6.1 NEW RECIPROCITY PRINCIPLES IN 3-D SOLAR RADIATIVE TRANSFER THEORY

Larry Di Girolamo*
Department of Atmospheric Sciences, University of Illinois at Urbana-Champaign, Urbana, IL

1. INTRODUCTION

Reciprocity principles in radiative transfer theory have been widely used in deriving analytical and numerical solutions of radiative transfer problems (*e.g.*, Van de Hulst 2000), in testing numerical models of radiative transfer (*e.g.*, Herman *et al*. 1980), and in remote sensing applications (*e.g.,* Stephens *et al*. 2000). In the most widely studied case, that of a horizontally homogeneous medium completely illuminated at the top boundary by a constant, unidirectional irradiance, the reciprocity principle states that the bidirectional reflectance distribution function (BRDF), f_r, at the top boundary of the medium is invariant under a change in the incident and outward directions; *i.e*, at the top boundary of the medium,

$$f_r(-\Omega_1;\Omega_2) = f_r(-\Omega_2;\Omega_1) \qquad (1)$$

where Ω represents the directional unit vector with the outward direction negative. For a unidirectional irradiance, F,

$$f_r(-\Omega_1;\Omega_2) = \frac{I(-\Omega_1;\Omega_2)}{\Omega_2 \; \mathbf{n} F(\Omega_2)} \qquad (2)$$

where I is the radiance and $\mathbf{n}$ is a unit vector that is outward normal at the top boundary (*i.e.*, $\mathbf{n} \; \Omega < 0$ represents a direction incident at the top boundary of the medium). Eq. (1) is the more popular form of the reciprocity principle used in optical and geophysical sciences and has recently been termed the principle of directional reciprocity (Davies 1994).

In general, 3-D radiative transfer solutions obey reciprocity principles that have both directional and spatial attributes (*e.g.*, Di Girolamo *et al*. 1998), whereas 1-D radiative transfer solutions obey a reciprocity principle with only directional attributes (*i.e.*, Eq. (1)). Over the past few years, the author has derived several new reciprocity principles that are appropriate for unidirectional illumination incident on a scattering and absorbing heterogeneous medium. Section 2 summarizes these reciprocity principles. Based on these principles, a new definition for the BRDF is introduced in Section 3.

2. NEW RECIPROCITY PRINCIPLES

For external unidirectional illumination, Di Girolamo (1999) derived the following reciprocity principle:

$$\Omega_2 \; \mathbf{n} \int_B F(\mathbf{r},\Omega_2) I(\mathbf{r},-\Omega_2;A,\Omega_1) d\mathbf{r} = \Omega_1 \; \mathbf{n} \int_A F(\mathbf{r},\Omega_1) I(\mathbf{r},-\Omega_1;B,\Omega_2) d\mathbf{r} \qquad (3)$$

where $I(\mathbf{r},-\Omega_2;A,\Omega_1)$ is the radiance at position $\mathbf{r}$ in direction $-\Omega_2$ caused by illuminating the surface *A* with an irradiance $F(\mathbf{r},\Omega_1)$ from direction Ω_1, $I(\mathbf{r},-\Omega_1;B,\Omega_2)$ is the radiance at position $\mathbf{r}$ in direction $-\Omega_1$ caused by illuminating the surface *B* with an irradiance $F(\mathbf{r},\Omega_2)$ from direction Ω_2, and surface integration is over surfaces *A* and *B*. Eq. (3) is general and applies to any absorbing and scattering media, regardless of heterogeneity. The only assumption used in its derivation is that the scattering phase function has time-reversal symmetry.

In the special case where the incident irradiance is independent of position (*i.e.*, uniform over the illuminated area), Eq. (3) can be rewritten as

$$\frac{\int_B I(\mathbf{r},-\Omega_2;A,\Omega_1) d\mathbf{r}}{\Omega_1 \; \mathbf{n} F(A,\Omega_1)} = \frac{\int_A I(\mathbf{r},-\Omega_1;B,\Omega_2) d\mathbf{r}}{\Omega_2 \; \mathbf{n} F(B,\Omega_2)} \qquad (4)$$

or

$$f_r(B,-\Omega_2;A,\Omega_1) = f_r(A,-\Omega_1;B,\Omega_2) \qquad (5)$$

Thus, the BRDF measured over area *B* in direction $-\Omega_2$ when area *A* is illuminated from direction Ω_1 will have the same magnitude as the BRDF measured over area *A* in direction $-\Omega_1$ when area *B* is illuminated from direction Ω_2.

