# ANALYSIS OF THE SINGULAR VECTORS OF THE FULL-PHYSICS FSU GLOBAL SPECTRAL MODEL

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# ABSTRACT

Analysis of singular vectors was performed on the Florida State University Global Spectral Model, and the linearization of the full physics model was used. It is demonstrated that physical processes, especially precipitation, fundamentally affect the leading SVs. It is also shown that the spatial filtering along with the projection operator of projecting the flow winds to the rotational wind, which was designed as a simple initialization process, improves the structural features of SVs and is beneficial in the suppression of spurious modes.

## 1. INTRODUCTION

We will examine the impact of physics on singular vectors (SVs). The availability of the linearization of full-physics version of the the Florida State University Global Spectral Model (FSUGSM) allows us to achieve our goal. Documenting the characteristics of the impact of physical processes on SVs with the FSUGSM and comparing the obtained results with those already published constitutes one of the major focus of this research, while to mitigate the effect of linearization on leading SVs, which leads nonmeteorlogical modes with spuriously large growth rates, constitutes the secondary major focus of this research.

A physically sound method for suppressing spurious modes is to introduce an initialization-like procedure, which is able to filter out the spuriously growing modes in the initial condition repeatedly during the interation process of computation of SVs. Here we will focus on the projecting operator introduced by Barkmeijer et al. (1992). However, the projecting operator is generalized to project variables not only onto a specified model area, but also onto a subspace of modes. This generalization immediately presents us with definitions of projection operators, such as spatial filterings that has been used to discuss the effect of spatial scale on linear growth of SVs, as well as balance constraints. We will simply refer to them as filterings. It will be shown that combining a spatially low-frequency passing filter with some balance constraints, will be beneficial in suppressing spurious modes, and more importantly will improve the features of computed SVs.

#### 2. THE MODEL

The FSU Global Spectral model (FSUGSM) has been used in numerical weather forecasts for operational purposes for more than a decade. Forecasts using this model especially emphasize tropical aspects such as monsoon and tropical storms.

The adjoint system of FSUGSM has been developed as a result of several years efforts. The adjoint system includes the dynamic core and all the model physical parameterizations. This model has been

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successfully applied to carry out both 4D-Var and optimal parameter estimation.

## 3. Basic formulations

In the analysis of dynamic instability and error growth, a standard way is to compute an amplification factor  $\lambda$  defined by

$$\lambda^{2} = \frac{\langle x_{t}, E_{t}x_{t} \rangle}{\langle x_{0}, E_{0}x_{0} \rangle} = \frac{\langle x_{0}, A^{T}E_{t}Ax_{0} \rangle}{\langle x_{0}, E_{0}x_{0} \rangle}, \quad (1)$$

where the superscript T denotes the transpose,  $x_t$  is the state of the system at time t, and  $x_0$  at time 0, that is, the initial condition. A(t,0) is the resolvent or propagator from time 0 to time t. Here  $\langle x, y \rangle = \sum x_i y_i$  for any vector pair x and y, that is, the Euclidean inner product. The subscripts 0 and t denote the initial time  $t_0$  and the end time of the window t. The weight matrix E is typically chosen as the total energy norm, thus being a positive diagonal matrix. In this case,  $\lambda$  is an energy amplification factor.

We introduce a projecting operator P, as well as a transformation operator F. Then (1) is generalized as

$$\lambda^{2} = \frac{\langle PF_{t}x_{t}, E_{t}PF_{t}x_{t} \rangle}{\langle F_{0}x_{0}, E_{0}F_{0}x_{0} \rangle}$$
(2)

If  $F_t$  and  $F_0$  transform the complete wind field to the rotational field, (2) is equivalent to the expression defined by Errico (2000). In the following computations, F will be temporal or spatial filtering operators combined with other constraints, aimed at suppressing spurious modes in leading SVs and at improving their features. We will simply call such a procedure filtering. For the generalized definition (2), the eigenvalue problem should then be defined

$$A^T F_t^T P^T E_t P F_t A v = \lambda^2 F_0^T E_0 F_0 v.$$
 (3)

The norms  $E_t$  and  $E_0$  in the present study are the so-called dry total energy norm, which do not involve the latent heat of condensation

$$< x, Ey >= \frac{1}{2} \int_{0}^{1} \int_{\Sigma} \left[ u_{x}u_{y} + v_{x}v_{y} + \frac{c_{p}}{T_{r}}T_{x}T_{y} \right]$$
$$(\frac{\partial p}{\partial \sigma})d\Sigma d\sigma + \frac{1}{2} \int_{\Sigma} R_{d}T_{r}P_{r}\ln\pi_{x}\ln\pi_{y}d\Sigma, \quad (4)$$

where u and v are wind components, T is temperature,  $\ln \pi$  is the logarithm of the surface pressure. There are some constants in (4).  $C_p$  is the specific heat of dry air at constant pressure,  $R_d$  the gas constant for dry air,  $T_r = 300K$  a reference temperature,  $P_r = 800hpa$  a reference pressure. In the following, the projecting operator is applied for restricting the computation of the energy norm (4) to the Northern hemisphere. The optimal time interval is taken as 36h. The linearization is carried out with the basic state of a model forecast. The forecast starts from the initialized analysis valid at 00UTC 03 September 1996, as well as at 12UTC 24 June 1994 when the Indian Monsoon was active. The full-physics model is always used in the forecasting model.

