5.5 END-TO-END ENSEMBLE FORECASTING: ENSEMBLE INTERPRETATION IN FORECASTING AND RISK MANAGEMENT

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1. INTRODUCTION

Good probability distributions of future outcomes are an important tool in risk management. The ideal forecast product for an end-user is not a probabilistic weather forecast but a probabilistic forecast of a weather dependent economic quantity (e.g. soft drink sales, electricity demand, wind energy production). Such *end-to-end* forecasts attempt to translate current uncertainty in weather variables to future uncertainty in variables relevant to the end user. Combined with the user's utility function, such forecasts enable rational decsion making.

Ensemble forecasts provide a starting point for generating probabilistic forecasts for risk management [Toth and Kalnay 1997; Palmer 2000]. In this paper, we present two different approaches to convert ensemble forecasts into probabilistic forecasts, and we illustrate these approaches using economic examples. The dependence of economic quantities of interest on the weather is generally nonlinear. This means that *the expected value of the economic quantity is not the value associated with the expected weather.* Under such circumstances, a crude probabilistic forecast can be substantially more valuable than a highly accurate prediction of the mean of the forecast distribution.

2. INTERPRETING ENSEMBLES

Ensemble forecasts do not sample the true distribution of uncertainty in the forecast. The initial conditions of the ensemble do not lie on the attractor of the model, let alone the system of which the model is an imperfect representation [Smith et al. 1999]. The ensemble does contain quantitative information about the forecast uncertainty. The optimum method for extracting this information depends on the type of forecast, and also on the application.

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2.1 *Conditioning Climatology*

Our method assumes that the forecast variables generated by a forecast model contain information about, but are not equivalent to actual weather variables. This is obvious if the forecast variable is not a weather variable (e.g. electricity demand), but is also true for weather variables such as temperature, due to the finite resolution and other imperfections of the model. Model output statistics (MOS) [Glahn and Lowry 1972] can be used as inputs into models that predict the forecast variables. These models can use the entire ensemble, or ensemble statistics, as conditioning variables and they can output information about the distribution of the forecast variable, not just its expected value.

2.2 *"Dressing" the Ensemble*

Dynamical ensembles can be convolved with statistical ensembles, constructed using the error statistics of the "best member" of the ensembles [Roulston and Smith 2001c]. This approach is most straightforward when the errors of the forecast quantity are quasinormal. This condition may arise with temperature but it is not true for quantities such as precipitation. Single, "best guess" forecasts can also be dressed with statistical ensembles based on their historical error statistics.

3. DECISION MAKING AND RISK MANAGEMENT

The central idea of decision making theory is *utility* [Lindley 1985]. The utility is a quantification of the desirability of a particular outcome, relative to alternative outcomes. Let $U(e,c)$ be the user's utility, where e is the set of possible events, and c is the set of choices open to the user. If p_e is the forecast probability of a particular event, then the optimum choice, c^*, is that which maximizes the user's expected utility, given by

$$E[U(e,c)] = \sum_e p_e U(e,c) \qquad (1)$$

In general, U is a nonlinear function of e so

$$E[U(e,c)] \neq U(E[e],c) \qquad (2)$$

To calculate $E[U(e,c)]$ for a given value of c requires knowledge of p_e. In business decision making, the utility is an increasing function of profit. The risk tolerance of the decision maker can be incorporated into the utility by making utility a nonlinear function of profit. The utility of a risk averse user will be a concave function of profit, such a function has diminishing marginal utility (see Fig. 1).

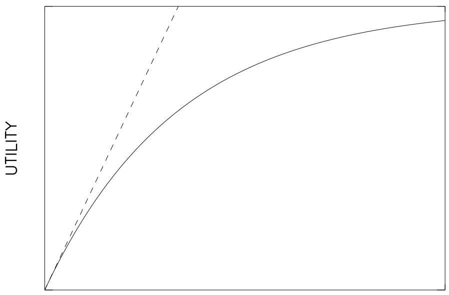

Figure 1: The utility as a function of profit of two different users. The dashed line is for a risk neutral user and the solid line is for a risk averse user.

