P2.6 ELEMENTS OF A MODEL FOR STRESS TRANSPORT IN THE URBAN ROUGHNESS SUBLAYER

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1. INTRODUCTION

Owing to the fact that the bulk of human activity occurs at or near street level, particularly significant issues relate to the mechanisms for momentum transport that result from the direct influence of the large scale roughness arrays (e.g., buildings, trees, etc.) that comprise the urban landscape. This region of the ASL is composed of the urban canopy below the nominal building height, H, and the urban roughness sublayer (URSL) that extends to a mean height of about $2.5H$. Flow characteristics in the URSL are particularly significant since they largely determine the rate at which, for example, pollutants are transported from the canopy layer. More generally, the properties of the URSL set the lower boundary condition for mesoscale numerical simulations. The complexity of the urban surface form and its poorly understood relation to URSL structure undoubtedly underlies the current lack of generally applicable models that faithfully capture the physical mechanisms of URSL transport.

In this paper, hypotheses and predictions relating to a physically tenable modeling strategy for URSL transport are presented and tested against experimental measurements. Specifically, analysis of the velocity/vorticity contributions to the generation of stress gradients indicates that the spectral overlap between specific velocity and vorticity components determines the Reynolds stress and turbulent kinetic energy gradients across the URSL. Spatially and temporally resolved measurements of the relevant velocity vorticity products over a range of surface conditions confirm the anticipated behavior in that the peak in the normalized spectra of these velocity vorticity products shift to lower frequencies with increasing surface roughness. The correlation between this shift in the spectral peaks and the underlying roughness morphometry provides a basis for parameterizing the stress gradients across the URSL. Implementation in the context of the vorticity transport model of Taylor (1932) is then briefly discussed.

2. PHYSICAL BASIS FOR THE MODEL

The classical closure problem of turbulence is most often stated as one for which functions for the Reynolds stresses, $\overline{u_j u_i}$, are sought. It is, however, not the Reynolds stresses but their spatial gradients, $\partial\overline{u_j u_i}/\partial x_j$ that actually appear in the time averaged momentum equations. The present modeling strategy utilizes this fact by directly exploring the nature of the Reynolds stress gradients via the tensor identity,

$$\frac{\partial\overline{u_j u_i}}{\partial x_j} = -\epsilon_{ijk}\overline{u_j\omega_k} + \frac{1}{2}\frac{\partial\overline{u_j u_j}}{\partial x_i}, \tag{1}$$

where ω_i are the vorticity components. For 2-D flow in the x, z plane, setting $i = 1$ yields,

$$\frac{\partial\overline{uw}}{\partial z} = \overline{w\omega_y} - \overline{v\omega_z} + \frac{1}{2}\frac{\partial\overline{u_j u_j}}{\partial x}, \tag{2}$$

and setting $i = 3$ gives,

$$\frac{\partial\overline{uw}}{\partial x} = \overline{u\omega_y} - \overline{v\omega_x} + \frac{1}{2}\frac{\partial\overline{u_j u_j}}{\partial z}. \tag{3}$$

Under the condition that $\partial/\partial z >> \partial/\partial x$, Eq. 2 indicates that the vertical Reynolds shear stress gradient is determined by the difference between $\overline{w\omega_y}$ and $\overline{v\omega_z}$, while Eq. 3 indicates that the vertical gradient of turbulent kinetic energy results from the difference between $\overline{u\omega_y}$ and $\overline{v\omega_x}$.

Given the above relations, the present modeling strategy hypothesizes that the URSL is usefully defined as the region between the nominal building height and the undisturbed ASL in which the vorticity field is modified by the separated wake vorticity shed from building arrays and/or ejected from the canopy layer, and that, in general, at high Reynolds number there is a wide disparity between the spectral peaks associated with the velocity and vorticity fluctuations. The substantial scale separation between the $\vec{V}$ and $\vec{\omega}$ fields at high Reynolds number necessarily confines their spectral overlap (i.e., the motions contributing to the correlation between the two) to a relatively narrow frequency range. Thus, within the undisturbed ASL (i.e., above the URSL) there is a frequency range over which the velocity and vorticity fields correlate. Within the URSL, however, this frequency range of this spectral overlap is modified owing predominantly to an increase in scale (decrease in frequency content) of the vorticity fluctuations associated with the surface roughness.

