

SPURIOUS RELATIONS AMONG RAINFALL PARAMETERS ARISING FROM INADEQUATE SAMPLING AND FITTING

A. R. Jameson*
RJH Scientific, Inc.
Arlington, Virginia

A. B. Kostinski
Department of Physics, Michigan Technological University
Houghton, Michigan

1. REMARKS

Marshall and Palmer (1948) were among the first investigators to develop power-law statistical correlations relating the parameters of the drop size distribution to the rainfall rate, R . In this talk we point out that the existence of such correlations is a statistical consequence of insufficient sampling of distributions of drop sizes that are decreasing functions of increasing diameter coupled with logarithmic fitting. To understand this it is important to recall that any drop size distribution can be written as the product of the total number of drops, n , times the probability density function (pdf) of the drop sizes, $p(D)$ (e.g., Kostinski and Jameson 1999; Jameson and Kostinski 2001a). $p(D)dD$ is then the probability that a drop selected at random has a diameter between D and $D+dD$. Because $p(D)$ is a decreasing function of drop diameter, the probability that a particular drop drawn at random lies between D and $D+dD$ is much greater for smaller drop sizes than for larger drops. It follows, then, that when the number of drops in a sample volume, n , is very small, few if any large drops will be included in a limited random sample of drop diameters. However, as n increases, it becomes more and more likely that larger drops will be included in a particular sample. This effect is exacerbated by peculiarities of logarithmic correlations (Thompson and Macdonald 1991).

In statistically homogeneous rain, there is a 'steady' drop size distribution independent of the measurement process (Jameson and Kostinski 2001a; 2001b). In statistically inhomogeneous rain, however, there is no such 'steady' drop distribution. Rather the distribution itself continually changes as more and more data are added (Jameson and Kostinski 2001b).

Even though a steady or 'constant' drop size distribution is present in statistically homogeneous rain, proper measurement of that distribution requires a number of drops sufficient to span or adequately represent the distribution from small to large drops. Yet, disdrometers and aircraft probes often measure only a small number of drops. In particular, the famous Marshall-Palmer (1948) relations are based upon data using dye paper exposed "...to obtain at least 100 drops per sample." We show in this talk however, this sampling appears to be wholly inadequate even in the most optimistic case of statistically homogeneous rain.

Monte Carlo experiments described in Jameson and Kostinski (2002) show that as the number of drops, n , in

the unit sampling volume increases, larger and larger drops appear in increasing numbers as dictated by $p(D)$ leading to power law relations similar to those reported in the literature. It is particularly sobering that the famous Marshall-Palmer (1948) Z - R relation can appear strictly as a consequence of inadequate sampling and logarithmic fitting.

How much has the sampling effect described here influenced observations in real rain? It is likely that most correlation relations are often derived in statistically inhomogeneous conditions (Jameson and Kostinski 2001a) that likely amplify the problem just described in part because there is never the rapid convergence like that found in statistically homogeneous rain. Moreover, in most real rain, the appearance of one drop is usually correlated with the appearance of another. Such drop clustering slows convergence (Kostinski and Jameson 1999, 115). It seems likely, therefore, that many past observations and correlation relations in real rain have been significantly affected by sampling that is inadequate to represent the true frequency distributions of drop sizes. It is impossible to know for sure, then, how much of the correlations are due to the statistics of the rain and how much are artifacts of the sampling and logarithmic fitting. For more details and complete discussion the interested reader is referred to the recent article by Jameson and Kostinski (2002).

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2. REFERENCES

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*Corresponding author address: A.R. Jameson, RJH Scientific, Inc., 5625 N. 32nd St., Arlington, VA 22207-1560; email: jameson@rjhsci.com