4. EXAMPLES

4.1 *Wind Energy Production*

To investigate the possible value of probabilistic forecasts to wind energy producers, a "toy" electricity market was used. In this market, the producer must promise an amount of energy, E_c, in advance for a given price, P_c. If the actual production, E_a, equals or exceeds the contracted amount then no penalty is incurred. If, however, the actual production falls short of the contracted production, the producer must purchase the shortfall on the spot market at a price, P_s. The income of the producer, I, is given by

$$I = \begin{cases} E_c \cdot P_c & \text{when} \quad E_a \geq E_c \\ E_c \cdot P_c - (E_c - E_a) \cdot P_s & \text{when} \quad E_a < E_c \end{cases} \tag{3}$$

The nonlinearity of I as a function of E_a means that to make the optimum choice of E_c requires a probabilistic forecast, $p(E_a)$; the expected at value of E_a is not sufficient. In practice, P_s, will also be unknown at the time of writing a contract and so a joint distribution, $p(E_a, P_s)$, is actually required. If P_s is independent of E_a then E_c should be set at the $100 \times P_c/\langle P_s \rangle$ percentile of the probability distribution of E_a. This shows that, in this context, having a probabilistic forecast of E_a is absolutely essential for rational decision making.

Windspeed data sampled at 1 minute intervals at the CLRC Rutherford Appleton Laboratory's Energy Research Unit was used to construct a synthetic time series of power production from a wind turbine. A climatological model of production, conditioned on the time of year, was constructed and compared with five other methods for generating power production forecasts:-
(a) A persistence forecast for which the last 48 half hourly productions are interpreted as a sample of the forecast distribution.
(b) Production associated with the ECMWF high resolution forecast, interpreted as a deterministic forecast.
(c) Productions associated with ECMWF ensemble of forecast treated as a sample of the true distribution.
(d) Climatology of power production conditioned on the high resolution forecast by using the half-hourly production levels in the 10 historical days which had forecasts closest to the current forecast.
(e) Climatology conditioned on the ensemble forecast, as in (d) but where "closeness" was defined by the Euclidean distance in the 3D space defined by the 10th, 50th and 90th percentiles of the windspeed ensemble forecast.

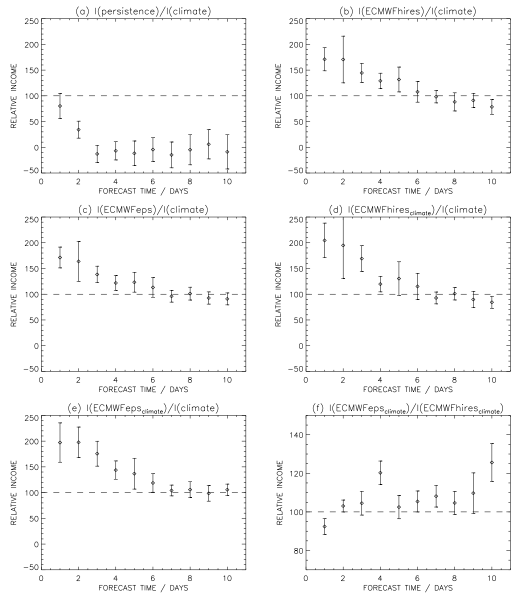

Figure 2: Weekly profits of generators using the different forecast generation strategies. (a) persistence, (b) ECMWF high resolution forecast, (c) ECMWF ensemble prediction scheme, (d) climatology conditioned on the high resolution forecast, (e) climatology conditioned on the ensemble. (a)-(e) are all relative to a climatological forecast (=100). (f) shows the climatology conditioned on the ensemble relative to the climatology conditioned on the high resolution forecast. The error bars were obtained by bootstrap resampling.

