9C.4 The Vertical Alignment of an Incipient Tropical Cyclone

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Observations indicate that strong vertical shear of environmental wind inhibits the intensification of a tropical cyclone (e.g., Gray 1968). This is because vertical shear tilts a vortex, and thereby diminishes its central pressure anomaly (e.g., Frank and Ritchie 2001). However, a vortex can oppose the destructive influence of vertical shear by its intrinsic drive toward an upright position. The physics of such vertical alignment continues to be an active area of research, and is the topic of this presentation.

For simplicity, we consider the alignment of a barotropic vortex, in the absence of friction and diabatic processes. The vortex is characterized by its mean tangential wind $\bar{v}(r)$, and the related variables

$$\bar{\Omega}(r) = \bar{v}/r, \qquad \bar{\zeta}(r) = \frac{1}{r}\frac{\partial(r\bar{v})}{\partial r}, \qquad \bar{\xi}(r) = f + 2\bar{\Omega}, \qquad \bar{\eta}(r) = f + \bar{\zeta}. \tag{1}$$

Here, f is the local Coriolis parameter, $\bar{\Omega}$ is the mean rotation frequency, $\bar{\zeta}$ is the mean relative vorticity, $\bar{\eta}$ is the mean absolute vorticity, and $\bar{\xi}$ is the average value of $\bar{\eta}$ within the radius r. The field variables in (1) are assumed to vary monotonically with r.

Figure 1 illustrates the process of vertical alignment. In this particular simulation, the vortex is quasigeostrophic and the environmental flow has no vertical shear. During alignment, the tilt precesses uniformly about the z-axis. Uniform precession occurs because a single azimuthally propagating vortex-Rossby-wave dominates the perturbation. A theory for the damping rate of this wave is tantamount to a theory for the observed alignment rate.

We have shown that a vortex-Rossby-wave is damped by a resonance with the fluid rotation frequency at a critical radius r_*. The critical radius is defined precisely by

$$\bar{\Omega}(r_*) = \omega/n, \tag{2}$$

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in which n and ω/n are the azimuthal wavenumber and phase-velocity of the wave, respectively. Schecter et al. (2002; SMR) presents the quasigeostrophic theory ($\bar{\zeta}/f << 1$) of resonant damping. These results have since been generalized, using the asymmetric balance (AB) model of Shapiro and Montgomery (1993). Formally, AB theory requires that

$$\mathcal{D}_I^2(r) = \frac{(\omega - n\bar{\Omega})^2}{\bar{\eta}\bar{\xi}} << 1. \tag{3}$$

This condition can be satisfied even if the Rossby number of the vortex is greater than unity.

The resonance at r_* causes the wave-amplitude a to exhibit an early stage of exponential decay; that is, $a(t) \propto e^{\gamma t}$, in which γ is negative. If the decay rate is small compared to the wave-frequency, then it is given by

$$\gamma = -\frac{n\pi}{\int_0^\infty dr r^2 |Q|^2/(d\bar{\eta}^{-1}/dr)} \left.\frac{d\bar{\eta}^{-1}/dr\,|\Phi|^2}{|d\bar{\Omega}/dr|}\right|_{r_*}. \tag{4}$$

Here, $Q(r)$ and $\Phi(r)$ are the pseudo potential vorticity (Ren 1999) and geopotential eigenfunctions of the vortex-Rossby-wave: Q is dimensionless, and Φ has units of energy over mass. The term "eigenfunction" is used loosely, since the wave is actually a quasi-mode of the vortex (SMR). Perhaps the most important result from (4) is that the decay rate is proportional to $d\bar{\eta}^{-1}/dr = -\bar{\eta}^{-2}d\bar{\zeta}/dr$, evaluated at

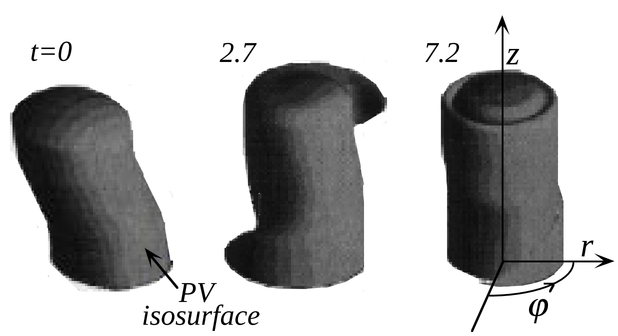

Figure 1: The conservative vertical alignment of a (quasigeostrophic) vortex. Time t is in units of $2\pi/\bar{\Omega}(0)$.

