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An Improved VTD Algorithm to Resolve Circulations of Wavenumber Two Tropical Cyclones

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1. INTRODUCTION

The primary circulation of tropical cyclones (TC) can be retrieved from single Doppler radar observations using the velocity track display (VTD) technique (e.g., Lee et al. 1994; Lee et al. 1999; Lee et al. 2000). In these studies, the VTD formulation was closed by assuming that the magnitude of the asymmetric radial winds is much smaller than their transverse (rotational) counterparts so the asymmetric radial winds can be ignored. The potential biases in both the phase and amplitude of the retrieved TC asymmetric circulation due to this assumption have been qualitatively discussed in Lee et al. (1999). The purpose of this study is to present an improved formulation to resolve the asymmetric radial winds.

2. IMPROVED FORMULATION

The improved formulation is based on the two-dimensional (2-D) flow model where small amplitude disturbances are superimposed onto a Rankine combined vortex (Lamb 1932, p231). The streamfunction of a 2-D vortex is:

$$\psi = \begin{cases} -\dfrac{1}{4}\omega(r_o{}^2 - r^2) - \dfrac{\omega r_o \alpha}{2n}\left(\dfrac{r}{r_o}\right)^n \cos[n(\theta-\sigma t)], r < r_o \\ -\dfrac{1}{2}\omega r_o^2 \log\dfrac{r_o}{r} - \dfrac{\omega r_o \alpha}{2n}\left(\dfrac{r_o}{r}\right)^n \cos[n(\theta-\sigma t)], \quad r > r_o \end{cases} \quad (1)$$

where r, r_o, θ, ω, t, n, σ, and α represent radius, radius of maximum wind (RMW) of the Rankine vortex, VTD azimuth angle, vorticity, time, wavenumber, angular velocity and a constant to be determined, respectively. It follows from (1) that the transverse and radial velocities VT and VR are:

$$\begin{aligned} VT &= \frac{1}{2}\omega r - \frac{\omega\alpha}{2}\left(\frac{r}{r_o}\right)^{n-1}\cos[n(\theta-\sigma t)] \\ VR &= -\frac{\omega\alpha}{2}\left(\frac{r}{r_o}\right)^{n-1}\sin[n(\theta-\sigma t)] \end{aligned} \quad r < r_o \quad (2)$$

$$\begin{aligned} VT &= \frac{\omega r_o^2}{2r} - \frac{\omega\alpha}{2}\left(\frac{r_o}{r}\right)^{n+1}\cos[n(\theta-\sigma t)] \\ VR &= -\frac{\omega\alpha}{2}\left(\frac{r_o}{r}\right)^{n+1}\sin[n(\theta-\sigma t)] \end{aligned} \quad r \ge r_o \quad (3)$$

The plane wave terms in these equations represent the disturbance that has an angular velocity of $\sigma = \omega\,(n\text{-}1)/2n$ around the Rankine-combined vortex (first terms shown for VT). Note that VR is continuous but VT is discontinuous across the RMW. For a given value of n, the amplitudes of the asymmetric transverse and radial components are equal but they have a specific phase lag. These relations between the amplitudes and phase provide the additional constraints needed to close the VTD formulation. The improved formulation is general to all wavenumbers, and it is also applicable to non-Rankine type profiles as well. This follows from the fact that VTD is able to resolve the magnitude of each wavenumber regardless of the specific flow pattern of the mean vortex; it is just the *relations* between the components of the asymmetries that are needed from (2) and (3).

3. RESULTS FOR WAVENUMBER TWO TC

For $n = 2$, the resulting circulation is elliptically shaped and consists of two pairs of counter-rotating vortices superimposed on the Rankine combined vortex (Figs. 1a and 1c). In this case, $\alpha = a - r_o$, where a is the semi-major axis of the ellipse which rotates counter-clockwise at the rate σ. The phase angle $\phi = \sigma t$ of a is measured from $\theta = 0°$; $\phi = 0°$ in Fig. 1. Thus, the maximum in VT leads (lags) that of VR by 45° inside (outside) $r = r_o$ as the elliptical circulation rotates.

Using equations (24) and (25) in Lee et al. (1999), and the aforementioned phase and magnitude relations, we obtain: $VTC2 = -VRS2 = -B3$ and $VTS2 = VRC2 = A3$ for $r \ge r_o$. It also follows that $\phi = \tan^{-1}(-A3/B3)/2$. In the region $r < r_o$, the same relations yield $A3 = B3 = 0$ so that the wavenumber two components of VT and VR can not be resolved from these GBVTD equations. However, ϕ is unchanged, and VT and VR can be obtained from

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(2) using the continuity of *VR* and the GBVTD estimates of *VT* and *VR* at $r = r_o$.

