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1. INTRODUCTION

Atmospheric attenuation is the rate at which a beam's energy is absorbed or scattered via interaction with the atmosphere. Depending upon environmental conditions, this attenuation can be significant, up to 15 dB/Km. Thus, the attenuation of weather radar signals has the effect of skewing hydrological estimates. Accurate accounting and correction of atmospheric attenuation effects must be undertaken in order to obtain accurate estimates of precipitation.

The classic texts of weather radar, Battan (1976) and Doviak (1993), and radar systems, Skolnik (1980) and Skolnik (1990), all refer to a table, graph, or even just a blanket statement that the level attenuation is known and is approximately constant. In presenting this data, these texts refer to the work of Bean and Dutton (1966) or Blake (1972). Their knowledge of the atmospheric attenuation as used in weather radar draws upon the work conducted at the MIT Radiation Laboratory during World War II, Kerr (1964).

This paper reviews the original work and briefly describes work conducted since 1947, primarily in the communications field. In addition, we provide a simple formulation and example calculation to allow the user to estimate the coefficient for his system.

2. CLASSICAL PHYSICS

Classically, the attenuation of energy traveling through some medium is due to two processes, dispersion and absorption. Dispersion concerns the bending of the radiation as function of frequency. Absorption concerns the molecules gaining energy from the beam through collision. Both of these processes are described classically through the dielectric constant,

$$\epsilon(f) = \epsilon_{\text{Re}}(f) + j\epsilon_{\text{Im}}(f), \quad (3)$$

where f is the frequency, $j = \sqrt{-1}$, and $\epsilon_{\text{re}}(f) - 1 \ll 1$. The real and imaginary components of dielectric constant are not independent rather are intimately connected. Thus, if we know the functional value of one, we may determine the other through the

mathematical technique of the Hilbert transform. For example, the imaginary component may be obtained from the real component via the transform,

$$\epsilon_{\text{Im}}(f) = -\frac{2}{\pi} f \int_0^{\infty} \frac{\epsilon_{\text{Re}}(f') - \epsilon_{\infty}}{f'^2 - f^2} df', \quad (4)$$

where ϵ_{∞} is the dielectric constant for infinite frequency.

The dispersion relation is described by the index of refraction and is simply the square root of the real component of the dielectric constant, i.e.

$$\eta(f) = \sqrt{\epsilon_{\text{Re}}(f)}. \quad (5)$$

The attenuation coefficient is a function of the imaginary component of the dielectric constant and the frequency, i.e.

$$a(f) = \left[\frac{2\pi 10^6 \log e}{c} \right] [f\epsilon_{\text{Im}}(f)]. \quad (6)$$

Therefore, by knowing the functional relationship for $\epsilon_{\text{Re}}(f)$, the absorption coefficient is,

$$a(f) = - \left[\frac{(4 \times 10^6) f^2 \log e}{c} \right] \int_0^{\infty} \frac{\epsilon_{\text{Re}}(f') - \epsilon_{\infty}}{f'^2 - f^2} df'. \quad (7)$$

This result implies than any small change in the index of refraction (much smaller than that required for ducting phenomena) would result in very high attenuation rates, as much as 3 dB/Km. This is physically unrealistic and thus we turn away from classical physics to modern physics to obtain a realizable model for the atmospheric attenuation.

3. QUANTUM PHYSICS

The quantum theory of nature implies only discrete amounts of energy can be absorbed rather than every energy in the spectrum. These discrete amounts

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are the quantum energy levels, a transition from one to the other is called a quantum leap. For example, if we have two energy levels, E_1 and E_2 , the quantum of energy that can be absorbed is,

$$\Delta E = E_2 - E_1 . \quad (8)$$

Albert Einstein related this quantum of energy to the frequency of radiation through the expression,

$$\Delta E = hf , \quad (9)$$

where h is Planck's constant, ($h = 6.626 \times 10^{-34} \text{ Js}$), and f is the frequency of radiation. In other words, to determine the frequency at which absorption will occur, the resonant frequency, we solve Eqn. (9) for f ,

$$f = \frac{\Delta E}{h} . \quad (10)$$

Thus, quantum theory places important restrictions upon the absorption of radiation. Rather than all of the energy being available for absorption as in the classical theory, only certain amounts of energy can be absorbed. This greatly affects the amount of radiation absorbed and hence the absorption coefficient.