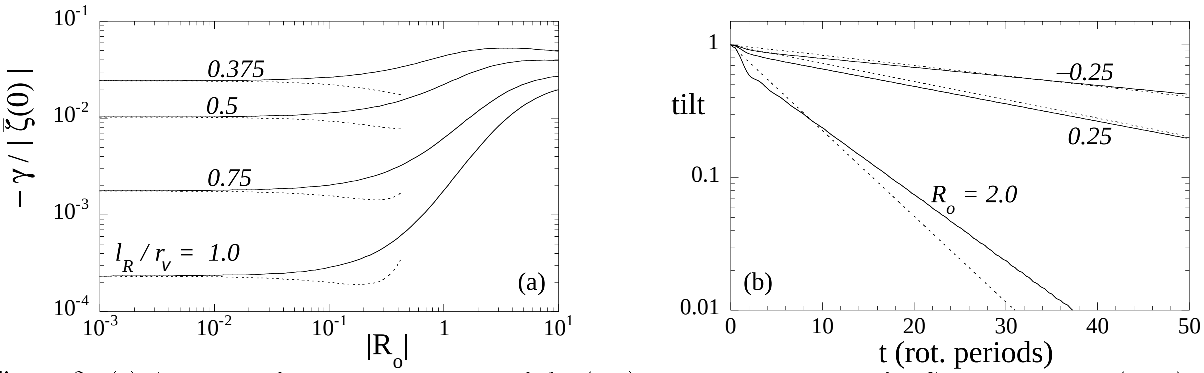

Figure 2: (a) AB theory for scaled decay rate of the $(1,1)$ vortex-Rossby-wave of a Gaussian cyclone (solid) and anticyclone (dashed). (b) Tilt versus time for cases in which $l_R/r_v = 0.75$. The solid and dashed curves are from simulations and theory, respectively. The tilt is normalized to its initial value, and time is in units of $2\pi/\bar{\Omega}(0)$.

r_*. Therefore, the damping rate increases with the relative vorticity gradient at the critical radius. If the decay rate is comparable to the wave-frequency, then Eq. (4) is inaccurate. In this case, "Landau's method" may be used to obtain γ (SMR).

As an example, we here consider a Gaussian vortex, for which $\bar{\zeta} = Z_o \exp[-9r^2/2r_v^2]$. The scaled decay rate, $-\gamma/|\bar{\zeta}(0)|$, of a vortex-Rossby-wave on a Gaussian vortex depends on two independent parameters: the Rossby number R_o and the internal Rossby deformation radius l_R, defined by

$$R_o = \bar{\zeta}(0)/f, \qquad l_R = NH/\pi m|f|. \qquad (5)$$

Here, N is the buoyancy frequency of the stably stratified atmosphere, H is the vertical thickness of the vortex, and m is the vertical wave-number.

Figure 2a plots the AB scaled decay rate of the $(m,n) = (1,1)$ vortex-Rossby-wave versus R_o, for several values of l_R. The solid curves are for cyclones ($R_o > 0$), whereas the dashed curves are for anticyclones ($R_o < 0$). The results do not extend to Rossby numbers less than minus one, where the vortex is symmetrically unstable, and balance models such as AB theory are invalid.

For the cyclones in Fig. 2a, the scaled decay rate increases as $|R_o|$ increases, and as l_R decreases. For the anticyclones, the scaled decay rate increases as $|R_o|$ decreases (below ~ 0.1), and as l_R decreases. These qualitative results are not universal; rather, opposite relations can be found on non-Gaussian vortices in the same region of parameter space. One of the more interesting results of Fig. 2a is that the scaled decay rate can increase by orders of magnitude as R_o increases from zero to order unity, say, by increasing the magnitude of $\bar{\zeta}(0)$. In quasi-geostrophic theory, the scaled decay rate does not vary with $\bar{\zeta}(0)$, and is equal to the $R_o \to 0$ limit of AB theory.

The AB theory of resonantly damped vortex-Rossby-waves has successfully explained the alignment observed in numerical experiments, performed with a linearized hydrostatic primitive model. In each simulation, the vortex is tilted as in Fig. 1. The initial tilt predominantly excites the $(1,1)$ discrete vortex-Rossby-wave. The time dependent tilt (wave-amplitude) is plotted in Fig. 2b, for $l_R = 0.75r_v$, and three different Rossby numbers. In all cases, there is good agreement between the simulations (solid), and the AB theory of resonant damping (dashed). Although the error of AB theory increases with R_o, it remains small for the case in which $R_o = 2$. At this Rossby number, the maximum of $\mathcal{D}_I^2$ is only 0.07.

Future work will examine the influence of moist convection and friction on the damping rate of vortex-Rossby-waves. Future work will also cover larger Rossby numbers (10-100), representative of intense hurricanes.

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