The value of the spot price was taken to be the U.K. system marginal price. A log-normal model for this price, conditioned on the time of day and year was used. For each half hour, a joint probability forecast for E_a and P_s was generated. The value of E_c which maximized the expected value of I was then chosen and the actual value of I calculated using the power production timeseries generated from the windspeed observations. The period Jan. 1999 to Aug. 1999 was used as the historical reference period, and the period Sept. 1999 to Aug. 2000 was used to simulate the performance of the producer. For the experiments, $E_c = 10$ GBP/MWh was used. Figure 2 compares the weekly profits of the producer relative to climatology as a function of lead time. The best performance is obtained when the ECMWF ensembles are used to condition the climatology. This result illustrates that even a relatively crude method for constructing probabilistic forecasts can be more valuable than a refined forecast of the mean of the forecast distribution. Figure 3 compares the daily income of a producer basing their choice of E_a on climatology, and one using the 4-day ECMWF ensemble to condition the climatology. It can be seen that the increase in income is due to both avoiding overpromising during periods of unusually low wind (e.g. Jan. 2000), and also to exploiting periods of unusually high wind (peaks in Fig. 3b).

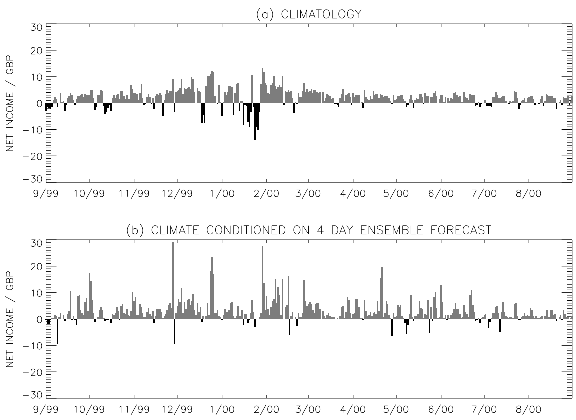

Figure 3: A comparison of daily net income of a generator basing decisions on the climatological forecasts and a generator using a climatological forecast conditioned on the 4-day ECMWF ensemble forecast.

A more detailed examination of the wind energy forecast experiments can be found in Roulston et al. [2001a,2001b].

4.2 Electricity Demand

Electricity demand is temperature dependent. This dependence, for the U.K., is shown in Fig. 4. In warmer climates, the demand curve rises again at high temperatures due to air conditioning load.

A similar toy market model was used to investigate the potential value of probabilistic forecasting to an electricity supplier. In the model, the supplier must decide in advance how much electricity, G, to contract from generators. This electricity costs P_c per unit. If the demand is D, the supplier can sell electricity onto consumers at a price of P_s per unit. However, should demand exceed contracted generation the supplier must pay an imbalance cost of P_i on the generation shortfall. The income of the electricity supplier is thus given by

$$I = \begin{cases} -P_c G + P_s D & \text{when} \quad D \leq G \\ -P_c G + P_s D - P_i(D - G) & \text{when} \quad D > G \end{cases} \tag{4}$$

The performance of an electricity supplier using different forecasting approaches was simulated. In all the simulations the supplier was contracting generation 4 days ahead of delivery. The cost price was $P_c = 1.0$ and the selling price was $P_s = 1.5$. The supplier's profits were calculated for a wide range of imbalance settlement prices, reflecting the observed volatility of electricity prices. The forecasting approaches used were:-
(a) Choosing G to maximize I averaged over demands drawn from a climatological distribution.
(b) Choosing G to match the demand associated with the temperature in the high resolution ECMWF forecast.
(c) Choosing G to maximize I averaged over the demands associated with each of the 51 members of the ECMWF ensemble.
(d) Dressing the ECMWF high resolution forecast with a statistical ensemble of historical errors (in the demand) and then choosing G to maximize I averaged over this ensemble.
(e) Dressing each of the 51 members of the ECMWF dynamical ensemble with a statistical ensemble of demand errors, and choosing G to maximize the average I.

The results of the simulations are shown in Fig. 5. Treating the single high resolution forecast as a deterministic forecast and contracting the generation appropriate for this single forecast is a *very bad* strategy. This is not because the high resolution forecast lacks skill, it has a high correlation with the observed temperature, but because this forecast is for the expected demand and given the nonlinearity of Eq. 4 the expected demand is *not* the optimal value of G to maximize the expected income. The bare ensemble provides better results because this forecast contains some information about the probability distribution. Creating a statistical ensemble by adding historical errors to the high resolution forecast yields an even better result, and creating the statistical ensemble around the dynamical ensemble provides further improvement, particularly at high P_i/P_c ratios when Eq. 4 is strongly nonlinear.

