P1.14 A MINIMUM VARIANCE APPROACH TO ESTIMATING WIND DIRECTION STATISTICS

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1. INTRODUCTION

At first glance, the calculation of statistical wind parameters, such as mean wind direction (*U*) and standard deviation of horizontal wind direction (σ_θ), appears almost trivial, however, on closer inspection it becomes apparent that this topic is problematic at best. Part of the problem lies with linear techniques being applied to circular data (which is beyond the scope of this work). Also, due to various limitations in data loggers, communications lines, amount of on-line storage, etc., it is not always possible to archive raw samples. For example, the network of meteorological towers at Vandenberg AFB, which covers much of the 100,000 acre property, is linked to the weather station via old telephone lines that are limited to extremely low data rates (at least by today's standards). Thus, raw samples are averaged on-site at each tower, and only the one-minute statistics are archived or used for operational support. This data is also input to several different safety models to produce estimates of various launch-related hazards, such as those due to exposure to air toxics, blast over-pressure, or debris. Since each of these models have their own unique input data requirements, the specification of appropriate averaging times in the data logger software can also be problematic.

Given these constraints, a flexible approach to estimating wind parameters such as σ_θ is required. This work presents one such approach to solving the problem of re-scaling wind data. The minimum variance approach requires only one-minute average wind direction data (i.e., *U* and σ_θ) and produces longer-term statistics comparable to several traditional methods that require all the raw direction data samples.

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2. WIND DIRECTION COMPUTATIONS

With the primary focus of estimating atmospheric stability, several standard algorithms (see Table 1), including the minimum variance estimator, were prototyped and compared using approximately 5 hours of raw 1-second samples collected on the morning of 17 November 1994 at meteorological tower 058, Vandenberg AFB, CA. The instruments are mechanical cup and vane anemometers, and only standard engineering unit conversions were performed on the raw data. All of the tested algorithms were used to calculate the required averaging times (1 min, 10 min, and 60 min) except the minimum variance estimator, which requires 1 average data as input.

Table 1. Tested Algorithms

Type	Reference
"Exact" Method	D. Skibin (1984)
Unit Vector Method	K. Mardia (1972)
Arithmetic Mean	Y. Mori (1986)
Minimum Variance Estimator	S. Arnold (present work)

All methods assume stationarity, and that the sampled wind direction data (for a specified averaging period) are normally distributed, however, Skibin's method makes no a priori assumptions about the statistical distribution of the wind direction samples. Mori's method is recommended in the EPA's On-site Meteorological Program Guidance document (USEPA, 1987), which also recommends a minimum of 60 samples for a reliable estimate of the mean, and a minimum of 360 samples for the standard deviation. All quantities described below were calculated with the full number of samples available for a given averaging period.

All of the tested methods can account for the 0° - 360° discontinuity and perform identically for the 1 minute averaging time. For averaging times greater than 1 minute, all of the methods yield large values of σ_θ when the wind distribution is non-normal or multi-modal. Significant veering or other changes in direction occurring within the averaging period also result in large values of σ_θ. Under these conditions, none of the tested methods yield a σ_θ value that is representative of the true atmospheric stability.

3. MINIMUM VARIANCE ESTIMATOR

The minimum variance estimator was adapted from a linear estimation technique described by Papoulis (1965) in *Probability, Random Variables, and Stochastic Processes*. In this technique, the mean directions are constructed from a weighted sum of the one-minute means; the standard deviations are similarly constructed. Given 10 one-minute means and standard deviations, calculated from 60 one-second samples[†], the j^{th} 10-minute mean can be constructed as follows: Let i = j – (n-1), where i and j index the one-minute and 10-minute means, respectively, and n is the desired averaging period. The i^{th} one-minute mean is given by m_i, with associated standard deviation σ_i. If the direction difference between adjacent m_i are suitably constrained, the 10-minute mean is given by:

$$D_j = \sum_{i=j-9}^{j} a_i \cdot m_i \tag{1}$$

where $$a_i = \sigma_i^{-2} \cdot \left[\sum_{i=j-9}^{j} \sigma_i^{-2} \right]^{-1} \tag{2}$$

and the 10-minute standard deviation is given by:

$$\sigma_j = \left[\frac{N}{N \cdot n - 1} \left[\sum_{i=j-9}^{j} \left[\left(m_i - D_j \right)^2 + \sigma_i^2 \right] \right] \right]^{1/2} \tag{3}$$

where N = number of raw samples in m_i and σ_i.

† Although dictated by the output of the instrument in this case, the choice of sampling rate is arbitrary.

