3.5 SCALING IN ICE CRYSTAL AGGREGATION

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1. INTRODUCTION

To understand and predict the evolution of cloud systems, knowledge of the particle size distribution (PSD) is necessary. The successful representation of ice PSDs by exponential size distributions for particle diameters greater than a few hundred microns has been well documented (e.g. Lo and Passarelli 1982). Particles smaller than a few hundred microns have PSDs which usually differ from that of the large particles and this has led to some workers choosing to use a modified gamma distribution to represent these distributions to capture the behaviour of the smaller particles (Mitchell 1991). We believe that difference in the PSD behaviour at small and large sizes indicates a difference in the physics controlling the evolution of the PSD. For small particles, particle production (nucleation and/or breakup) and diffusional growth dominates the evolution of the PSD while for larger particles the evolution of the PSD is dominated by aggregation (Field, 2000). In this paper we will demonstrate that the large particle ice PSD mode satisfies a scaling relation exhibited universally by aggregating systems and that the exponential distribution is a natural consequence of aggregation.

The descent of ice crystals is controlled by aerodynamic effects that lead to differential sedimentation throughout the cloud layer. This allows ice crystals to come into contact and aggregate to form larger crystals. Ice crystals tend to approach each other on ballistic or straight line trajectories, but every collision does not necessarily result in a sticking event. Therefore, to some extent aggregation of ice crystals may be limited by the number of collisions it requires to form an aggregate rather than the time required for the particles to come into contact with one another. This type of aggregation is known as Reaction Limited Aggregation (RLA). If the sticking efficiency upon contact between ice crystals were unity then it would be the time it takes for the particles to come into contact which would be important in controlling the aggregation rate. This type of aggregation is known as Diffusion Limited Aggregation (DLA)(Meakin, 1992 and references therein provide an overview). Computer simulation and experimental evidence (Lin et al, 1990) suggest that aggregates generate fractal structures that have a fractal dimension, d_f=2.1 for RLA and 1.8 for DLA. This value is equivalent to the exponent in ice crystal mass--dimension relationships and is in good agreement with values of ~2 found for ice aggregates. Researchers (see Meakin, 1992) looking at colloidal systems and computer simulations also discovered that the PSDs studied in those cases evolving through the aggregation process could be scaled successfully using:

$$N(M,t) = M_0(t)^{-\theta} f(M/M_0(t)) \qquad (1)$$

where $N(M,t)$ is the time dependent PSD as a function of particle mass, $M_0(t)$ is a mean mass representative of the PSD (e.g. the number weighted mean mass), θ is a scaling exponent and $f(M/M_0(t))$ is a scaling function that is common to all $N(M,t)$ and independent of time. Furthermore they find that by multiplying eq. 1 by $M\mathrm{d}M$ and integrating over all M that for conservation of mass θ=2. So, given $M_0(t)$ and the scaling function $f(M/M_0(t))$, the PSD $N(M,t)$ can be found.

We have used data acquired from 14 lagrangian spiral descents carried out during the TRMM (Tropical Rainfall Measurement Mission), FIRE-1 (First ISCCP Regional Experiment) and ARM (Atmospheric Radiation Measurement) campaigns. During the spirals the aircraft drifted with the wind and descended at ~1 m s^{-1}. The analysis of the data relies on being able to treat the data as *if it were* obtained in a steady state cloud. In reality clouds vary both spatially, temporally and are affected by sedimentation, but "...for a source which varies slowly in time, there can be a region below that behaves as if the source were infinite and steady…" (Lo and Passarelli, 1982) when sampled with a lagrangian descent. For each ~1 km of aircraft track ice water content and precipitation rate were computed using a 'two-parameter' method described by Heymsfield et al. (2002a) that makes use of ice particle projected area as well as diameter.

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2. CONSERVATION OF ICE MASS FLUX DENSITY

Bearing in mind the assertion that we are able to treat a region of spatial and temporally varying cloud as if it was in steady-state when we sample it with a lagrangian descent then we can proceed as follows. Consider the following scenario: in an ideal steady-state cloud, mass is continually replaced at cloud top at a constant rate and transported to lower altitudes by sedimentation while aggregation is occurring. For this scenario no diffusional growth, sublimation or melting is considered. It can be seen that aggregation will lead to larger particles and hence greater fall speeds lower down in the cloud and so ice water content (IWC) will decrease with increasing depth, but ice mass flux density will be conserved .

