}{2}(\mathbf{H}Cv - d)^T R^{-1}(\mathbf{H}Cv - d)\ . \quad (5)$$

The Hessian of J_{inc} is

$$\nabla^2 J_{inc} = \mathbf{I} + C^T \mathbf{H}^T R^{-1} \mathbf{H} C\ , \quad (6)$$

where **I** stands for the identity matrix. Comparing (6) with (2), we see that the smallest eigenvalue of Hessian matrix from (6) will be at least larger than one, so that the condition number will not become infinite (Lakshimivaranhan, 1999). This new Hessian matrix is much better conditioned than the Hessian matrix of original problem (1).

The matrix C defined in (5) is realized as,

$$C = DF, \quad (7)$$

where D is a diagonal matrix of standard deviation of the background error. For simplicity, we assume that D has diagonal elements specified by the error estimation of numeri-

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cal experimentations. F is a recursive filter (Hayden and Purser 1995, Lorenc 1992) defined by

$$\begin{aligned} Y_i &= \alpha Y_{i-1} + (1-\alpha) X_i \qquad for \;\; i = 1, \cdots, n \\ Z_i &= \alpha Z_{i+1} + (1-\alpha) Y_i \qquad for \;\; i = n, \cdots, 1 \end{aligned} \tag{8}$$

where, X_i is the initial value at grid point i, Y_i is the value after filtering for i=1 to n, Z_i is the initial value after one pass of the filter in each direction and α is the filter coefficient. This is a first-order recursive filter, applied in both directions to ensure zero phase change. Multi-pass filters are built up by repeated application of (8). This filter is applied in all three directions.

We also assume that the observation errors are independent, that is, the observation error covariance matrix R is also a diagonal matrix with constant diagonal elements given by estimated error of each type of observation.

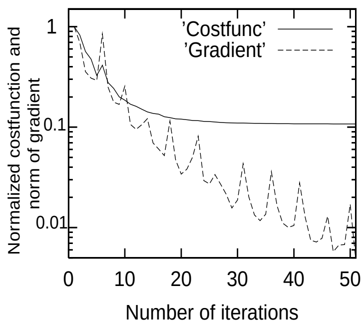

Fig 1: The scaled cost function (J/J_0) *(solid line), and scaled gradient norm (*$\|g\|/\|g_0\|$*) (dashed line) as a function of the number of iterations.*

3. Test results

As a preliminary example, the case of June 8, 1995 is used to test the 3DVAR scheme. It was a major day during the 1995 Verification on Onset of Rotation in Tornadoes Experiment (VORTEX 95) as several damaging tornadoes were produced by storms in the eastern Texas Panhandle. In the experiment, the first guesses before minimization are zero.

Figure 1 shows that the cost function starts to level off after 20 iterations in the control experiment. After 20 iterations, the curve of the cost function becomes essentially horizontal, although the norm of the gradient is still decreasing.

The quality of variational analysis can be ascertained by comparing the analysis fields with the ADAS analysis. In Fig 2, we show the contours of u-component of wind field. Comparing the 3DVAR with the ADAS, we can conclude that the quality of the analysis is reasonable although more careful evaluations, especially numerical forecast experiments, are needed to justify the result of analysis.

In another experiment, a single wind profile is used in the middle of analysis domain. It is found that the isotropic spread of the observation information when using a single pass of the filter agrees with our expectation (figure not shown). The influence area of this single observation depends on the number of passes used and the correlation scale. More detailed results will be presented at the conference.

4. Conclusion

In this paper, we described an incremental 3DVAR system for the ARPS model. The system is preconditioned by the background error covariance matrix, which is based on a recursive filter. Numerical experiments show that a reasonable reduction in the cost function is achieved in the minimization process and the quality of the analysis is reasonable. The single-observation experiment shows that the recursive filter performs adequately in spreading the observational information. New data types will be added to this assimilation scheme and numerical forecast experiments will be performed to further test the quality of this system in the future.

