BAROCLINIC EKMAN LAYER

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Introduction

The depth of the turbulent Ekman boundary layer (usually referred to as the planetary boundary layer–PBL) is one of its most fundamental properties strongly required in a number of practical problems within meteorology and oceanography. The PBL depth exhibits strong variability, which is partially explained by the effects of the density stratification and the earth's rotation. In spite of nearly a century of continuous efforts (since Ekman, 1905) and strong practical motivation, no consensus is achieved in the determination of the PBL depth as dependent on the PBL governing parameters. Essential disagreement between alternative formulations is hardly surprising in the light of a rather uncertain correspondence between known formulations and observational data. Experimental testing of all known PBL depth eqautions exhibits large spread of data. This suggests that additional mechanisms, besides the static stability and rotation, should be seriously considered. This paper focuses on the effect of baroclinicity.

Baroclinic PBL depth equations

On the assumption that the vertical gradient of the geostrophic wind $\Gamma = \nabla_z u_g =$ constant, conventionally neutral PBL depth is immediately extended to the baroclinic regime by inserting $u_T = u_*(1 - \sqrt{Ri_c/Ri})^{-1/2}$ instead of the usual stress velocity u_*:

$$h_E = C_R \frac{u_*}{|f|} \left[\left(1 - \sqrt{\frac{Ri_c}{Ri}} \right) \left(1 + \frac{C_R^2 C_{uN}}{C_S^2} \mu_N \right) \right]^{-1/2} \quad (1)$$

Here, $Ri = (N/\Gamma)^2$ is the gradient Richardson number above the PBL, N is the Brunt-Väsäla frequancy, $\mu_N = N/|f|$ is the imposed-stability parameter and f is the Coriolis parameter. Constants C_R, C_{uN}, C_S are to be determined from empirical data.

The truly neutral PBL could only be barotropic, as the presence of baroclinic shear would inevitably make turbulent the whole neutrally stratified flow.

LES validation of Eq. (1) is shown in Figure 1. The theoretical curve is calculated after Eq. (1) taking the a priori value of $Ri_c = 0.25$. Figure 1 demonstrates good performance of Eq. (1) over the whole range of Ri typically observed in the earth's atmosphere.

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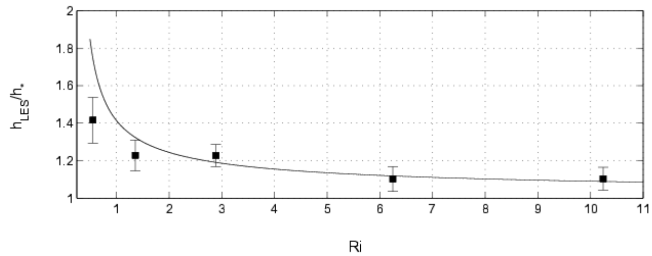

Figure 1: The dependence of the dimensionless baroclinic PBL depth, h_{LES}/h_*, on the free flow Ri. Here, $h_* = C_R u_*/|f|(1 + C_R^2 C_{uN} C_S^{-2} \mu_N)^{-1/2}$. Black boxes are LES data. The line represents Eq. (1).

Accounting for baroclinicity, the Monin-Obukhov turbulent length scale $L = -u_*^3/F_{bs}$ and the internal-stability parameter $\mu = |f| u_*/L$, become

$$L_{baroclinic} = L \left[1 - \sqrt{\frac{Ri_c}{Ri}} \right]^{-3/2}, \quad (2)$$

$$\mu_{baroclinic} = \mu \left[1 - \sqrt{\frac{Ri_c}{Ri}} \right], \quad (3)$$

where F_{bs} is the surface buoyancy heat flux. Introducing these corrections, baroclinic stable stratified PBL depth equation becomes

$$h_E = C_R \frac{u_*}{|f|} \left[1 - \sqrt{\frac{Ri_c}{Ri}} \right]^{-1/2} \times \\ \times \left(1 + \frac{C_R^2 C_{uN}}{C_S^2} \mu_N + \frac{C_R^2}{C_S^2} \mu \left[1 - \sqrt{\frac{Ri_c}{Ri}} \right] \right)^{-1/2} \quad (4)$$

LES validation of Eq. (4) is sown in Figure 2.