Aggregation phenomena have been studied extensively using computer simulations (see Meakin, 1992 for an overview) and it has been shown that the evolution of PSDs controlled by aggregation can be scaled successfully using eq. 1 which can be rewritten as

$$N(D,t) = D_1(t)^{-\theta} g(D/D_1(t)) \tag{2}$$

where, $N(D,t)$ is the PSD at time t, $D_1(t)$ is an average diameter for the PSD at time t, θ is a scaling exponent, and $g(D/D_1(t))$ is the scaled function. In this paper the average diameter, $D_1(t)$, is defined as the ratio of the first and zeroth moments of D. In a non-sedimenting system a value for θ can be found by conservation of mass, but in a sedimenting system that has reached steady state it is ice mass flux density that is conserved. So, plotting $D_1(t)^{\theta} N(D,t)$ versus $D/D_1(t)$ will result in the collapse of the PSDs onto a single curve.

Given the scaling relation the exponent θ can be found from conservation of ice mass flux density, ϕ

$$\phi(t) = \int_0^{\infty} N(D,t) m(D) v(D) \mathrm{d}D \tag{3}$$

where $m(D)$ and $v(D)$ are the mass and fall velocity of a particle with diameter, D. Assuming the following power laws represent the mass and fall velocity: $m=\alpha D^{\beta}$, $v=aD^{b}$ and letting $\alpha a=\eta$ eq. 3 becomes

$$\phi(t) = \int_0^{\infty} \eta D_1(t)^{-\theta} D^{\beta+b} g(D/D_1(t)) \mathrm{d}D \tag{4}$$

and on letting $x=D/D_1$ and rearranging we obtain

$$\phi(t) = D_1(t)^{\beta+b+1-\theta} \int_0^{\infty} \eta x^{\beta+b} g(x) \mathrm{d}x. \tag{5}$$

The integral on the right hand side is constant and so the mass flux, ϕ, will be constant if

$$\theta = \beta + b + 1. \tag{6}$$

We will see in section 3 that if we use the scaling suggested by eq. 5 then we will get an adequate collapse of the data without any assumptions being made about the form of $g(x)$. However, the function $g(x)$ is a solution to the Smoluchowski equations and the results of numerical simulation (e.g. Leighton 1980) show that $g(x)$ can have the general form,

$$g(D/D_1(t)) = AD^{\nu} \exp(-D/D_1(t)), \tag{7}$$

where ν controls the shape of the PSD at small sizes. The normalisation factor, A, can be obtained by substituting eq. 6, 7 into 4 to give

$$A = \frac{\phi D_1(t)^{-\nu}}{\eta \Gamma(\theta+\nu)} \tag{8}$$

which results in the following expression for N(D,t) when eq. 2, 7 and 8 are combined

$$N(D,t) = \frac{\phi}{\eta\Gamma(\theta+\nu)} D_1(t)^{-\theta} \left(\frac{D}{D_1(t)}\right)^{\nu} \exp\left(-\frac{D}{D_1(t)}\right) \tag{9}$$

where Γ is the gamma function. For simplicity we set ν=0 to obtain

$$N(D,t) = \frac{\phi}{\eta\Gamma(\theta)} D_1(t)^{-\theta} \exp\left(-\frac{D}{D_1(t)}\right). \tag{10}$$

Eq. 10 can be compared directly with standard form of the exponential distribution used for PSDs that was introduced by Gunn and Marshall (1958) to represent observed snow PSDs.

$$N(D,t) = N_0(t) \exp(-\lambda(t) D) \tag{11}$$

where N_0 is the *y*-intercept of the exponential distribution and λ is the gradient in log-linear space. By inspection it can be seen that

$$\lambda = \frac{1}{D_1(t)} \tag{12}$$

$$N_0 = \frac{\phi D_1(t)^{-\theta}}{\eta\Gamma(\theta)}. \tag{13}$$

If we had initially assumed an exponential distribution (eq. 11) for the PSD and integrated to obtain the ice mass flux density we would have arrived directly at eq. 10 (*cf* Mitchell, 1988 eq. 15), but by not assuming a form for the PSD we were still able to determine the value that is required for the exponent θ to conserve ice mass flux density.

To obtain θ from the observations we use the assumption that for a constant ice mass flux density, or to use the more familiar term - precipitation rate, a plot of N_0 verus λ in log space will have a gradient of θ.

