P1.19 ISSUES SURROUNDING LINEARITY SPECIFICATION CONSTRICTION

James L. Alford*, James J. Stagliano, Jr., James Helvin
Enterprise Electronics Corporation

1. INTRODUCTION

Current radar digital receivers have a linear dynamic range greater than 100 dB with a linearity of ±1.0 dB. Some receivers on the market claim a linearity of ±0.1 dB, post digitization. A recent inquiry to change the linearity specification to 0.25 dB resulted in system analysis to obtain the desired linearity.

The desire for constricting the linearity specification is a direct result of error analysis. With the current linearity specifications, the possible error is dramatically large, allowing the conclusion that radar based hydrology measurements are highly suspect. By increasing the linearity of a system, the possible measurement error is greatly reduced, allowing accurate hydrological products.

This paper addresses the pertinent issues concerning reducing the linearity specification, including the definitions of linearity, measurement error analysis, dynamic compensation of system variations due to temperature and age, and the impact upon meteorological products. Finally, we discuss the cost-benefits of such a design, i.e. is the market willing to bear the dramatically increased expense to achieve a ±0.25 dB linearity.

2. LINEARITY SPECIFICATION

A radar manufacturer specifies their systems are linear to $\pm\delta$ throughout the dynamic range. An obvious question for the customer, what does the manufacturer mean by this statement? How can the validity of the statement be tested efficiently? To answer these questions, we need to explore quantitative measures of linearity.

All measures of linearity revolve around comparing a set of calibration data, $S \equiv \{(x_1, y_1), (x_2, y_2), \cdots, (x_N, y_N)\}$, with a linear model,

$$F(x) = mx + y_0 \,, \tag{1}$$

where m is the slope and y_0 is the intercept. The fundamental unit in these comparisons between the data and the model is the deviation between them, i.e.

$$\Delta(x_i) = y_i - F(x_i) \,. \tag{2}$$

These comparisons may take the form of p-norms or of some statistical measures of the deviations between the data and the model.

The simplest comparison is by far the p-norm, (Stoer (1992)). The p-norm is simply,

$$\|\Delta\|_p = \left\{ \sum_{i=1}^{N} |\Delta(x_i)|^p \right\}^{1/p} \,, \tag{3}$$

with $p = \infty$ being the maximum deviation magnitude (Stoer (1992)),

$$\|\Delta\|_\infty = \max_{1 \le i \le N} |\Delta(x_i)| \,. \tag{4}$$

For a specified δ, the associated linearity definition becomes

$$\|\Delta\|_p \le N^{1/p} \delta \,, \tag{5}$$

for $0 < p < \infty$ and with $p = \infty$ being the maximum deviation magnitude,

$$\|\Delta\|_\infty \le \delta \,, \tag{6}$$

for $p = \infty$.

This definition has a major disadvantage. By judicious selection of data points, the system can appear to meet or surpass specification even if otherwise it would normally fail miserably. For example, consider the data in Table I. From this table we would conclude that the system meets the $\delta = 1.5$ dB linearity specification with the ∞-norm. Yet, at higher resolution as in Table II, the system has failed at numerous points.

Using the p-norm definition, the only method to ensure the system meets required specification is to obtain calibration data for the resolution of the processor. For a processor whose output is eight bit reflectivity data, this implies a resolution of 0.5 dB or 200 points for a 100 dB dynamic range. Sixteen bit

* *Corresponding Author Address:* James L. Alford, Enterprise Electronics Corporation, 128
S. Industrial Blvd., Enterprise, AL 36330;
e-mail: alf@eecradar-idt.com

reflectivity data has a resolution of 0.01 dB, requiring 10,000 points. This is not practical.

P_{Input} (dBm)	Z_{Theory} (dBZ)	$Z_{Measured}$ (dBZ)	Deviation (dB)
-30.0	25.0	24.1	-0.9
-35.0	22.5	21.9	-0.6
-40.0	20.0	21.1	1.1
-45.0	17.5	17.3	-0.2
-50.0	15.0	13.6	-1.4
-55.0	12.5	12.2	-0.3
-60.0	10.0	9.9	-0.1
-65.0	7.5	6.5	-1.0
-70.0	5.0	5.3	0.3

Table I. Table of data values in linear regime, input power from -30.0 dBm to -70.0 dBm. At this interval, 5.0 dBm the system appears to meet ∞-norm specifications. $r = 0.994$ and $\sigma = 0.89$.