MIUU LES model

The LES model developed at the Dept. of Earth Sciences solves numerically filtered equations for the Boussinesq fluid in the horizontally periodic domain. The subfilter stress tensors are expressed in terms of filtered velocity and potential temperature using a dynamic mixed subfilter model (DMM) introduced by Vreman et al. (1994). It links the subfilter and the resolved variables assuming that the LES partially resolve the Kolmogorov's inertial subrange of scales.

LES provide detailed information about turbulence in simulated boundary layers. Since the PBL depth is controlled by the largest turbulent eddies, it is hardly

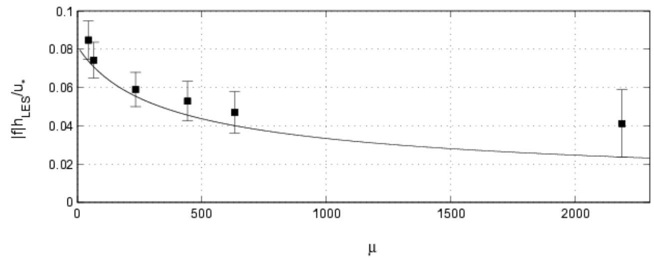

Figure 2: The dependence of the dimensionless depth, $|f|\ h_{LES}/u_*$, of the simulated baroclinic stable PBL ($\mu_N = 340$, $Ri = 5.1$) on the internal stability parameter, μ. Black boxes are LES data. The line represents Eq. (4).

strongly dependent on fine feature of the small-scale part of turbulent spectrum. Accordingly, numerical experiments were performed on relatively coarse mesh. The computational domain was taken relatively large. The point is that the simulated PBL depth is sensitive to the domain size. Earlier investigation of non-local features of the stable PBL disclosed their dependence on the internal-wave field in the adjacent free flow (Zilitinkevich, 2002). Then appropriate resolution of the major internal-wave harmonics is required. It should also be sufficiently high to allow undisturbed development of the steady-state regime.

In new MIUU-LES numerical experiments, the height of the domain is always chosen sufficiently high so that the simulated steady-state PBL depth would be less than two third of the domain height. LES runs cover 5 to 8 hours. Before each set of experiments, an 18-hour run is done to ensure that the steady state is achieved.

Conclusions

To the best of our knowledge, the role of baroclinicity was not considered in earlier formulations for the neutral and stable PBL depth. It is shown that the contribution from the baroclinic shear Γ increases the PBL turbulent velocity scale u_T, which in turn increases the equilibrium PBL depth h_E. This effect is fully determined by the free-flow Richardson number Ri.

The proposed diagnostic formulation, Eq. (4), for the equilibrium neutral or stable PBL depth accounts for the following mechanisms and governing parameters (in brackets): earth's rotation (Coriolis parameter f), surface-layer stability (surface buoyancy flux F_{bs}), free-flow stability (Brunt-Väisäla frequency N), and baroclinicity (geostrophic-wind shear Γ). Accordingly, it presents the dimensionless PBL depth as a function of the three dimensionless numbers, namely, the imposed-stability parameter μ_N, the internal-stability parameter μ and the free-atmosphere Ri.

Taking $\Gamma = 0$, Eq. (4) reduces to the barotropic PBL depth equation already derived and tentatively verified against atmospheric data by Zilitinkevich et al. (2002). LES provide an efficient complementary empirical-validation tool, which allow considering separately different essential dependencies included in Eq. (4). Results from the MIUU-LES are consistent with earlier LES. They fully validate the proposed theory, give quite certain estimates of the empirical constants $C_R = 0.5, C_S = 0.9, C_{uN} = 0.45$, and show full consistency with the traditional estimate of the critical Richardson number $Ri_c = 0.25$.

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