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## 1. INTRODUCTION

Current practices of 3D variational analysis in many operational centers are constructed in spectral space, which has the advantage that the statistics of background error, both structure and amplitude, can be easily obtained and applied in the analysis procedure. It is simpler to apply a diagonal background error covariance in spectral space than to convolve the corresponding smooth kernel with the innovations in the physical space. However, one has little control over the spatial variation of the error statistics when a simplified background error covariance in spectral space is used. With diagonal covariance in spectral space the structure function is limited to be homogeneous geographically (at least in the zonal direction) and isotropic in shape (Parrish and Derber 1992, Courtier et al 1998).

Hayden and Purser (1995) following up on the work of Purser and McQuigg (1982) show how a very simple and computationally cheap family of recursive filters can be combined to yield empirical smoothers which are locally isotropic but retain the freedom of spatial inhomogeneity. Recent developments of spatially recursive filters (Purser et al 2001) enable the construction of a variational analysis in physical space which allows more degrees of freedom in defining the error statistics adaptively. Using recursive filters, we show an analysis system constructed on physical space with latitude dependent structure functions and error statistics.

## 2. GLOBAL ANALYSIS ON GRID SPACE

In order to incorporate as much of the existing work in the global analysis system in NCEP, the version on physical space is formulated to be similar to the spectral statistical-interpolation analysis system (SSI, Parrish and Derber 1992). A preconditioned conjugate gradient algorithm (Gill et al. 1980; Navon and Legler 1987; Derber and

Rosati 1989) is used to minimize the functional. The analysis variables are stream function ( $\psi$ ) velocity potential ( $\chi$ ) unbalanced part of temperature (T), surface pressure (P) and humidity (q), defined on the grid instead of in spectral coefficients. The amplitudes and scales of the background error are defined as function of latitude and height.

For 3D Var in physical space, the multi-variate design between the analysis variables of mass and wind is a challenge. Since the variables are defined on physical space it is not easy to apply the linear balance operator (Parrish & Derber 1992) which includes the inverse of the Laplacian. Yet the relation between the mass field and the stream function is linear, so that statistical regression between the two is possible. The balanced part of the temperature increment is defined as  $T_b = G \psi$  where matrix G projects stream function increments at one level to a vertical profile of balanced part of temperature increments. The balanced part of the surface pressure increments are defined as  $P_b = W \psi$ , where matrix W integrates the contribution of stream function from each level. The balanced part of the temperature (surface pressure) increments explains about 50% (80%) of the variance in the troposphere. A similar relation was also reported in Gustafsson et al, 1999. Since the variables are defined on a grid these statistics can be latitude dependent. We found that the balance design is crucial; without it the assimilation degrades quickly. The fit to the data for the surface pressure of the guess field is doubled in amplitude within two days (8 cycles) of assimilation when compared with the results of SSI.

## 3. APPLYING RECURSIVE FILTERS TO A GLOBAL DOMAIN

An efficient self-adjoint version of the numerical recursive filters can be applied to the task of convolving a spatial distribution of innovations with a smoothing kernel which is interpreted to be a covariance function of background error.

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To apply the recursive filter on the global domain the grid is divided into three pieces: two Cartesian polar patches and a zonal band in between. For the zonal band patch, both the inhomogeneity of the Gaussian grid and the shrinking of the zonal grid toward polar area is treated as scale variation.

For polar patches the transform routine between the Cartesian grid and the lat-lon grid, and its adjoint are needed. The stereographic projection is used to project the Gaussian grid on to a plane. The observational residual field is converted with the adjoint of the transform program from Gaussian grid to the Cartesian grid, recursive filters are applied, and the forward routine is used to transform the field back to the lat-lon grid. Two blending zones between the polar patches and the zonal band allow a smooth transition when the three parts are merged back to the global Gaussian grid.

#### 4. ESTIMATION OF BACKGROUND ERROR COVARIANCE

The error variance is estimated in grid space with what has become known as the "NMC method" (Parrish and Derber 1992, Rabier et. al. 1998). The error statistics are estimated with the differences of 24 and 48-hour forecasts for 49 cases distributed over a period of one year. Only the results for stream function are shown. Fig 1 shows that the amplitude of the error variance is largest in mid-latitudes and in the southern hemisphere.