3. EXPERIMENTAL VALIDATION

Spatially and temporally resolved measurements of velocity and vorticity acquired using multi-element hot-wire sensors are used to test the present modeling hypotheses. Figure 1, which is derived from data acquired at the Surface Layer Turbulence and Environmental Science Test (SLTEST) facility in Utah's west desert shows

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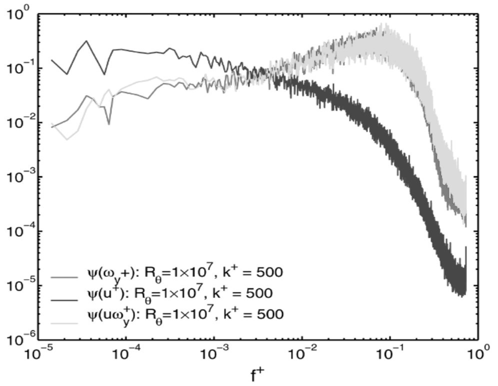

Figure 1. Inner normalized spectral functions of u, ω_y and $u\omega_y$.

that the peak in the spectral function (pre-multiplied spectra) for u occurs at orders of magnitude lower frequency than that for ω_y. The results in figure 1 also show that the inner normalized spectral function for $u\omega_y$ closely tracks that for ω_y alone. This result (which consistently holds independent of surface condition and for all velocity vorticity products examined) indicates that the spectral overlap between u and ω_y is dictated by the spectral content of ω_y. The smooth and small-scale roughness data are derived from measurements at the SLTEST site, while the MUST data (Mock Urban Setting Test, see Biltoft 2002) were taken in the RSL over and array of large scale shipping containers. Consistent with the creation of lower frequency vortical motions, the peaks of the curves in figure 2 consistently shift to lower frequency with an increase in the size of the surface roughness. Similarly, and consistent with the scale of the vorticity field dictating the frequency range of spectral overlap, the data of figure 3 indicate a nearly identical trend in the inner normalized $w\omega_y$ spectral functions. Together the data of figures 1, 2 and 3 provide direct support for all of the present modeling hypotheses.

4. MODEL DEVELOPMENT

The present physical assumptions provide a basis for model development. These results show that the spectral content of the motions underlying stress gradient generation are predictably modified owing to the surface roughness condition, and thus have potential for directly connecting URSL transport to surface morphometry. Specifically, scaling arguments proposed by Taylor (1932) allow conversion of Eq. 2 to the following form,

$$\frac{\partial \overline{uw}}{\partial z} \approx u_c l_c \frac{\partial \Omega_y}{\partial z} - u_c \Omega_y \frac{\partial l_c}{\partial z}, \tag{4}$$

where u_c and l_c are characteristic velocity and length scales and Ω_y is the mean vorticity. The first term on the right of Eq. 4 reflects the physics of gradient transport, while the second term (believed to be dominant in the URSL) represents change of scale effects. The present results suggest that a characteristic length scale distribution based upon the observed behavior of the spectral overlap of the velocity and vorticity fields holds promise for improved URSL parameterizations that relate surface morphometry to URSL transport.

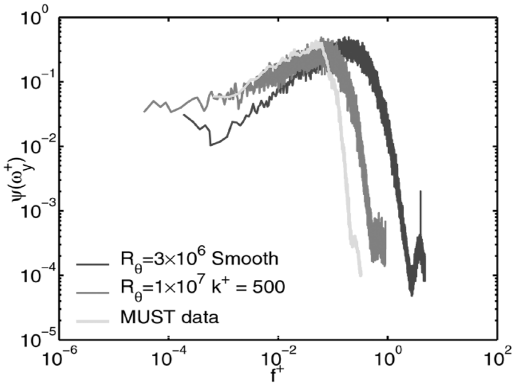

Figure 2. Inner normalized spectral functions of ω_y for varying surface roughness conditions.

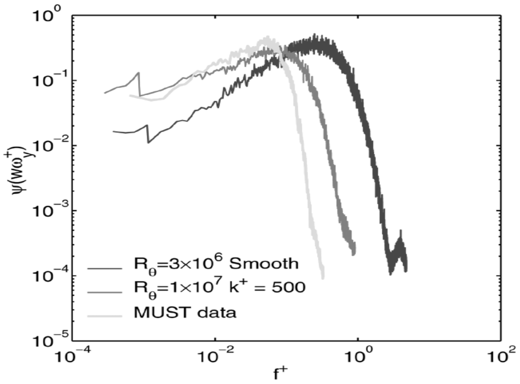

Figure 3. Inner normalized spectral functions of $w\omega_y$ for varying surface roughness conditions.

5. ACKNOWLEDGEMENT

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6. REFERENCES

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