10D.6 High order numerical schemes for simulating tropical cyclone dynamics.

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1. INTRODCUTION

This study arises from the development of a new non-hydrostatic model to study the physics and dynamics of Tropical Cyclones. The two major components of a numerical weather prediction (NWP) model are the dynamics and physics kernels. Here, we have concentrated on implementing and testing numerical schemes that have appeared in the recent literature. The focus of this study is on the accuracy and efficiency of using high-order difference schemes. We also assess the performance of two non-oscillatory schemes for scalar advection of positive definite quantities. The results from a range of test cases are presented, culminating in a high resolution simulation of an idealized tropical cyclone. Tests were performed with advection schemes up to 10^{th} order to confirm the need for higher order schemes but also to demonstrate the diminishing returns using schemes beyond 6^{th} order. The scheme with the best overall performance was the 5^{th} order upwind advection scheme with similar order non-oscillatory schemes for fields that have positive definite constraints.

2.NUMERICAL ADVECTION SCHEMES.

We consider the advection of some scalar quantity ϕ which can be written in advective form as

$$\frac{\partial f}{\partial t} + U\nabla.f = 0, \qquad (1)$$

or in flux form as

$$\frac{\partial f}{\partial t} + \nabla.F = 0, \qquad (2)$$

Where F=**U**ϕ and **U** = (u,v,w) is the cartesian velocity vector. Equation (1) is linear if **U** is a function only of the independent variables. Eq. (1) and (2) are equivalent for non-divergent flows and for one-dimensional linear advection can be written in semi- discrete form as

$$\frac{\partial f}{\partial t} + uL(f) = 0,$$

and

$$\frac{\partial f}{\partial t} + \frac{1}{\Delta x}(F_{i+1/2} - F_{i-1/2}) = 0.$$

where L(ϕ) is the advection operator and $F_{i+1/2}$ is the flux at the east boundary of the cell centered at point .

We compare two non-oscillatory advection schemes. The first is the flux corrected transport (FCT) algorithm proposed by Boris and Book (1973) and extended to multi-dimensions by Zlaesak (1979). The FCT scheme approximates the solution with a high order scheme in regions where the solution is smooth while using a low order (usually first order) monotone schemes where the solution is poorly resolved, or discontinuous. The algorithm is flexible and allows the use of any number of high order schemes. We have implemented the FCT algorithm with third and fifth order upwind schemes, and fourth and sixth order centered schemes.

The second scheme is referred to as the Weighted Essentially Non-Oscillatory (WENO) scheme that has gained popularity for use in the study of high mach number flows. The WENO scheme is based on ENO shock capturing schemes (Shuand Osher,1988,1989) and was developed by Liu et al. (1994) and further refined by Jiang and Shu (1996), and extended to high orders by Balsara and Shu (1996). Shu (2000) provides a succinct summary of the WENO scheme. The WENO scheme performs a linear combination of polynomial curve fits using stencils of width r to achieve an accuracy of 2r+1. The weighted combinations is what gives the scheme it's name.

3. RESULTS.

For all the test cases, we used both the advective form and the flux form discretizations and found that they both produced similar results, hence we will show the results from the advective form discretization. For 1-D advection we compared performance of the first to tenth order basic advection schemes using centered differencing for the even ordered schemes and upwind differencing for the odd ordered schemes. The L_2 error norms verify the order of convergence for each of the basic advection schemes, with the L_2 errors tending towards 10^{-13} for high order schemes, and high resolution. This appears to be due to floating point

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truncation errors, as the time truncation errors for third order Runge Kutta scheme should be much lower than that. The first order upwind scheme is, as expected highly dampening, and the third and fifth order upwind schemes exhibit some dampening at low resolutions, but this is alleviated at higher order schemes. The second and forth order centered schemes exhibit trailing oscillations but again these are reduced using higher order schemes. Using a rectangular pulse as a test case the Gibbs phenomena is evident in all schemes above first order, because the leading truncation term in the first order scheme is the second order diffusion operator. These oscillations are not as significant in the upwind schemes as compared to the centered schemes. From the results of the one dimensional cases we restricted our 2 dimensional numerical solutions to using first to six order schemes. The test case here is rotating a cone through 360 degrees and comparing the numerical solution with the analytical solution. The first order scheme almost completely dampened the cone out after one rotation. The second order scheme maintained the amplitude better but had a trailing wake due to large dispersion error for third and fifth order upwind schemes the amplitude of the cone was increasingly preserved, and for forth and sixth order centered the wake was reduced, although still evident. The L_2 errors are similar for the one dimensional case, and the fifth and sixth order schemes had very similar L_2 error. Results for two dimensional deformational flow were similar. The first order scheme was highly dampening, and the even order centered schemes were highly oscillatory, especially when there were fine scale features. This is due to enhanced Gibbs oscillations from the coarse resolution of the fine scale features. One the other hand the third and fifth order upwinding scheme maintained the form of the exact solution quite well. The L_2 error for the schemes showed that for coarser resolutions the third and fifth order upwind schemes outperformed the fourth and sixth order centered schemes. There is, however, a cross over point with respect to resolution after which the centered schemes do better than the upwind schemes. Figures supporting these results will be presented during the presentation, however the small area allowed for figures means they would be indistinguishable here.

4. **CONCLUSIONS.**

In terms of performance over a comprehensive set of test cases ranging for advection of simple 1-D through to two dimensional multi scale flows, it is suggested that high order upwind or centered schemes provide arguably the best simulations. Fifth and sixth order are recommended as moving to higher order schemes produces diminishing returns for significant coding and computational cost. When properties such as positvity and non-oscillatory behavior are required, such as for moisture variables, it was found using FCT and WENO schemes that firth or sixth order again produced the best results.

5. Acknowledgment.

This research was supported under ONR Grant N00014-00-1-0288, for which the authors are appreciative.

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