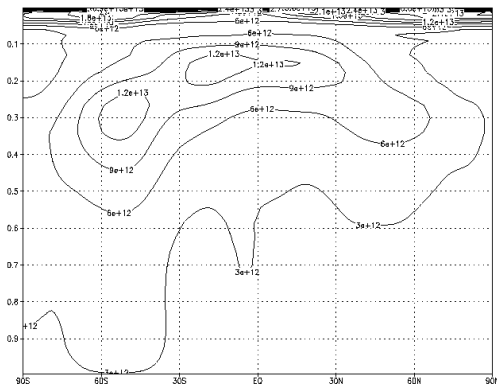


Figure 1. Background error variance (\*100km) of the stream function as function of latitude and height.

The horizontal scale information from the background error in spectral space is retrieved by using the convolution effect in spectral space. In spectral space the dot product of the error

statistics and the spectrum of an impulse at each latitude is taken and the result is transferred back to physical space to produce an isotropic structure at each latitude for each variable and at each height. A table of recursive filter results is used to fit the structures and produces the scales in recursive filter unit. Fig. 2 shows the horizontal scales of stream function are largest in the tropics and increase with height.

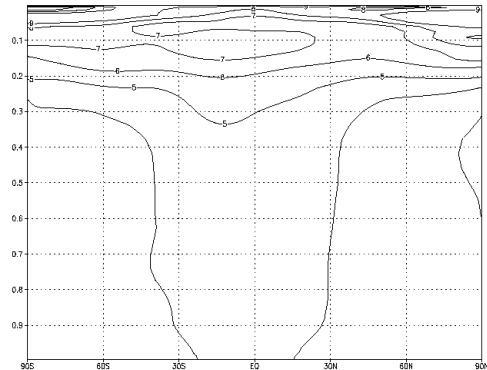


Figure 2. Horizontal scale of the structure function in grid unit for the background error covariance of the stream function.

It has been recognized that objective analysis using the Gaussian shape to model the covariance severely hampers the ability of the analysis to assimilate the smallest scales. The fat-tailed feature in the spectra of error covariance is also observed when the error covariance is defined in spectral space as in the current operational SSI. It is straight forward to apply a background error covariance with fat-tailed spectrum in spectral space 3D Var since the error covariance is defined in the spectral space. To achieve a fat-tailed spectrum when using the recursive filters, a linear combination of multiple recursive filters is needed.

In our procedure, two horizontal scales are applied. The second horizontal scales are set to be a half of the first and the estimated scale from the NMC method fall between the scales applied. In the MPP computational setup horizontal smoothing is done when the domain is divided into horizontal slices and the vertical smoothing is done when the domain is in vertical columns. For computation efficiency, single recursive filter is used in the vertical direction.

The vertical scales are estimated with the vertical correlation of each variable. The correlation is fitted locally with a table of results of recursive filters. The scale of the best fit out of the table is assigned as the scale of the variable at the vertical level for each latitudinal grid. Fig. 3 shows

the vertical scales used for the stream function. Note that the vertical scales are also locally defined so that the negative correlation further away is not included. For unbalanced temperature the localized vertical correlation may introduce hydrostatic imbalance. The vertical scales are largest near the surface and stream function has the largest vertical scales among the variables. In general, the vertical scales are smaller near the tropics. These results are consistent with what is reported in Rabier et al (1998) and Ingleby (1999).

