2.1 RANDOMNESS OF RADIANCES FROM BROKEN CLOUD FIELDS

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1. INTRODUCTION

In the measurement of Earth radiation budget by satellite radiometers, fields of broken clouds present a special problem. In order to compute the radiant flux from a measured radiance, it is necessary to know the bidirectional reflectance distribution function (BRDF), which describes the anisotropy of the radiation from the scene. The randomness of the clouds in terms of size, shape, optical depth and spacing causes them to have varying distributions of directional reflectance. Many of the problems of attempting to treat broken cloud fields arise from the three-dimensional characteristics of the clouds. McKee and Cox (1976, 1979) showed that 3-dimensional effects of cloud shapes on radiative transfer create far greater variability than do differences of cloud microphysical parameters. Because of the importance of 3-dimensional cloud effects in remote measurements, 3-D radiative transfer have become an active area of research, as can be seen from the site: http://climate.gsfc.nasa.gov/I3RC

Because the BRDF is a random variable, flux computed from the measured radiance is in error due to the difference of the model BRDF from the true but unknown BRDF for each realization. One aspect of the computation of the BRDF of a broken cloud field is the field of view of the radiometer.

This paper discusses the measurement of radiation over a partly cloudy or broken cloud field. A number of simulations have been made of the measurement process with cloud fields described by high resolution satellite imagery. The purpose of this paper is not to present another simulation but to examine the parameters of the problem so as to gain an understanding of their roles in the measurement process. First the nature of the measurement is considered, then an empirical description of cloud fields is reviewed which leads to a probabilistic model of cloud fields. The measurements are then expressed in terms of this model.

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2. MEASUREMENT

The measurement of a radiometer may be expressed as

$$m = \int_{FOV} L(x, y, \theta, \phi) P(\alpha, \beta) d\Omega$$

where L denotes radiances from the Earth-atmosphere system at point x,y and the view zenith angle θ and relative azimuth ϕ from the Sun define the direction of the ray of reflected sunlight from the point to the instrument. The response of the instrument to radiance at a given point in the field of view is not constant but is defined by the point spread function of instrument $P(\alpha,\beta)$, which is, described in instrument coordinates as angles along-scan α and cross-scan β as measured from the optical axis. The integration is over the field of view FOV of the instrument.

The field of view of the instrument is projected onto the Earth-atmosphere system as distance x alongtrack and distance y crosstrack so that the measurement is written as

$$m = \int_{FOV} L(x, y, \theta, \phi) P(\alpha, \beta) J dx dy$$

where J is the Jacobean of the transformation from instrument to Earth coordinates and the integration is now over the projection of the field of view onto the Earth-atmosphere system, often called the footprint of the measurement. The projection of the instrument coordinates α,β onto the Earth-atmosphere system x,y varies as the instrument scans; the size of the projection of the FOV onto the Earth-atmosphere system increases with view zenith angle θ.Typically the characteristic size of the field of view is 10 to 60 km at nadir. The angles $d\alpha$ and $d\beta$ at the instrument project onto the Earth-atmosphere as distances dx and dy as

$$dx = \rho \csc\theta d\alpha \ , \quad dy = \rho d\beta \ ,$$

where ρ is the slant distance from the spacecraft to the scene and can be computed from the altitude of the spacecraft, the view zenith angle θ and the radius of the Earth. The Jacobean is then

$$J = 1/(\rho^2 \csc\theta)$$

J is a function of the view zenith angle, but may be taken to be constant over the field of view, i.e. for a single pixel.

The shortwave radiance L is related to the albedo of a point by

$$L(x, y, \theta, \phi) = \pi^{-1} R(x, y, \theta, \psi) a(x, y) H \cos\varsigma$$

where $a(x, y)$ is the albedo of the point, $R(x, y, \theta, \psi)$ is the bidirectional reflectivity of the point x,y in direction θ, ψ , H is the solar flux at the "top of the atmosphere" and ζ is the solar zenith angle.

We will assume that the scene is partial clouds over a dark surface, e.g. ocean away from the Sun-glint direction, so that radiance from the surface through the cloud can be neglected. These effects can be added later. It is assumed further that the clouds are in one layer, so that they do not overlap. Under these approximations the measurement can be expressed in terms of the contributions of individual clouds as

$$m = \pi^{-1} HJ \cos\varsigma \text{ x}$$
$$\sum_i \int_{FOV} R(x, y, \theta, \psi) a(x, y) P(\alpha, \beta) dx dy$$

For a broken cloud field, the clouds will assumed to be small relative to the instrument FOV. With this approximation the measurement becomes

$$m = \pi^{-1} HJ \cos\varsigma \sum_i P_i R_i a_i A_i$$

where A_i is the area of cloud i, a_i is its albedo, R_i is its BDRF and P_i is the value of the point spread function at the cloud center. The BDRF and the albedo are considered here to apply to a cloud as an entity, as by McKee and Cox (1979), rather than as a quantity distributed over the area of the cloud. This concept is suitable for a viewpoint far from the cloud. The case of clouds which are large compared to the FOV must be treated using the integral. Because the parameters R_i, A_i, a_i and the location of the cloud within the field of view are random variables, the measurement is likewise a random variable.

In the analysis of data, the scene is usually assumed to be uniform and the pixel level and the albedo is computed by use of a scene dependent R_i model which accounts for the anisotropy of the radiance field. Because R_i is a random variable which manifests itself in a summation, the relation of the average albedo within the pixel is a random number rather than the model value, resulting in an error in the retrieved albedo value.

The point spread function PSF may have a "tail" which decreases exponentially and thus extends a large distance (e.g. Smith, 1994). A judicious choice of the limit of the FOV is made, e.g. a line is defined beyond which the point spread function is below a threshold value, so that the FOV is limited. Then the FOV has an area A and the set of clouds in the field of view is finite. The FOV is symmetric about the x-axis and its 2 sides are given by

$$y = \pm f(x)$$

The area of the FOV can be computed from this expression.

