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**1. A GENERALIZED POTENTIAL VORTICITY INVERSION**

Many important synoptic scale processes can be understood via the concept of "Potential Vorticity Thinking". In this framework potential vorticity inversion plays a central role. The invertibility principle states that the full state of the atmosphere can be recovered for a given equilibrium, knowing a boundary condition and the potential vorticity field  $P$  by solving the following equation

$$\frac{1}{\rho} \zeta_3 \cdot \nabla_3 \Theta = P$$

where  $\rho$  is the density,  $\zeta_3$  the 3 dimensional vorticity vector and  $\Theta$  the potential temperature. The balance conditions used so far are based on truncations of the divergence equation. The expansion of the divergence equation in Rossby power leads to the geostrophic balance –the 0-order expansion– and the non-linear balance equation –the first order expansion. Another way to derive a more general balance condition could be to split the atmospheric motion in the Rossby and gravity modes and to retain all the Rossby modes and some physically relevant gravity modes that do not amplify by using dynamical initialization (McIntyre and Norton, 2000). Such a decomposition is introduced in the present paper. The approach of McIntyre and Norton (2000) is extended to a stratified primitive equation model (ARPEGE) by using the Ertel potential vorticity and the digital filtering approach of the dynamical initialization as the balance assumption. The balance condition could be formally expressed as follows:

$$(\zeta, D, \Theta) = \mathcal{I}(\zeta, D, \Theta). \quad (1)$$

where  $\mathcal{I}$  projects a particular state of the model onto the slow manifold and the wind is known through the vertical component of the relative vorticity  $\zeta$  and the divergence  $D$  and the potential temperature  $\Theta$ . Eq. 1 means that the solution is lying on the slow manifold and is therefore supposed to be already initialized. Thus it is invariant by a subsequent initialization. It must be stressed first that any dynamical initialization algorithm can be used.

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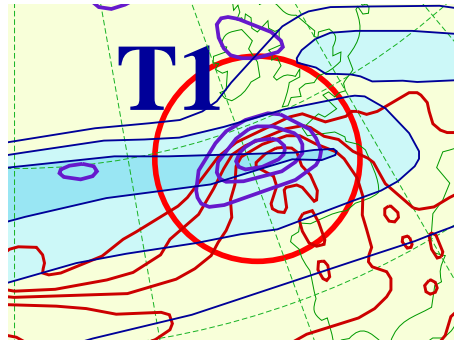


Figure 1: Synoptic overview of the first storm valid at 0 UTC 26 December 1999. Relative vorticity (one isoline every  $5 \cdot 10^{-5} \text{ s}^{-1}$ ),  $\theta'_w$  at 900 hPa and wind velocity on the 1.5 potential vorticity unit surface (The dark shading is associated to winds in excess of 80 m/s).

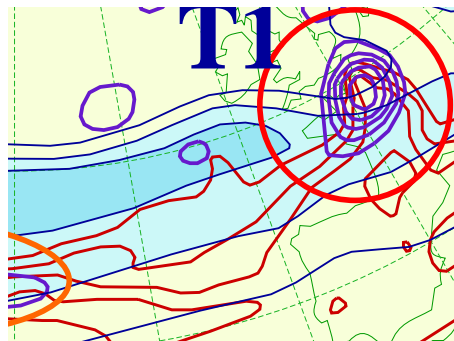


Figure 2: Same as Fig.1 but valid at 6 UTC 26 December 1999.

The initialization of the french operational spectral global model ARPEGE used in the full scale application of this technique is based on a nonrecursive digital filter applied to both a backward and forward integration of the model. Periods less than 3 h are filtered out and the attenuation of periods longer than 12 h remains small. The frequency response of the filter is shown in Fig. 5 of Lynch et al. (1997).