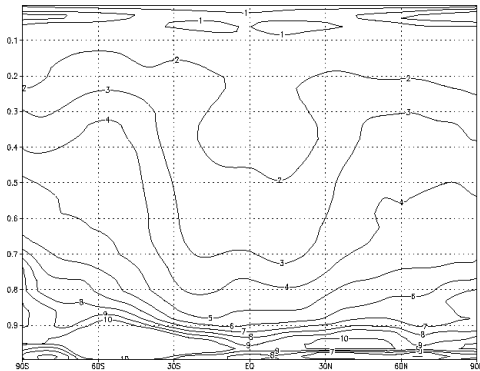


Figure 3. Vertical scales, in grid unit, of the structure function for the background error covariance of the stream function

## 5 ANALYSIS AND ASSIMILATION RESULTS

The analysis system is tested against the operational SSI in NCEP. Two sets of low resolution T62 data assimilation are cycled for 19 days to produce 2 weeks verifiable 5-day forecasts. Figure 4 shows the anomaly correlations in the extra-tropics (latitudes 20-80 north and south) for 500 hPa height. Each experiment verified against its own analysis. The 14-case mean for northern hemisphere is 0.742, 0.750 for the experiment and the control and 0.714 and 0.718 for the southern hemisphere. The experimental analysis system produces a small negative (1%) impact in northern hemisphere and a slightly (.5%) negative impact in the southern hemisphere over the 2 week period. The impact in the tropics, however, is more consistent and positive. The day-3 RMS vector wind error at 200 and 850 hPa is shown in fig. 5. The mean for the 850 wind error over the period is 3.920 and 4.443 for the experiment and the control respectively and 7.242 and 8.227 for the error at 200 hPa. The improvement is 12% and 10% over the period for 200 and 850 hPa respectively.

## 6 CONCLUSION

We propose an alternative way of defining background error covariance in 3D Var. By using recursive filters in physical space, the covariance can be changed with geographical location. This degree of freedom comes with a price: limited freedom of the error statistics in wave number space. This limitation is partially overcome by applying multiple recursive filters for the structure functions.

In the experiments that we report in this study, the error structures are similar to those in the NCEP SSI since the scales of the background error structure are estimated with the NMC method and are homogeneous in the zonal direction. The small impact in the extra-tropics indicates that the 3D Var formulated in the physical space can be as effective as in spectral space. The consistent positive impact in the tropics indicates that the new-gained freedom in spacial variation (in current setup: latitude dependent) of the background error statistics is beneficial to forecasts compared with the freedom in wave number space (as in SSI) in which the statistics represent global characteristics.

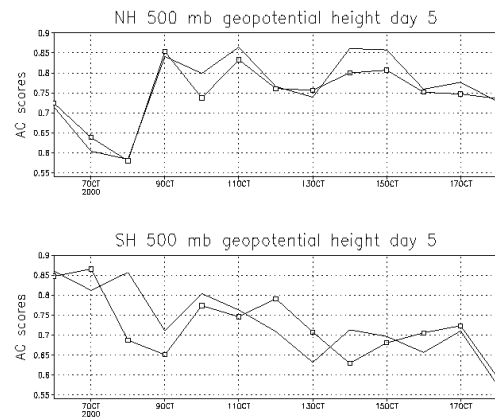


Figure 4. The anomaly correlations in the extra-tropics for northern (above) and southern (below) hemisphere 500 hPa height. Experiment: line with open square; Control: solid line

Cutting up the global domain for recursive filters has its limitation. The problem is more severe in the stratospheric layers where the scales of the structure function are large compared to the sub-domain. A possible solution is to solve the largest few wave numbers in spectral space so that when the domain is divided, the largest overlapping can be defined by the lower bound of the solution in spectral space.

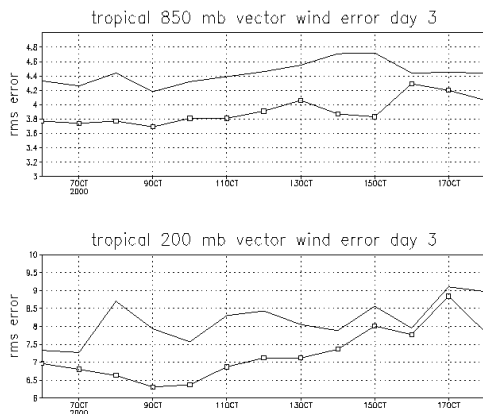


Figure 5. RMS vector wind error at 850 (above) and 200 (below) hPa for 72 hours forecasts. Experiment: open square; control: solid line.

It's straight forward to apply this 3D variational analysis to a regional domain, and, we plan to work to adopt a fully inhomogeneous and anisotropic background error covariance in the system.

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