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ADAPTIVE TUNING OF OBSERVATION ERROR PARAMETERS IN A VARIATIONAL DATA ASSIMILATION

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1. INTRODUCTION

Modern assimilation schemes basically rely on linear estimation theory, or on an extension of this formalism. In such a formulation, each observation is given a weight that is proportional to the inverse of its specified error variance measuring the confidence given to this particular observation. Practical implementations of operational analysis schemes are also based on the use of background fields, that can be seen as another source of observations (Talagrand 1997) with a given confidence corresponding to the forecast error covariances. The analysis is known to be very dependent on the specification of the errors of the different observations, and because these errors are not perfectly known, a large potential for improvement on analyses is offered by methods allowing their tuning.

On the other hand, large operational centers are now using, or have planned to use, assimilation schemes based on a 3D or 4D variational approach, that especially allows the use of a wide range of observations (Courtier and Talagrand 1987). Diagnostics based on statistics of departures between observations and the minimizing solution have been proposed in the variational framework (Talagrand 1999). In this paper, we present a method based on these diagnostics, but that aims to perform an adaptive tuning of the error parameters from a single batch of observations (Desroziers and Ivanov, 2001).

2. VARIATIONAL FORMALISM

The principle of the incremental formulation of 3D/4D-Var algorithms (Courtier *et al.* 1994) is to seek the increment $\delta\boldsymbol{x}$ to be added to the background $\boldsymbol{x}^b$ - so that the analysis is given by $\boldsymbol{x}^a = \boldsymbol{x}^b + \delta\boldsymbol{x}$ - which minimizes the cost function

$$\begin{aligned} J(\delta\boldsymbol{x}) &= J^b(\delta\boldsymbol{x}) + J^o(\delta\boldsymbol{x}) \\ &= \frac{1}{2}\delta\boldsymbol{x}^T\boldsymbol{B}^{-1}\delta\boldsymbol{x} \\ &+ \frac{1}{2}(\boldsymbol{d}-\boldsymbol{H}\,\delta\boldsymbol{x})^T\boldsymbol{R}^{-1}(\boldsymbol{d}-\boldsymbol{H}\,\delta\boldsymbol{x}). \end{aligned}$$

The background term J^b measures the distance between the analysis $\boldsymbol{x}^a$ and the short-range forecast $\boldsymbol{x}^b$, with $\boldsymbol{B}$ the forecast error covariance matrix. In the observation term J^o, $\boldsymbol{H}$ is the linearized observation operator, $\boldsymbol{R}$ stands for the observation error covariance matrix, including representativeness error (Lorenc 1986), and $\boldsymbol{d} = \boldsymbol{y}^o - H(\boldsymbol{x}^b)$ the innovation vector, with H the observation operator that allows the computation of the model equivalents in the space of the observations.

The solution of the minimization of J is given by

$$\delta\boldsymbol{x}^a = \boldsymbol{K}\boldsymbol{d} = \boldsymbol{K}(\boldsymbol{y} - H(\boldsymbol{x}^b)),$$

where $\boldsymbol{K} = \boldsymbol{B}\boldsymbol{H}^T(\boldsymbol{H}\boldsymbol{B}\boldsymbol{H}^T + \boldsymbol{R})^{-1}$ is the gain matrix.

Following Talagrand (1997), the complete vector of observations $\boldsymbol{z}^o$ can be seen as the two components vector of proper observations $\boldsymbol{y}^o$, with dimension p, and of background estimate $\boldsymbol{x}^b$, with the same dimension n as the true state $\boldsymbol{x}^t$: $\boldsymbol{z}^o = (\boldsymbol{x}^b, \boldsymbol{y}^o)^T$ with

$$\boldsymbol{x}^b = \boldsymbol{x}^t + \boldsymbol{\epsilon}^b,$$

where $\boldsymbol{\epsilon}^b$ is the vector of unknown forecast errors with covariance matrix $\boldsymbol{B}$ and

$$\boldsymbol{y}^o = H(\boldsymbol{x}^t) + \boldsymbol{\epsilon}^o,$$

where $\boldsymbol{\epsilon}^o$ is the vector of unknown observation errors with covariance matrix $\boldsymbol{R}$. Thus $\boldsymbol{z}^o$ can also be written

$$\boldsymbol{z}^o = \Gamma(\boldsymbol{x}^t) + \boldsymbol{\epsilon},$$

with Γ the complete *observation operator* and $\boldsymbol{\epsilon}$ the vector of forecast and observation errors, with dimension $n+p$.

An important result pointed out by Talagrand (1999) is that, if J_j stands for a term of J, which is the sum of m_j elements, then the expectation of J_j at the minimum is

$$E[J_j(x^a)] = \frac{1}{2}[m_j - Tr(\boldsymbol{\Gamma}_j^T\boldsymbol{S}_j{}^{-1}\boldsymbol{\Gamma}_j\boldsymbol{P}^a)],$$

where $\boldsymbol{\Gamma}_j$ and $\boldsymbol{S}_j$ are respectiveley the linearized observation operator and the observation error covariance matrix associated with these m_j elements. Here $\boldsymbol{P}^a$ stands for the estimation analysis error covariance matrix resulting from the analysis with the whole set of m pieces of observations ($m = \sum_j m_j$ and $J = \sum_j J_j$).