Eq. (3) or (5) can be used to test whether a particular 3-D radiative transfer model obeys reciprocity. Most models that solve for 3-D solar radiative transfer in the Earth-atmosphere system assume uniform irradiance over the model domain's top boundary, making Eq. (5) applicable. Even so, these models are typically not set up to vary the areas of illumination and measurement; rather, the entire top boundary of the model domain is illuminated. In addition, periodic boundary conditions are often applied to the side-walls of the model domain. For these models, Di Girolamo (2002) derived the following reciprocal relationship:

$$\langle f_r(-\Omega_2;\Omega_1) \rangle = \langle f_r(-\Omega_1;\Omega_2) \rangle \qquad (6)$$

where $<\bullet>$ signifies spatial averaging over the top boundary of the model domain. Thus, for these models, the domain-averaged BRDF is directionally reciprocal, regardless of the domain size and the heterogeneity of the medium.

A special class of 3-D radiative transfer problems is the searchlight problem. In the searchlight problem, the incident radiation is applied to a single point from a single direction at the top of a horizontally homogeneous medium, giving way to a 3-D radiative transfer solution. In a 1-D radiative transfer problem, the

**Corresponding author address:* Larry Di Girolamo, Dept. Atmos. Sci., Univ. Illinois, Urbana, IL 61801-3070; email: larry@atmos.uiuc.edu.

entire horizontally homogeneous medium is uniformly illuminated, giving way to a 1-D radiative transfer solution. Di Girolamo (1999) showed that, for a given horizontally homogeneous medium, the solution to these two problems are linked by the following reciprocal relationship:

$$f_r(r,-\Omega_2;\infty,\Omega_1) = f_r(\infty,-\Omega_2;r,\Omega_1) \qquad (7)$$

The left-hand side of Eq. (7) is simply the BRDF of the 1-D radiative transfer solution. The right-hand side of Eq. (7) is the BRDF integrated over the entire horizontal domain (which stretches to infinity in all horizontal directions). Note, the directions of illumination and measurement are not switched in this equation; a purely spatial reciprocal relationship exists between 1-D radiative transfer and the 3-D searchlight problem.

3. REDEFINING THE BRDF

There have been many reports that Eq. (1) is invalid based on experimental evidence (*e.g.*, Davies 1994). A complete explanation as to why it is possible to obtain experimental failure of Eq. (1) is given in Di Girolamo *et al*. (1998). However, there remains a debate in the community on whether the BRDF should be reciprocal (Liang and Strahler 1999). In trying to understand why this debate continues, the author has realized that the definition of the BRDF needs to be reexamined.

In the 1960's and 70's, numerous papers were published recommending that the scientific community standardize the definition and nomenclature of a wide variety of reflectance quantities. This reached its apex in 1977 when the National Bureau of Standards published its recommended definition and nomenclature for these reflectance quantities (Nicodemus *et al*. 1977). The most basic of these quantities is the BRDF, from which all other standardized reflectance quantities can be derived. The standard notation that was recommended is that given in Eq. (1).

Most textbooks on radiative transfer and remote sensing now include the definition (but with their own notation) of the BRDF as the basic quantity that characterizes the reflecting properties of a surface. However, most of these textbook do not include the basic conditions used by Nicodemus *et al*. (1977) in deriving the BRDF. These conditions are (1) the surface must be horizontally homogeneous, (2) uniform irradiance from a single direction exists over a large enough area such that the radiance leaving the top of the surface does not vary with horizontal position, and (3) the BRDF is defined at a point. These are simply the conditions used in 1-D radiative transfer theory. Eq. (1) is strictly obeyed under these conditions.

In practice, these conditions are difficult to meet: measurements are never made at a point, but are made over a finite field-of–view, and most natural surfaces are not horizontally homogeneous. So it should not be surprising that violations in Eq. (1) are found based on experimental evidence.

Equation (5) suggests a new definition for the BRDF, whereby the areas of illumination and measurement are considered. This new definition has one restrictive condition: uniform unidirection irradiance over the illuminated area. Without this condition, no reflectance quantity can be defined that obeys reciprocity and we are left with the reciprocal relationship of Eq. (3) to contend with. The new definition has several advantages over the original definition: (1) it has less restrictive conditions, making it more amenable to real world situations, (2) it has the areas of illumination and measurements as dependent variables, which reminds us that there are spatial scales to contend with when quantifying reflectance, (3) it allows us to quantify differences in measured BRDF of the same sample using different experimental apparatuses (*e.g.*, Venable 1985), and (4) it maintains a simple reciprocity principle.

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