P_{Input} (dBm)	Z_{Theory} (dBZ)	$Z_{Measured}$ (dBZ)	Deviation (dB)
-30.0	25.0	24.1	-0.9
-31.0	24.5	25.5	1.0
-32.0	24.0	23.7	-0.3
-33.0	23.5	22.2	-1.3
-34.0	23.0	20.6	-2.4
-35.0	22.5	21.9	-0.6
-36.0	22.0	20.1	-1.9
-37.0	21.5	21.5	0.0
-38.0	21.0	21.6	0.6
-39.0	20.5	21.5	1.0
-40.0	20.0	21.1	1.1
-41.0	19.5	21.1	1.6
-42.0	19.0	18.8	-0.2
-43.0	18.5	15.5	-3.0
-44.0	18.0	15.2	-2.8
-45.0	17.5	17.3	-0.2
-46.0	17.0	15.6	-1.4
-47.0	16.5	17.9	1.4
-48.0	16.0	16.6	0.6
-49.0	15.5	15.8	0.3
-50.0	15.0	13.6	-1.4

Table II. Table of data values in linear regime, input power from -30.0 dBm to -50.0 dBm. At this resolution, 1.0 dBm the system fails to meet the ∞-norm specifications at numerous points. $r = 0.91$ and $\sigma = 1.48$.

Alternatives to this brute force methodology utilize ideas from statistics to validate claims more efficiently and at the same time provide the customer with a measure of linearity performance as well as specification.

The first foray into statistical techniques is the correlation coefficient. The correlation coefficient is a measure of how well the quantities relate to each other (Yamane (1964)). Mathematically, the correlation coefficient is given by,

$$r = \frac{N\sum_{i=1}^{N} x_i y_i - \left(\sum_{i=1}^{N} x_i\right)\left(\sum_{i=1}^{N} y_i\right)}{\sqrt{N\sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2}\sqrt{N\sum_{i=1}^{N} y_i^2 - \left(\sum_{i=1}^{N} y_i\right)^2}} \tag{7}$$

The closer that $|r|$ is to 1.0, the closer the relationship. If $r \sim 0$, x and y do not relate to one another.

Performing correlation analysis on the data in Table II, we obtain $r = 0.91$. It is clear that the data is highly correlated, i.e. it is very linear based upon this analysis. Yet, we do not have a measure for the error.

The next excursion into the statistical analysis is to determine the parameters (slope and intercept) that best fits the data. In this process, an estimate for the standard deviation (error) can be obtained that quantifies the linearity of the system. The method used is to minimize a function of the deviations squared, i.e. the least squares method. The function to be minimized is written (Stoer (1992)),

$$G(x, y; F(x)) = \sum_{i=1}^{N} [y_i - F(x_i)]^2 \tag{8}$$

The parameters of $F(x)$ are the slope and intercept and are given by (Yamane (1965)),

$$m = \frac{N\sum_{i=1}^{N} x_i y_i - \left(\sum_{i=1}^{N} x_i\right)\left(\sum_{i=1}^{N} y_i\right)}{N\sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2}, \tag{9}$$

and

$$y_0 = \frac{\sum_{i=1}^{N} y_i - m\left(\sum_{i=1}^{N} x_i\right)}{N}. \tag{10}$$

The measure of the error, the sample standard deviation, is given by (Yamane (1965)),

$$\sigma = \sqrt{\frac{G(x, y, F(x))}{N-2}}. \tag{11}$$

This measure quantifies the linearity by specifying error bounds with confidence intervals. For example, we can say with 68% confidence the error at any one point will not be greater than σ, 95% confidence that the error will not be greater than 2σ, and 99.5% confidence that the error will not be greater than 3σ, (Baird (1994)).

The linearity definition based upon a multiple of σ is known as the multisigma linearity definition. The system approaches the ∞-norm definition as the multiple of σ increases. The minimum multiple we should consider is 3, i.e. 3-σ linearity $\left(\sigma \geq \delta/3\right)$.

For the data in Table II, we find the standard deviation to be σ = 1.5 dB. Thus, we have 68% confidence that any point in the range will fall within ±1.5 dB from the curve, 95% confidence of ±3.0 dB, and 99.5 % confidence of ±4.5 dB.

Finally, the root mean square (rms) definition of linearity is very similar to the 1-σ definition. The only difference is the term in the denominator (Yamane (1965)), i.e.

$$\sigma_{rms} = \sqrt{\frac{G(x, y, F(x))}{N}}\ . \qquad (12)$$

3. 0.25 DB LINEARITY

Specifying a δ = 0.25 dB linearity using the 3-σ definition means that $\sigma \leq$ 0.083 dB. This, of course, has some serious implications upon the design, implementation, and testing of any system with this specification.

A requirement of 0.25 dB linearity over a 100+ dB dynamic range requires every component to be scrutinized, selected, and/or modified with extreme care. The test signal generators need to be level across the output power range and across the frequency range of the generator. Another consideration is the SMA connectors. Variance in a connector can be as much as 0.25 dB with each cycle of connection and disconnection. The variance is even greater when water vapor (humidity) effects are considered. Assuming that once the system is assembled, no maintenance requiring a disconnect are performed, the loss will not vary greatly.