Assuming a Doppler radar located at (0,0) observes this idealized TC, the VTD-derived circulation and the orientation of the ellipse, illustrated in Figs. 1b and 1d, show very good agreement with the idealized TC with weaker amplitude in the retrieved asymmetric disturbances. Limitations of this approach and its application to Typhoon Herb (1996) will be presented in the conference.

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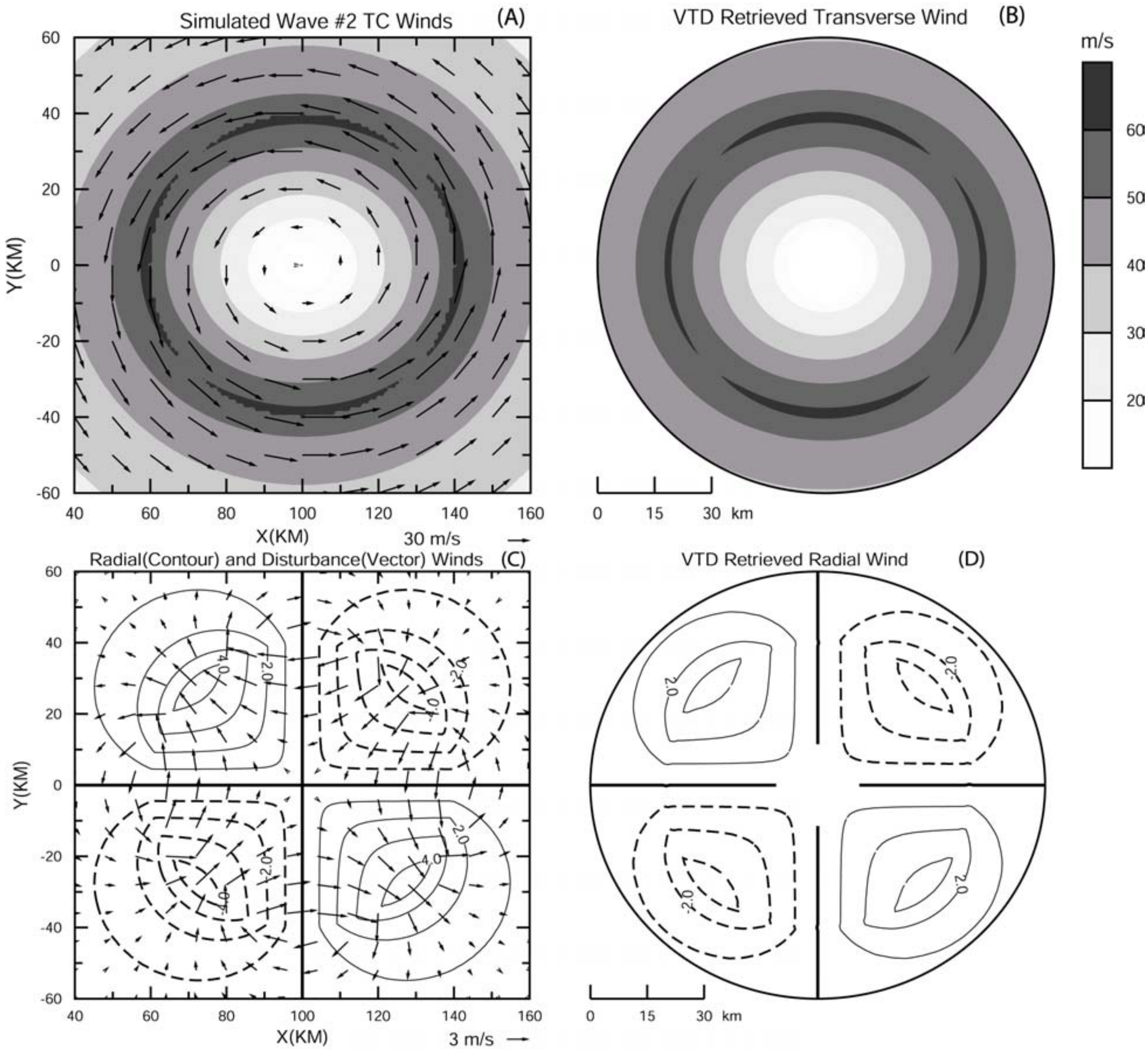

Figure 1: (A) The total winds (vector) and their magnitude (gray shades) of a simulated wavenumber two TC, (B) the magnitude of the VTD retrieved transverse winds (gray shades), (C) the simulated wavenumber two disturbance (winds, in vector) and the magnitude of the radial winds (contour), (D) the magnitude of the VTD-retrieved radial winds for the wavenumber two disturbance. The wavenumber two TC is constructed by superimposing wavenumber two disturbances on a Rankine combined vortex. The Doppler velocities are computed from the simulated wavenumber two wind fields for hypothesized radar located at (0, 0). The VTD algorithm is applied to the Doppler velocities to retrieve the transverse and radial winds.