4. Acknowledgement

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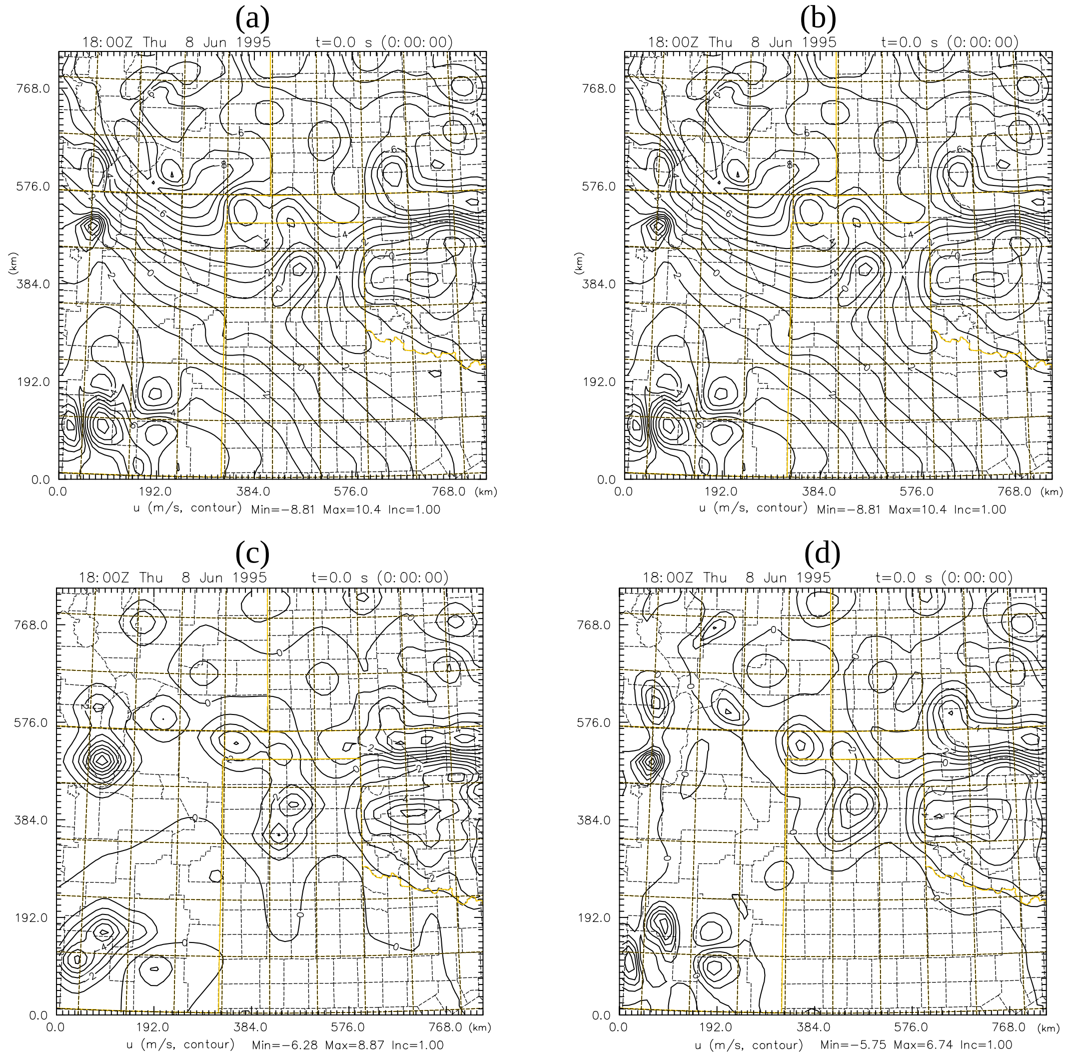

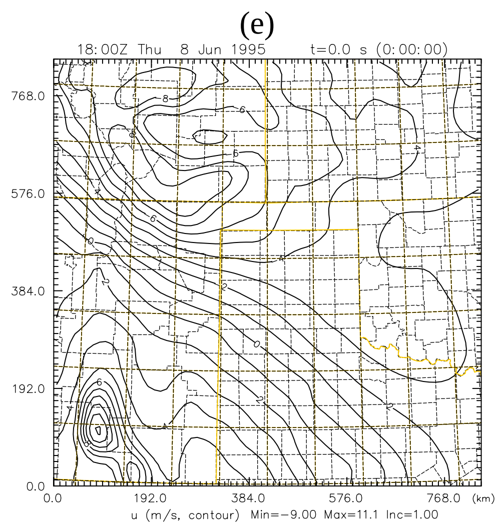

Fig. 2. Comparison of 3DVAR analysis and ADAS analysis of east-west velocity (u) field at the surface, for 18 Z, June 8, 1995. (a) the 3DVAR analysis, (b) the ADAS analysis, (c) the 3DVAR analysis increment, (d) the ADAS analysis increment, and (e) the analysis background. Close to this time, tornadic supercell storms occurred in the Texas Panhandle area.