Noise reduction in the system can also assist in improving linearity (as well as dynamic range). The obvious choice for reducing noise is to cool the components. Cooling the LNA's is relatively simple with Peltier junctions. The addition of a Peltier junction can reduce the noise figure as much as 1 dB. Similarly, cooling the mixer will reduce its thermal noise. In addition, the output of the mixer typically has spurious peaks outside of baseband due to the nonlinear effects of overdriving (performed to achieve maximum dynamic range). Cooling the mixer will reduce these nonlinear contributions, thus reducing the error in the mixer.

Each amplifier has a gain curve specific to that particular device. Each one will have different little bumps and wiggles, deviations from perfect linearity that must be corrected in the data processing. Small nonlinearities in the analog section of the can be corrected in the digital section of the receiver.

At least one manufacturer of digital receivers claims to be linear to within 0.1 dB (theoretical not tested). Correcting for small linearities in the analog section of the receiver must occur in the digitization section. The addition of weighted noise and temporal integration into the digitization process has the effect of smoothing the small bumps and wiggles. This process averages a number of data samples with noise added, the data is dithered.

Another technique for correcting the nonlinearities is the use of a "smart" digitizer. During the calibration mode, a smart digitizer determines the deviation between the digitized data and the linear model for the A/D's. These deviations are stored in a lookup table for real-time correction of the data. The theoretical limit for a smart digitizer is the bit-weight of the A/D devices.

Building a system for performance specifications to be linear within 0.25 dB over a 100 dB dynamic range is accessible, expensive but possible. Once the system is built, the system needs to be calibrated and tested. In otherwords, how do we verify the system meets the desired performance specifications? High performance test and maintenance equipment must be obtained. To meet the specifications, assuming a 3-σ specification, the calibration and test equipment should have a variance no greater than 0.083 dB over the entire dynamic range. To illustrate, a mid-performance test signal generator ($100,000) offers a nonlinear error specification of 1.0 dB for output signal levels from 90 dBm to +10 dBm and 2.5 dB for signal levels below 90 dBm. Hence, even this generator would not be a suitable candidate for calibrating our system.

4. PRODUCT IMPLICATIONS

The implications for the meteorological products greatly depends upon the product resolution. Most products have an 8 bit (256 levels) or less resolution. With 256 levels, the effective resolution of reflectivity data is 0.5 dB (with a range from -31.5 to 95.5). Constricting the linearity specification to δ = 0.25 dB effectively reduces the error to within one reflectivity quantum. The current δ = 1.5 dB encompasses six quanta of reflectivity.

One product area where these errors may have an impact is in rainrate evaluation via the Marshall-Palmer relation. In the former, the maximum error in the rainrate is,

$$\Delta R = \left(10^{2\delta/10b} - 1\right) R, \tag{13}$$

where b is the exponent in the Marshall - Palmer relationship and R is the rainrate. Using b = 1.6, we find $\Delta R = 0.54R$ for δ = 1.5 dB and $\Delta R = 0.075R$ for δ = 0.25 dB. In otherwords, for δ = 1.5 dB the relative error in rainrate measurement (assuming the Marshall-Palmer model fits the event exactly) is ~54% and ~7.5 % for δ = 0.25 dB.

Another product area is hydrometeor classification using Z and Z_{DR}. Hydrometeor classification is highly sensitive, particularly for "dry" hydrometeors, to the value of Z_{DR} which is the difference between the horizontal and vertical reflectivity values. Since it is a difference, the possible errors will add, thus a δ = 1.5 dB system allows up to 3 dB error in Z_{DR}. This error is enough to dramatically change the hydrometeor classification. A system with δ = 0.25 allows an error of 0.5 dBZ in Z_{DR}, resulting in more accurate classifications.

5. CONCLUSION

There are many possible definitions of linearity. One is the p-norm with the ∞-norm being the easiest to determine and visualize. However, the p-norm provides little information concerning the distribution of the calibration data and requires that every digitization point within the linear be tested. Statistical measures using multiples of the standard deviation (multi-sigma) provide a measure by which the certainty of the calibration points can be estimated. This allows for calibration with fewer points and a measured certainty with the results.

From clearly a scientific perspective, the weather radar industry should continuously evolve to higher and higher precision to reduce the errors. However, the financial expense of producing such a system is dramatic, easily adding several million dollars to the price of a weather radar system. Only a few of the well funded, government research laboratories have the available resources to purchase such a system.

6. REFERENCES

D. Baird, (1994): **Experimentation: An introduction to measurement theory and experiment design**, Prentice-Hall, pp. 224.

J. Stoer and R. Bulirsch, (1992): **An Introduction to numerical analysis**, Springer-Verlag, pp. 660.

T. Yamane, (1964): **Statistics:An introductory analysis**, Harper & Row, pp.734.