3. CLOUD DISTRIBUTION OBSERVATIONS

The description of the cloud sizes and spacing and their relation to the field of view of the radiometer is an exercise in stochastic geometry. This aspect will be simplified immensely by considering them to be circular in the horizontal plane and characterized by a diameter.

Plank (1969) investigated the size distributions of fair weather cumulus clouds using photographs taken from aircraft. He found that for representative Florida cumulus clouds, the cloud base diameters are between d = 50 to 200 ft. (80 to 320 m) and D_m = 3.0 miles (5 km), and their size is distributed exponentially:

$$n = K \exp(-\gamma D) , \qquad d < D < D_m$$

where γ is between 2 to 22 miles^{-1} (1.2 to 13 km^{-1}) and K is in km^{-2}. The minimum cloud size is related to the break in the spectrum at small sizes noted by Cahalan et al. (1994). The total number of clouds is then given by integrating this expression over the range d to D_m:

$$N = \frac{K}{\gamma}(e^{-\gamma d} - e^{-\gamma D_m})$$

The total sky cover is given by integrating the number of clouds times their size, giving

$$S_C = \frac{\pi K \chi}{2\gamma^3}$$

where

$$\chi \cong 1 - e^{-\gamma D_m}\left[1 + \gamma D_m + \frac{(\gamma D_m)^2}{2}\right]$$

For a given γ and cloud fraction, K can be computed and then the total number of clouds N. The average cloud area is then $A_C = S_C / N$.

4. PROBABILISTIC MODEL

The cloud distribution is considered to be a Poisson process, so that N is the mean number per unit area. The mean number of clouds in the FOV is AN and probability that there are n clouds in any given FOV is

$$p_n = e^{-AN} \frac{(AN)^n}{n!}$$

Given that a cloud is present, the cumulative probability distribution of diameter D is

$$P(D) = \frac{1 - e^{-\gamma(D-d)}}{1 - e^{-\gamma(D_m - d)}}$$

The point spread function P varies across the FOV so that it is necessary to specify the location of each cloud within the FOV. The location of the clouds will be assumed to be random over the domain and independent for each cloud. The probability distribution for x is then

$$p(x) = \frac{2f(x)}{A}$$

and y is uniformly distributed between $[-f(x), f(x)]$.

It is useful to consider the case of the point spread function being constant over the FOV, in which case the measurement is independent of the position of clouds within the FOV. If the BDRF and albedo for each cloud is the same, the mean measurement reduces to

$$\langle m \rangle = \pi^{-1} HJPRaS_C \cos\varsigma$$

The second moment of the measurement is given by

$$\langle m^2 \rangle = (\pi^{-1} HJPRa)^2 \langle \sum_i A_i^2 \rangle$$

The number of clouds in the footprint is independent of the size of the clouds, so that $\langle \sum_i A_i^2 \rangle = N \langle A^2 \rangle$ and the mean square cloud area is given by

$$\langle A^2 \rangle = \gamma^{-5} \{ e^{-\gamma D} [24 - 24(\gamma D) - 12(\gamma D)^2 - 4(\gamma D)^3 - (\gamma D)^4 \} \Big|_d^{D_m}$$

The variance of the measurements can be computed as $\sigma_m^2 = \langle m^2 \rangle - \langle m \rangle^2$ and the dispersion (squared) is

$$\sigma_m^2 / \langle m \rangle^2 = N \langle A^2 \rangle / S_C - 1$$

This result is based on the assumption of a PSF which is constant over the field of view. A realistic PSF is peaked near its center. The effect of this peak will be to weight the clouds near this peak heavily and others less, so that the variance will not decrease as $N^{-1/2}$ as in the case of a constant PSF, but more slowly. For a PSF which is not constant, one would have to evaluate the second moment of measurement using the PSF.

These variations are due only to the amount of cloudiness is a given field of view and are not due to variations of the bidirectional reflectance function R or albedo a of cloud. A simple model of this effect is now presented.

The albedo of the cloud will increase with its diameter D because less light leaves the sides in a downward direction. Also, as D increases, the height and thus optical depth increases, which increase albedo. The radiance from the Sun-lit side of a cloud is quite high, while the radiance from the shady side is low. As D increases, these effects of the edges decrease relative to the total radiance in a given direction from the cloud. Thus R and a are functions of D. The mean measurement then becomes

$$\langle m \rangle = \pi^{-1} HJPN \cos\varsigma \times \int^{D_m} R(\theta, \psi; D) a(\zeta; D) A(D) p(D) dD$$

The second order moments can be also computed in like manner.

This equation shows that the relations of BRDF and albedo to cloud size and the distribution of cloud sizes determine the distribution of measurements. The R and a are not simply functions of diameter D but also of a number of other random variables. This can be taken into account by writing the probability density function as being dependent on these other variables in addition. One would then integrate over these added variables as well

.In the model discussed here, the number of clouds, their size distribution and locations are assumed to be independent. For large cloud fractions, these approximations will break down as the clouds would overlap.

5. CONCLUDING REMARKS

A simple model is presented for the random nature measurement of radiances from broken cloud fields. This model is based on observations and demonstrates that the basic variables of the problem are the instrument field of view and point spread function, the distribution of clouds and the dependence of the radiances on the cloud sizes. The basic cloud distribution parameters are the cloud fraction, the range of cloud sizes and the rate of decrease of number with size. The relations developed here provide a framework for parametric studies e.g. of the effect of field of view size on retrieved fluxes for cloud fields.

6. ACKNOWLEDGEMENT

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7. REFERENCES

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