The inversion algorithm outlined above has been applied in the frame of a model with few degrees of freedom but supporting Rossby and gravity waves (Lorenz, 1980). It is found that the solution of this generalized inversion is closest to primitive equations solutions than solutions of quasi-geostrophic inversions. Moreover, the accuracy of the balance is com-

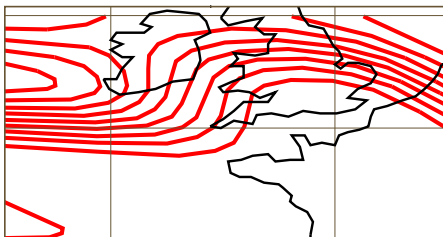


Figure 3: Ertel potential vorticity field inverted valid at 0 UTC 26 December 1999. The level is 300 hPa (contour interval: 0.5 pvu).

parable to the accuracy of the non-linear balance of Charney. The vertical velocity is a direct product of the method unlike other inversion methods that require the solving of an omega equation. It is also found that the vertical velocity provided by the generalized inversion and the vertical velocity of a flow that is solution of the primitive equations are very close and seem in particular as accurate as quasi-geostrophic and semi-geostrophic vertical velocity.

## 2. APPLICATION OF THE PV INVERSION METHOD TO THE FIRST STORM OF DECEMBER 1999

Just after Christmas 1999, two exceptionally severe storms have crossed France and western Europe causing heavy damages and human losses. The first one results from a weak low that forms over the western Atlantic on 24 Dec. During the following 36 hours the low propagates very rapidly towards Europe and is followed by a rapid intensification phase between 21 UTC 25 Dec and 06 UTC 26 Dec (see Fig. 1 and 2).

The potential vorticity inversion method is used to address the contribution of the upper level potential vorticity features associated to the jet outflow to the development during the maximum growing phase of the low level vorticity maximum. Fig. 3 displays the potential vorticity distribution which is inverted. Note the high potential vorticity patch at the level of the tropopause just north of the core of the jet and the low potential vorticity area over southern England. In the generalized inversion of that potential vorticity field, a vertical velocity field attributable to a potential vorticity structure is a direct outcome of the inversion. It is shown on Fig. 4. In that case the maximum of ascending vertical velocity associated to the potential vorticity maximum is as usually slightly downstream but far to the west from the actual vertical velocity pattern of the 4DVAR analysis displayed on Fig. 5. Therefore, the upper level features do not interact with the cyclone as in the *type-B* paradigm of cyclone development.

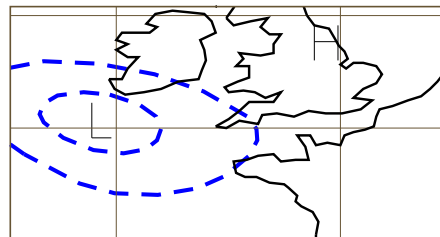


Figure 4: 700 hPa vertical velocity produced by inversion of the potential vorticity distribution displayed in Fig. 3. Contour interval: 0.2 Pa/s, the dashed lines stand for ascending motions

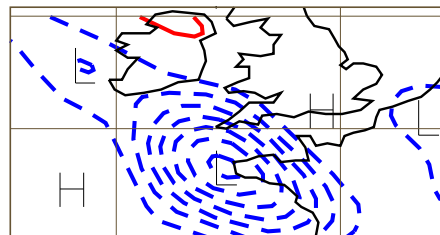


Figure 5: 700 hPa vertical velocity from the 4DVAR analysis valid at 0 UTC 26 December 1999. (contour interval: 0.2 Pa/s, the dashed lines stand for ascending motions, the isoline 0 is not shown).

## 2. CONCLUSION

A new method inspired by the work by McIntyre and Norton (2000) in the frame of the shallow water equations has been implemented and tested successfully in the context of a low order model and of a 3D model. It is based on the Ertel formulation of the potential vorticity and the implicit balance provided by the primitive equations themselves through the use of the digital time filtering of the model. The generalized approach of attribution presented above turns out to be a valuable tool to understand mid-latitude cyclogenesis. The computation of interactions between potential vorticity structures are straightforward thanks to this approach.

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