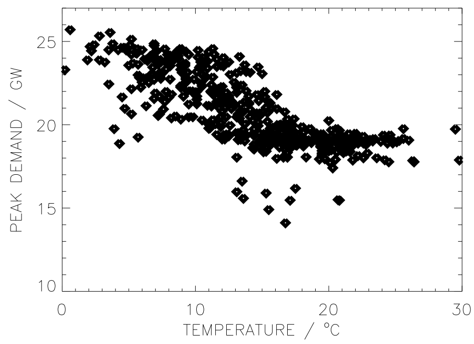

Figure 4: Weekday peak electricity demand in the U.K. as a function of temperature.

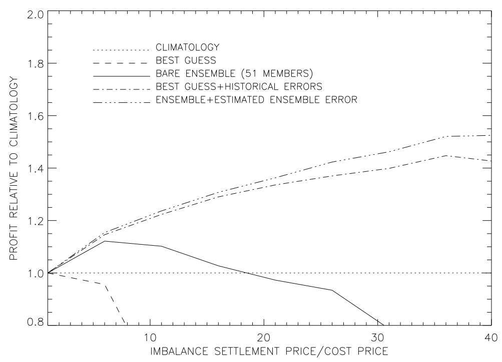

Figure 5: Simulated profit of an electricity supplier in Washington D.C. The supplier contracts generations 4 days in advance and a comparision of different forecast methods is shown. The profit is plotted as a function of the imbalance price to the cost price (P_i/P_c).

The large increase in performance when historical errors are added to the forecasts (high resolution and ensemble) is partly because these were historical errors in demand, incorporating not just uncertainty in the temperature forecast, but also in the demand forecast.

A more detailed description of the electricity demand forecasting experiments, including results for other locations, can be found in Smith et al. [2001].

5. SUMMARY

The nonlinear nature of many users' utility functions make probabilistic forecasts essential for rational decision making. The construction of probabilistic weather forecasts requires the interpretation of the dynamical ensembles produced by operational forecast centres such as ECMWF and NCEP. Two different approaches to interpreting ensembles have been discussed in this paper. Each has been illustrated using an economic example example. From the examples it can be appreciated how important probabilistic forecasts are, when dealing with nonlinear utility functions, for a wide range of applications.

References

Glahn, H.R. and Lowry, D.A., 1972: The use of Model Output Statistics (MOS) in objecctive weather forecasting, *J. Applied Meteorology, 11,* 1203-1211.

Lindley, D.V., 1985: *Making Decisions,* John Wiley and Sons, London.

Palmer, T.N., 2000: Predicting uncertainty in forecasts of weather and climate, *Rep. Prog. in Phys., 63,* 71-116.

Roulston, M.S., Kaplan, D.T., Hardenberg, J. and Smith, L.A., 2001: Value of the ECMWF ensemble prediction system for forecasting wind energy production, *European Wind Energy Conference (Abstract OD3.5) Copenhagen, 2001.*

Roulston, M.S. Kaplan, D.T., Hardenberg, J. and Smith, L.A., 2001: Using medium range weather forecasts to improve the value of wind energy production, in preparation.

Roulston, M.S. and Smith, L.A., 2001: Combining dynamical and statistical ensembles, submitted to *Tellus A.*

Smith, L.A., C. Ziehmann and K. Fraedrich, 1999: Uncertainty dynamics and predictability in chaotic systems, *Quart. J. Royal Met. Soc., 125,* 2855-2886.

Smith, L.A., Roulston, M.S. and Hardenberg, J., 2001: End to end ensemble forecasting: Towards evaluating the economic value of the ensemble prediction system, *ECMWF Technical Memorandum, No. 336.*

Toth, Z. and E. Kalnay, 1997: Ensemble forecasting at NCEP and the breeding method, *Mon. Wea. Rev., 125,* 3297-3319.