4. RESULTS AND DISCUSSION

In the figures that follow, the 10 and 60-minute calculations are updated each minute, and the horizontal axes are time-centered (e.g., the first 10-minute mean direction is centered on minute 5). Figure 1 shows the 10-minute mean wind direction for the first 2 hours calculated by the four different algorithms, while the 10-minute standard deviations are shown in Figure 2. Figures 3 and 4 show the same results for the 60-minute averaging time. For the 10-minute averaging period, all the algorithms yield similar results, however, the minimum variance estimator does not calculate the same quantities as the standard algorithms, so it produces a slightly larger σ_θ value in some cases. Note the differences between all the methods at the left side of all the Figures 3 & 4. This illustrates the effect of the large direction change in the first hour of the data. The main difference between the minimum variance estimator and the other methods is that, while the other methods weight all samples equally, the minimum variance estimator applies weights to the 1 minute mean directions based on the magnitude of the σ_θ associated with that direction. In other words, the more tightly grouped sets of directions are given correspondingly more weight in the variance estimate. The minimum variance estimator is thus a better estimate of the mean because it is a minimum error estimate. Also, it approaches the arithmetic mean in the limit.

It is interesting to note that the 10 and 60-minute minimum variance estimates of the standard deviation, although based on one-minute data composed of only 60 one-second samples, compare very favorably to the traditional results calculated from 600 and 3600 samples, respectively.

5. SUMMARY AND CONCLUSIONS

A new method of estimating wind direction means and standard deviations that uses short-term statistics instead of raw direction samples has been developed and tested. Observed performance to date has been quite good. Although tested with several different data sets from multiple instruments and locations in and around Vandenberg AFB, the algorithm should be tested with more data collected under different conditions. Aside from test and evaluation issues, the new algorithm has several desirable properties, and shows potential utility in areas such as re-analyses of historical data sets and air quality modeling studies.

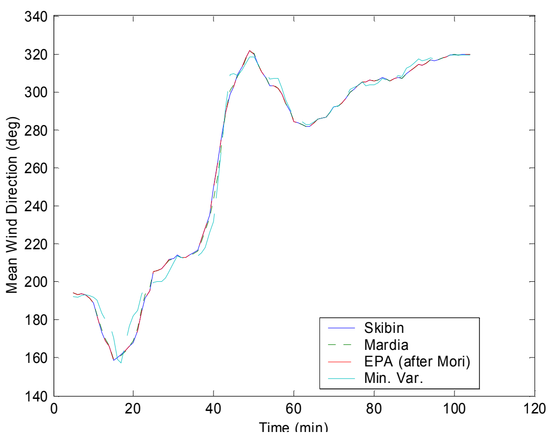

Figure 1. Comparison of Mean Wind Direction – 10-Minute Averaging Period.

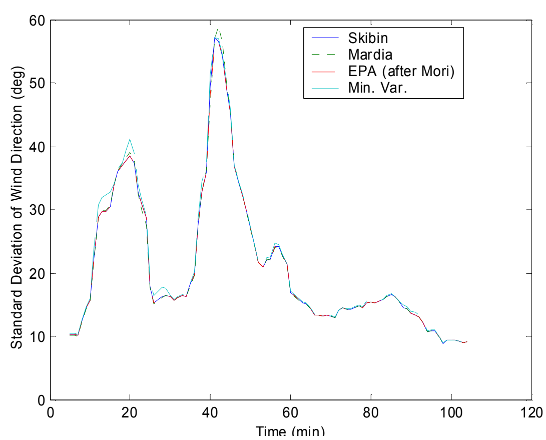

Figure 2. Comparison of Standard Deviation – 10-Minute Averaging Period.

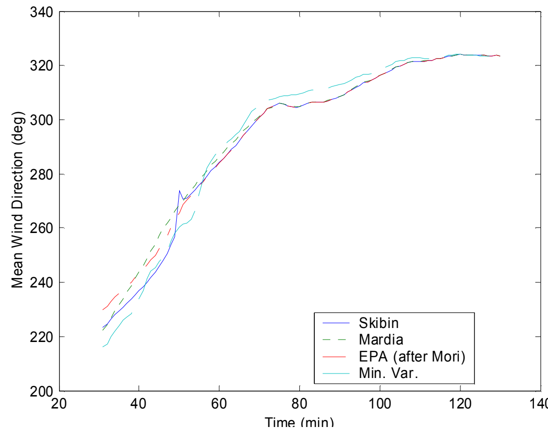

Figure 3. Comparison of Mean Wind Direction – 60-Minute Averaging Period.

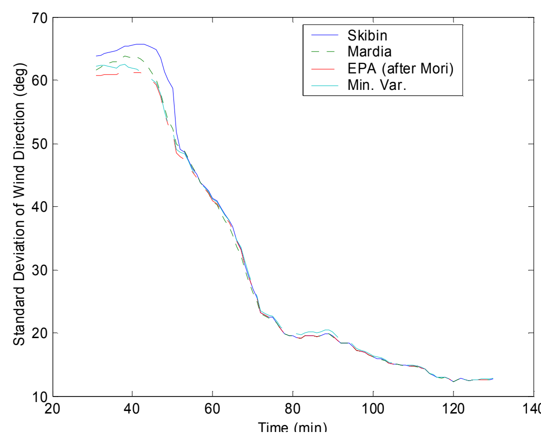

Figure 4. Comparison of Standard Deviation – 60-Minute Averaging Period.

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