The quantum mechanical absorption coefficient function is given by

$$a(f) = \left(\frac{Af\eta}{T} \right) \left(\frac{\sum_q \sum_s |\mu_{sq}|^2 p(f_{sq}, f) e^{-E_q/kT}}{\sum_q e^{-E_q/kT}} \right) , \quad (11)$$

where A is a constant, μ_{qs} is the dipole moment connecting states q and s , f_{qs} is the frequency corresponding to the transition between the two states, η is the molecular density, f the frequency of the incident radiation, and $p(f_{qs}, f)$ is a structure factor describing the shape of the absorption line. The structure factor is given by,

$$p(f_{qs}, f) = \left(\frac{f\Delta f}{\pi f_{qs}} \right) \times \left(\frac{1}{(f_{qs} - f)^2 + \Delta f^2} + \frac{1}{(f_{qs} + f)^2 + \Delta f^2} \right) , \quad (12)$$

where Δf is the line breadth constant.

To obtain an accurate measure of the atmospheric attenuation, we need to know the molecular constituents of the atmospheric, dipole moments, and the corresponding spectroscopic information. Table I summarizes the main constituents of the atmosphere and the above information.

The primary atmospheric constituents are nitrogen, oxygen, and water vapor. Nitrogen has no dipole moments and no energy transitions in the microwave region. Hence, nitrogen will not be a major factor in the attenuation.

Oxygen, on the other hand, does have a permanent magnetic dipole moment. Therefore, O_2 will have an interaction with a microwave pulse. Of the hundreds of spectral lines associated with oxygen, only a couple of dipole moments are sufficient to provide nonnegligible attenuation. The relation describing this attenuation as a function of frequency at standard pressure and temperature is given by,

$$a_{O_2} = 8.5 \times 10^{-3} f^2 \times \left\{ \left[(60 - f)^2 + 0.5625 \right]^{-1} + \left[(60 + f)^2 + 0.5625 \right]^{-1} + \left[f^2 + 0.5625 \right]^{-1} \right\} , \quad (13)$$

The final major contributor is water vapor. Water vapor is highly variable in the atmosphere ranging from 0-0.4% of atmospheric content. This variable content coupled with the strong interaction of water's electric dipole moment with the EM radiation, and we have the largest contributor to atmospheric attenuation. The quantum mechanical selection rules for energy transitions of the H_2O molecule forbid nonresonant transitions. Thus, the rotational quantum number J can only change by one unit. Thus, for $J \in [0, 6]$, there are approximately 156 allowed transitions, of which only one will provide a significant contribution in our frequency range, $f \sim 22.2 \text{ GHz}$. The relation describing attenuation by water vapor as a function of frequency at standard pressure and temperature is given by,

$$a_{H_2O} = 3.50 \times 10^{-3} \rho f^2 \times \left\{ \left[(22.2 - f)^2 + 9.00 \right]^{-1} + \left[(22.2 + f)^2 + 9.00 \right]^{-1} \right\} , \quad (14)$$

where ρ is the water vapor density in g/m^3 .

Performing some algebraic manipulation on Eqns. (13) and (14), the relations for these two components of the attenuation coefficient become,

$$a_{O_2} = 8.5 \times 10^{-3} f^2 \times \left\{ \frac{1.297 \times 10^7 + 3f^4 + 5.625f^2}{[3600 - (f^2 + 0.5625)]^2 (f^2 + 0.5625)} \right\}, \quad (15)$$

and

$$a_{H_2O} = 7.00 \times 10^{-4} \rho f^2 \times \left\{ \frac{f^2 + 502.8}{f^4 - 969.65f^2 + 252835} \right\}. \quad (16)$$

The total atmospheric attenuation will be the linear combination of the contributions from oxygen and water vapor

$$a(f) = a_{O_2}(f) + a_{H_2O}(f). \quad (17)$$

Assuming a standard atmosphere at sea level (1 atm pressure at a temperature of 20°C) water vapor content of 7.5 g/m³ (100% relative humidity) the atmospheric attenuation for S, C, and X band frequencies are plotted in Figure 2.

Oxygen's contribution to the attenuation (dashed line) dominates in the S and C bands. Water vapor (dotted line) is not a major factor simply because of the small proportion of air that is composed of water vapor. The total atmospheric attenuation (solid line) is the linear combination of the two and is essentially the shape of the water vapor line translated by the value of the oxygen attenuation.

For Figure 1, we chose a set of parameters for the atmospheric environment (pressure, temperature, water vapor content). In actuality, these values can change considerably and thus the atmospheric attenuation will change continuously. The dynamic attenuation model seeks to adjust the attenuation based upon atmospheric conditions.

