13.5 Modelling the Urban Flow Field and Pollution Dispersion using Digital Elevation Models

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1. INTRODUCTION

In recent years there has been an increasing availability of high-resolution 3-D urban databases, which can be used for the analysis and measure of city geometry (urban morphometry). In the context of urban morphometry, Ratti et al. (2001) described a method to extract various parameters from urban Digital Elevation Models (DEMs) by using image-processing techniques. As DEMs are becoming increasingly common some questions arise on how best to use them in the modelling of urban wind field and dispersion.

In this paper we discuss some applications of DEMs to: (1) the calculation of the aerodynamic roughness length z_0 for the city as a whole and its spatial variation $z_0(x, y)$ on the neighbourhood scale; (2) the estimation of the flow over the city from $z_0(x, y)$ and using both linearised perturbation modelling and computational fluid dynamics (CFD); (3) the modelling of mean wind profile within and above the urban canopy.

2. AERODYNAMIC ROUGHNESS LENGTH AND ITS SPATIAL VARIABILITY FROM DEMS

DEMs contains a full 3-D description of the urban surface on a 2-dimensional support (the image). Urban DEMs can be analysed with image processing techniques using simple packages like the Matlab Image Processing Toolbox.

The aerodynamic roughness length can be calculated from the built to total area ratio at ground level (λ_p), the frontal area density (λ_f) and the average building height by using various formulas as reported in Grimmond and Oke (1999). We use a slight modification of Macdonald et al. (1998) where we replaced the average building height (H) with the average building height weighted with the frontal area of the building (z_H). Table 1 reports, as an example, some parameters calculated from a DEMs of Salt Lake City (see Fig. 1). This built in-house database covers an area of 1600x1600 m^2.

Table 1: Parameters from Salt Lake City for the entire area.

$\lambda_P = 0.25$
$\lambda_f = 0.11$
$z_H = 26.75$ m
$z_0 = 1.3$ m

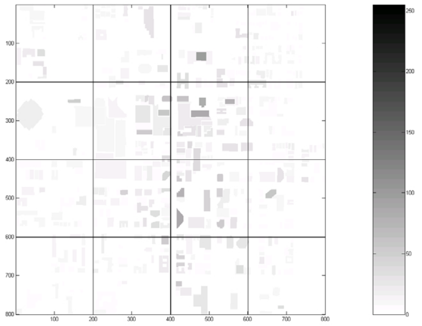

Figure 1: DEMs of Salt Lake City; each square has an area of 400x400 m^2.

The total area covered by the database has been divided into 16 neighbourhoods as shown in Fig. 1, consistent with a previous analysis of DEMs of European cities (Ratti et al. 2001). The calculated parameters are reported in Table 2; the numbers in each square corresponds to the calculated parameters for each neighbourhood in Fig. 1.

Table 2: Parameters from DEMs of Salt Lake City for each neighbourhood.

$\lambda_P = 0.150$ $\lambda_f = 0.077$ $z_H = 18.60$ m $z_0 = 1.0612$ m	$\lambda_P = 0.202$ $\lambda_f = 0.098$ $z_H = 17.58$ m $z_0 = 0.992$ m	$\lambda_P = 0.197$ $\lambda_f = 0.149$ $z_H = 31.60$ m $z_0 = 2.838$ m	$\lambda_P = 0.208$ $\lambda_f = 0.114$ $z_H = 11.23$ m $z_0 = 0.726$ m
$\lambda_P = 0.194$ $\lambda_f = 0.054$ $z_H = 23.51$ m $z_0 = 0.624$ m	$\lambda_P = 0.503$ $\lambda_f = 0.205$ $z_H = 27.68$ m $z_0 = 0.497$ m	$\lambda_P = 0.397$ $\lambda_f = 0.349$ $z_H = 40.52$ m $z_0 = 2.573$ m	$\lambda_P = 0.199$ $\lambda_f = 0.101$ $z_H = 15.22$ m $z_0 = 0.902$ m
$\lambda_P = 0.284$ $\lambda_f = 0.101$ $z_H = 10.70$ m $z_0 = 0.370$ m	$\lambda_P = 0.409$ $\lambda_f = 0.207$ $z_H = 23.12$ m $z_0 = 0.832$ m	$\lambda_P = 0.321$ $\lambda_f = 0.264$ $z_H = 42.19$ m $z_0 = 3.257$ m	$\lambda_P = 0.211$ $\lambda_f = 0.107$ $z_H = 21.35$ m $z_0 = 1.267$ m
$\lambda_P = 0.184$ $\lambda_f = 0.067$ $z_H = 9.86$ m $z_0 = 0.379$ m	$\lambda_P = 0.198$ $\lambda_f = 0.097$ $z_H = 18.95$ m $z_0 = 1.077$m	$\lambda_P = 0.292$ $\lambda_f = 0.160$ $z_H = 25.20$ m $z_0 = 1.425$ m	$\lambda_P = 0.128$ $\lambda_f = 0.060$ $z_H = 13.81$ m $z_0 = 0.660$ m