It can also be shown that this expression can be re-written with

$$E(J_j^o) = \frac{1}{2}[p_j - Tr(\boldsymbol{P}_j(\boldsymbol{H}\boldsymbol{K})\boldsymbol{P}_j{}^T)],$$

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where $\boldsymbol{P}_j$ is the projection operator that allows to go from the complete set of observations to the subset of p_j observations, and $\boldsymbol{R}_j$ is the observation error covariance matrix associated with this subset.

On the other hand, it can be proved that this expression can be evaluated with

$$(\boldsymbol{P}_j \delta \boldsymbol{y})^T \boldsymbol{P}_j [\boldsymbol{H} \delta \boldsymbol{x}^a_{(\boldsymbol{y}+\delta \boldsymbol{y})} - \boldsymbol{H} \delta \boldsymbol{x}^a_{(\boldsymbol{y})}],$$

where $\boldsymbol{x}^a_{(\boldsymbol{y})}$ and $\delta \boldsymbol{x}^a_{(\boldsymbol{y}+\delta \boldsymbol{y})}$ are the analyses respectively computed with the complete set of true values $\boldsymbol{y}^o$ of the observations, and with the whole set of perturbed observations $\boldsymbol{y}^o + \delta \boldsymbol{y}$ (where $\delta \boldsymbol{y}$ is a p-dimensional vector of small perturbations with Gaussian distribution).

The expectation of parts J^b_j of the background term can be determined with the same kind of procedure.

3. ADAPTIVE TUNING

The previous cost function J can be re-written with

$$J(\delta \boldsymbol{x}) = \sum_j \frac{1}{{s^b_j}^2} J^b_j(\delta \boldsymbol{x}) + \sum_j \frac{1}{{s^o_j}^2} J^o_j(\delta \boldsymbol{x}),$$

where ${s^b_j}^2$ and ${s^o_j}^2$ are respectively the background and observation error weighting parameters supposed to be homogeneous for a given subset of observations j. The rationale behind the procedure that we propose is that if the ${s^b_j}^2$ and ${s^o_j}^2$ are the proper weights to intoduce, then the values $\frac{1}{{s^b_j}^2} J^b_j(\delta \boldsymbol{x})$ and $\frac{1}{{s^o_j}^2} J^o_j(\delta \boldsymbol{x})$ should be close to their expected values which can be computed as shown previously. This leads to a non-linear problem with respect to parameters ${s^b_j}^2$ and ${s^o_j}^2$, that can be solved with a fixed-point iterative method.

4. APPLICATION IN 3D-VAR

The previous procedure has been applied in the framework of the French Arpège 3D-Var, based on a spectral global model and an incremental formulation. The test is performed with both simulated background and radiosounding measurements with true locations : the observation errors are produced by using the operational variances for geopotential, temperature, wind and humidity. The initial error profiles have been deliberately chosen very different from the true ones (respectively dotted and solid lines in Fig. 1). Figure 1 shows that the convergence of the iterative procedure is extremely fast : the profiles obtained after a single iteration of the fixed-point procedure are very close to the true ones.

5. CONCLUSION

The statistical expectation of parts of the cost function minimized in a variational analysis can be computed with a randomization method. These computations allow to propose a new method for determining observation error parameters in an assimilation scheme. The application of such a method on simulated radiosounding observations is very promising. The possibility to adapt the procedure for the determination of TOVS radiance errors is currently investigated.

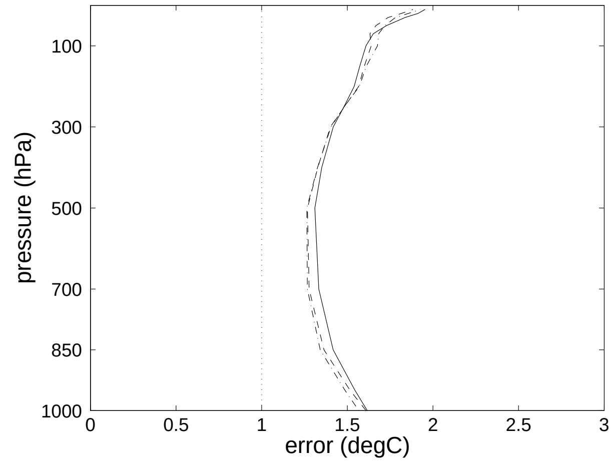

Figure 1: Temperature error profiles used for the simulation of observations (solid line), imposed at the beginning of the iterative procedure (dotted line), after 1 iteration (dashdot line) and after 5 iterations (dashed line).

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