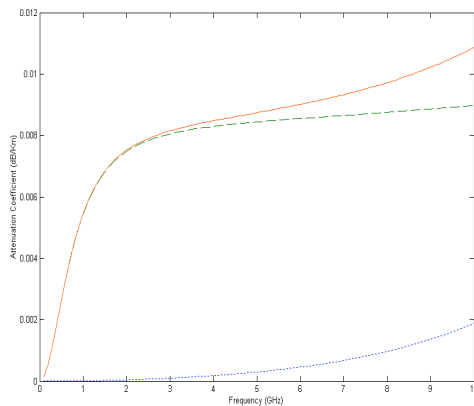


Figure 1. Atmospheric attenuation (solid line) is the linear combination of the contributions due to oxygen (dashed line) and water vapor (dotted line). Note how water vapor becomes significant at as the frequencies enter the X-band. This is due to the proximity of the 22.2 GHz water vapor line.

4. DYNAMIC ATTENUATION MODEL

The atmosphere is not a steady-state system, but rather a system that is in continuous flux, particularly the water vapor content. The parameters we measure, temperature, barometric pressure, and relative humidity, characterize these fluctuations.

The obvious adjustment to be considered is the dynamic variation of the water vapor content in Eqn. (16). Recall that water vapor content is related to the relative humidity, ϕ , through a relation,

$$\rho = \phi \rho_s, \quad (18)$$

where ρ_s is the maximum water vapor content available and is given by the relation,

$$\rho_s = \left[\frac{13.24}{T + 273.15} \right] \left[10^{\left(\frac{7.5T}{237.3+T} \right)} \right] \quad (19)$$

where T is the temperature in °C. Substituting this into Eqn. (16), a dynamic model for the attenuation due to water vapor is given by,

$$a_{H_2O} = \left\{ \frac{8.08 \left[10^{\left(\frac{5.5T - 474.6}{T + 237.3} \right)} \right] \phi f^2}{T + 273.15} \right\} \times \left\{ \frac{502.8 + f^2}{f^4 - 969.65f^2 + 252835} \right\}. \quad (20)$$

where the frequency, f , is in GHz, the temperature, T , is in °C, and the relative humidity, ϕ , is dimensionless.

5. EMPIRICAL MODELS

The models discussed in Sections 2-4 are essentially theoretical models with some discussion of physical effects and validity. Driven by the communications industry's need for very precise attenuation data, new empirical models have been developed. These models are based upon mathematical fitting of experimental data concerning attenuation of satellite communications.

Liebe and Layton (1987) fit the spectral data, obtaining the a model for the total attenuation due to the atmosphere,

$$\begin{aligned}
a(f, \alpha, T, P, e) = & S_{H_2O}(T, e, P)F_{H_2O}(f, \Delta f) \\
& + S_{O_2}(T, e, P)F_{O_2}(f, \Delta f) \\
& + \alpha(f, T, P, e)
\end{aligned} \quad (21)$$

$F(f, \Delta f)$ is the line shape, given by,

$$F(f, \Delta f) = \frac{\Delta f}{(f - f_0)^2 + \Delta f^2} + \frac{\Delta f}{(f + f_0)^2 + \Delta f^2}, \quad (22)$$

where Δf is the linewidth parameter. The line width parameter is modeled by,

$$\Delta f = \frac{A_1 P}{T^{x_1}} + \frac{A_2 e}{T^{x_2}}, \quad (23)$$

where A_j are fitting constants, P and e are the total and species partial pressures respectively, and x_1 and x_2 are also fitting constants. Liebe and Layton used $x_1 = 0.6$ and $x_2 = 1.1$. $S(T, e, P)$ is the line strength and is given by,

$$S(T, e, P) = \frac{A_3}{T^{x_3}} \exp\left(A_4 \left[1 - \frac{A_5}{T}\right]\right), \quad (24)$$

where again A_j and x_3 are fitting constants, $x_3 = 3.5$. The third term of the empirical model describes self-broadening contributions, i.e. water vapor molecules colliding with other water vapor molecules. This term is given by,

$$\alpha(f, T, P, e) = f^2 \left\{ \frac{B_f e P}{T^{x_f}} + \frac{B_s e^2}{T^{x_s}} \right\}, \quad (25)$$

where B_j and x_k are fitting constants.

6. CONCLUSION

A weather radar's performance is highly dependent upon the frequency. An X-band radar (~8 -12 GHz), demonstrates much larger attenuation than a C or S band system. In this paper, we reviewed the state of knowledge concerning atmospheric attenuation of radar energy, deriving the theoretical base for these effects and developed a simple dynamical model for the attenuation based upon environmental factors.

Future work involves experimentally validating the models, applying the empirical model described in section 5 to the radar range equation and thus obtaining a dynamical, empirical formulation for returned signal.

Once a satisfactorily sufficient model has been obtained, inverting the problem and obtain estimates for environmental parameters, i.e. temperature, pressure, humidity from the radar data.

7. REFERENCES

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