3. URBAN FLOW FIELD

Real cities are characterised by inhomogeneous distribution of buildings. This inhomogeneity will result in different local values of the aerodynamic roughness length within the city. As shown above the spatial

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aerodynamic roughness variation $z_0(x,y)$ is easily available from DEMs. This can be used as input in airflow linearised models (Hunt et al. 1988) and CFD models to calculate the flow field over the city. Here, we use the airflow perturbation model FLOWSTAR (Carruthers et al. 1988) which uses linearised analytical solutions of momentum and continuity equations to calculate the wind field. FLOWSTAR also allows topographic perturbation so no increase in computing is required to incorporate topographic features as well. CFD calculations are being done but not presented here. Figure 2 shows the vector plot of the simulated wind field at 10 m height using FLOWSTAR and the same spatial distribution of aerodynamic roughness as reported in Table 2.

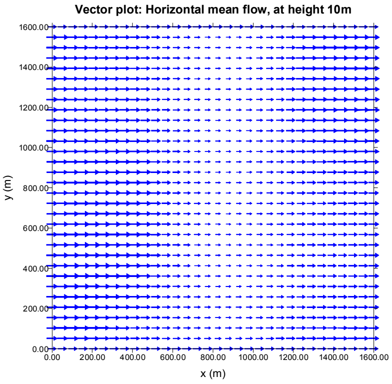

Figure 2: Wind field similation using FLOWSTAR and the aerodynamic roughness length values in Table 2.

4. A SIMPLE MODEL FOR THE WIND PROFILE

As pointed out in Macdonald (2000), mean wind profiles in urban areas can be obtained from the modification of simple model for the mean wind in a vegetative canopy (Cionco, 1972).

In this model the wind profile is obtained by solving an equation for the shear stress variation due to the drag forces on elements of the buildings. It reads as:

$$\rho A_d d\tau = \frac{1}{2}\rho C_D U^2(z) dA_f dz \qquad (1)$$

where A_d is the underlying surface of a building and $dA_f = L_f dz$ with L_f the overall width of the buildings in the direction perpendicular to the flow. The other symbols in Eq. 1 have the usual meaning.

Equation (1) can be re-written using a mixing length model for the turbulent transport in the canopy as:

$$\frac{d}{dz}(l\frac{dU}{dz})^2 = \frac{1}{2}C_D U^2(z)\frac{\lambda_f(z)}{H} \qquad (2)$$

where we have used $\lambda_f(z) = \frac{HL_f}{A_d}$.

Equation 1 is an ordinary non-linear differential equation with non-constant coefficients. Cionco (1972) proposed an exponential solution to his equation obtained by using as boundary conditions $U(0)=0$ and $U(H)=U_H$. However, U_H may be unwise boundary condition as this is in a region with strong gradients. The use of $\lambda_f(z)$ provided by DEMs is expected to avoid using U_H as boundary condition. A numerical solution of Eq. (2) has been obtained (see Fig. 3) using a linear variation of $\lambda_f(z)$ and with $l(z)$ as in Macdonald (2000).

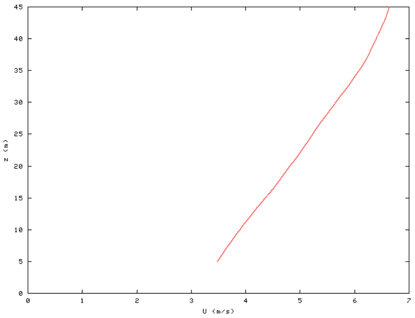

Figure 3: Numerical solution of Eq.(1).

5. CONCLUSIONS AND FUTURE WORK

This paper reports on some examples of application of DEMs. A new range of parameters can be easily available; there is still need for the identification of those parameters that can be useful to improve the understanding of the fluid mechanics of the urban area.

6. AKNOWLEDGEMENTS

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