5.7 A QUASI-ONE-DIMENSIONAL LAGRANGIAN STOCHASTIC MODEL OF RELATIVE DISPERSION IN TURBULENT FLOWS

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1. INTRODUCTION

The statistical approach for the description of concentration fluctuations is based on the statistical modeling of the simultaneous trajectories of pairs of particles (see, e.g., the recent review by Sawford 2001).

We present a stochastic model for the separation between particles of a passive contaminant effectively released into a homogeneous, isotropic and stationary turbulent flow at high Reynolds numbers. In the atmosphere, a model solely for the magnitude of the separation distance effectively averages over all orientations of the separation vector and the properties for the distance are essentially equivalent to those in isotropic turbulence. Therefore we model only the scalar separation, that is the *magnitude* of separation, instead of the three components of the separation vector, while the orientation of the pairs will be accounted for by implicit averaging. This averaged statistic is a robust indicator of typical separations within a cloud and can be expected to be a measure of local dilution or levels of internal fluctuations in the cloud.

The first model for scalar separation was proposed by Durbin (1982). More recently, Kurbanmuradov and Sabelfeld (1995) gave an example of a scalar separation model which satisfies the well-mixed condition. To formulate our model we use a direct new formulation for the conditional acceleration which satisfies the well-mixed condition as well as consistency of the model with mean, variance, skewness and kurtosis of the Eulerian distribution of velocity differences. The simple model that results from this approach has the advantage of very efficient computational times, which permits opportunities for new applications, particularly embedded within complex atmospheric flow models.

This simple relative dispersion model describes properties of the internal mixing within a cloud such as the dissipation of concentration fluctuations due to the continuous entrainment of uncontaminated fluid into the volume of the cloud, which is most effectively carried out at scales of motion of the same order as the cloud size. Larger scale incursions of clean air are mostly meandering effects, and can be accounted for with meandering plume models (Gifford 1959; Luhar et al. 2000; Yee and Wilson 2000).

2. MODEL DEFINITION

The mean square concentration near plume and puff centres in terms of scalar separations in a frame of reference moving with the cloud centre of mass can be written as:

$$\overline{c^2}(t) = \int_0^\infty p(r_o;t_o\,|\,0;t)\,q(r_o)\,dr_o \qquad (1)$$

where *r* is the scalar separation between two particles, the subscript "o" indicates conditions at the source, $p(r_o;t_o|0;t)$ is the probability distribution function (pdf) of the separation at the source of particles that are coincident at time *t*, and $q(r_o)$ is the covariance of the concentration at the source. This expression was obtained after averaging over all angles of the initial separation vector. The scalar separation *r(t)* between two particles is modeled by the following system of stochastic differential equations for *r* itself, and for its rate of change $u = dr/dt$:

$$du(t) = a(u,r)dt+(2C_o e)^{1/2}dW \qquad (2)$$

$$dr(t) = u(t)dt \qquad (3)$$

where *dW* are the random increments of a Wiener process with zero mean and variance *dt*, *e* is the mean energy dissipation rate, C_o is a universal constant which was set equal to 6 and the drift term *a(u,r)* is determined from a Fokker-Planck equation. The model is derived first by assuming the following functional-form closure for *a(u,r)*:

$$a(u,r) = a(r) + b(r)u + g(r)u^2 \qquad (4)$$

and then by imposing the well-mixed criterion to determine the coefficients *a*, *b* and *g* This form, a quadratic function of *u*, is of the same type as the one used by Franzese et al. (1999) in the context of one-particle models for the atmospheric convective boundary layer.

3. MODEL RESULTS

The simulations were made releasing 10^5 particle pairs, and using a variable time step $dt = 10^{-3} \times s_u^2(r)/(2C_o e)$. The function $q(r_o) = c_o^2 exp(-r_o^2/2s_o^2)$ was assumed as source term, where s_o is a characteristic length scale of the source and c_o is the concentration of the cloud centre of mass at the source. By definition, $c_o \sim Q/(U s_o^2)$ for a plume, where *Q* is the amount of

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contaminant released per unit time and U is the mean wind velocity at the source, while $c_o \sim M/s_o^3$ for a puff, where M is the amount of contaminant released. Since the scale of the fluctuations at the source is known to have an important effect on the second-order statistics of concentration, the simulations are performed for plume and puff releases from five different source sizes, and focus on the parameterization of this size dependence. The source size s_o ranges over five orders of magnitude, namely from $s_o = 10^{-4}L$ to L, where L is the turbulence integral length scale.

The results include the simulations of the pdf of relative separation (Fig. 1), mean square relative separation $\overline{r^2}$, mean square concentration field $\overline{c^2}$, mean concentration $\bar{c}$, standard deviation of concentration s_c (Fig. 2), and intensity of concentration fluctuations $s_c/\bar{c}$.

The evolution of the above concentration statistics with time depends on the source size and on source characteristics such as the emission rate and the mean wind for a plume and the amount of material released for a puff, as well as on the turbulence characteristics. We propose a scaling law that eliminates this dependence on the source size. By virtue of such a scaling we found a universal behavior to account for source size effects for the concentration statistics and performed some simple analysis which led to the derivation of general formulae for the decay of $\overline{c^2}$, $\bar{c}$, and s_c with time, and to the estimation of the value and location of the maximum s_c. For example, an interesting consequence of this representation is the result that the maximum s_c, normalized over the initial concentration, is a constant, i.e. it does not depend on the source size. The results show a very good agreement with the predictions of the similarity theory. For instance, the model reproduces the prescribed -9/2 exponent in the time decay law for $\overline{c^2}$.

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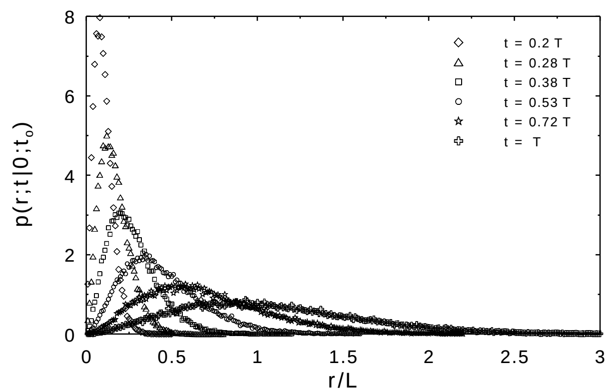

Fig. 1 Probability density function *p(r;t|0;t_o)* of the particle scalar separation *r(t)*, for particles with zero initial separation, as a function of *r/L* for several times *t* after release.

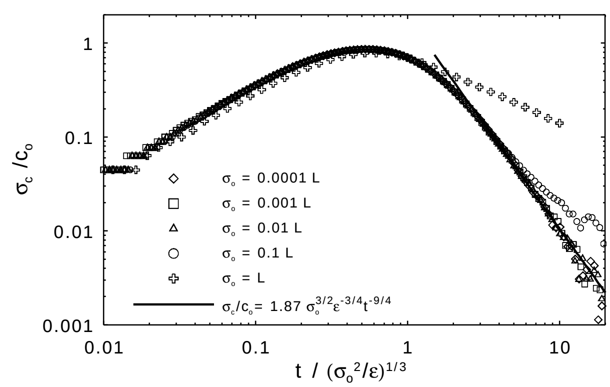

Fig. 2 Nondimensional standard deviation of concentration s_c/c_o as a function of $t/(s_o^2 e)^{1/3}$ for simulated plumes, predicted by the model for five values of s_o. The solid line represents the formula $s_c/c_o = 1.87\, s_o^{3/2} e^{-3/4} t^{-9/4}$.