#### 4. Model without physics

The model without physics means in our setting that the model consists of the dynamic core with simple horizontal diffusion and surface drag (Case NP). In this case, the spurious modes are dominant amongst the leading SVs. The spurious modes are characterized by vertical and horizontal two-grid size wavelength perturbations. In the horizontal, the perturbation may exist only at some isolated model grid points, and is mainly located over high mountains. Seven of the 10 SVs turn out to consist of such spurious modes.

#### 4.1 Model with the full physics

We first include the boundary layer physics and vertical diffusion into the model (Case BP). The introduction of the boundary physics and vertical diffusion shows a limited improvement in this model. Out of the 10 leading SVs, there are still five SVs consisting of spurious modes. The three reasonable SV in the case without physics still exist, while a new additional reasonable SV is obtained. The vertical diffusion and the boundary layer physics do not satisfactorily suppress spurious modes in our case.

When the full physics is introduced into the models (Case FP), the computed SVs are substantially improved. The most remarkable aspects include that three SVs mainly consist of tropical modes, and there is only one SV consisting of spurious mode. It is noteworthy that the first SV is dominated by spurious modes.

For the SVs located in mid-latitudes and obtained with all the three cases, that is, NP, BP, and FP, physics does not appear to have an evident impact on their singular values. That means that SVs presenting a similar structure are associated with close singular values irrespective of whether the model with simplified or full physics is used.

For the modes over the tropics, we can deduce that precipitation increases the growth rate of the mode. Here we do not suggest that there is the excitation of new modes, since the mode is baroclinic. The combination of the baroclinic and precipitation latent heating causes the modes to have larger singular values and then become leading SVs. Only when the growing mode is geographically coupled with precipitation, does the precipitation enhance its growth rate.

## 5. Singular vectors with filtering

In this model, spurious modes are basically suppressed in the computed leading SVs when the surface-drag scheme, vertical diffusion and the truncation at resolution T21 is applied to the initial condition, along with projecting the wind fields onto the rotational projector Errico (2000).

The filtering affects the meteorological modes significantly. For the SVs consisting of the meteorological modes the associated singular values are reduced by about a half. As expected, their structure has larger scales. The combined perturbations of all the ten SVs appear to well cover the extra-tropics, which is desired by ensemble forecasts.

It is interesting to note that all the reasonable meteorological SVs at middle latitudes in Case NP have a counterpart in terms of both the geographical location and spatial structure. Those modes excited by physics in the tropics and subtropics in Case FP also have a counterpart, but only in terms of the geographical location. The tropical perturbation with filtering presents well-organized wave train structure, which may be attributed to the use of only the rotational wind.

## 6. Summary and discussion

The results obtained have basically verified the findings of the impact of physical processes on SVs by Erhendorfer et al. (1999) and Barkmeijer et al. (2000). The physical impact of SVs can be substantial in this global model even for an optimal time interval as short as 24h. The growth rates are substantially enhanced and the structures are profoundly changed. Note that this FSUGSM version has a relatively low resolution, with truncation at T42 and with only 12 levels in the vertical. For a model of higher resolution, the impact of model physics is expected to be stronger. However, we have demonstrated that physical processes, especially precipitation, affect leading SV fundamentally only when SV perturbation is geographically coupled with precipitation.

We have shown that the spatial filtering along with the projector operator of projecting the flow winds to the rotational wind improves the structural features of the computed SVs and is beneficial in the suppression of spuriously growing modes. On the contrary, the temporal filtering seems to be enhancing spuriously growing modes.

## References

- Barkmeijer, J, 1992: Local error growth in a barotropic model, *Tellus*, **44A**: **(4)**, 314-323.
- Barkmeijer, J., R. Buizza, T.N. Palmer, K. Puri and J-F. Mahfouf,2000: Tropical singular vectors computed with linearized diabatic physics.
  ECMWF Technical Memoranda, No 297.December 1999. [Available from ECMWF, Shinfield Park, Reading, Berkshire RG2 9AX, United Kingdom.]
- Buizza., R. 1994: Sensitivity of optimal unstable structures. Q. J. R. Meteor. Soc., 120, 429-451. c Ehrendorfer, M., Errico, E.R., and Raeder, K.D., 1999: Singular vector perturbation growth in a primitive equation model with moist physics. J. Atmos. Sci., 56, 1627-1648.
- Errico, E.R., 2000: Interpretations of the total energy and rotational energy norms applied to determination of singular vectors. *Quart. J. Roy. Meteorol. Soc.*, **126**, 1581-1599.