3. AIRCRAFT OBSERVATIONS

Data from a lagrangian spiral descent carried out on the 9th March 2000 for the ARM field program are presented to demonstrate the scalability of ice PSDs. This flight was carried out in cirrus uncinus and started at cloud top (-50°C, 9500m) and descended to cloud base (-28°C, 6600m).

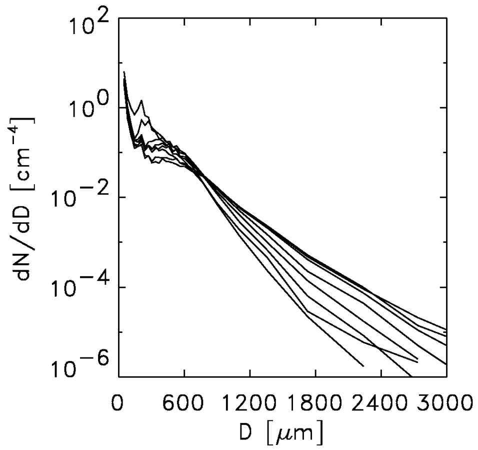

Fig.1 Particle size distributions obtained at different altitudes. The narrowest PSD was obtained at the highest altitude in the cloud, the broadest near cloud base.

Figure 1 shows PSDs averaged over each loop below 9000 m where the aircraft was in cloud (~15 km of aircraft track). Using a value for θ (=3.9) obtained from a log-log plot of N_0 verus λ the scaled spectra collapse onto a well defined function and even the mode between 200--500 µm lines up quite well (fig. 2). It can be seen that just using the scaling factor D_1^{θ} results in the data collapse onto a well defined exponential curve for $D/D_1>2$.

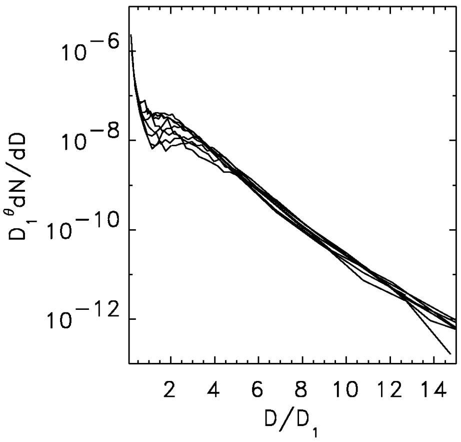

Fig.2 Scaled PSDs from fig. 1.

By using the estimates of precipitation rate, P, computed using the 'two-parameter' method and the measured PSDs we looked at how θ varied. For each descent the ~1km PSDs were binned according to P. For each P bin on each descent plots of N_0 versus λ generated estimates of θ. The result of this exercise are shown in fig. 3.

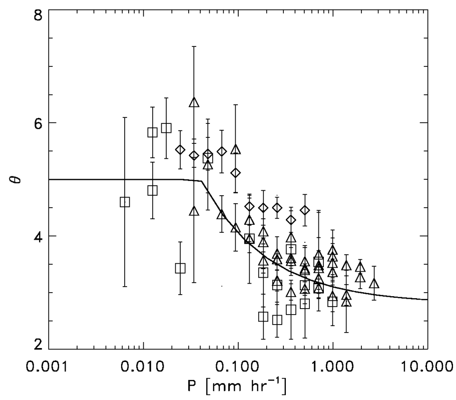

Fig.3 plot of θ as a function of P for the 14 flights. Triangle: TRMM, diamonds: ARM, squares: FIRE1. The vertical line represents two standard deviations.

What fig. 3 shows is that it is possible to choose a θ value derived from a set of m-D, v-D power law relations combined with an exponential PSD that will give estimates of P consistent with those determined from the two-parameter method using the actual measured PSD. We know that P and θ are physically linked because the value of θ is dependent upon the conservation of ice mass flux density. However we cannot necessarily say beforehand how θ will vary with P if at all. Figure 3 shows that θ is correlated with P and that θ values start off high at low P values and eventually attain a value of 3-4 for P>0.5 mm hr^{-1}. We suggest that the value of θ is controlled directly by the particle habit which may be selected to some extent by the process of binning P (e.g. synoptically generated cirrus has low P values) and the amount of aggregation that has occurred. The scatter in θ probably reflects the variation in primary crystal habit and initial growth conditions soon after nucleation. An additional factor is that high precipitation rates tend to be correlated with larger D_1 values. Hence, mass and fall velocities are weighted by the larger particles that also tend to have the lower β and b values.

Using expressions derived for η, θ as functions of estimates of P obtained from the two parameter method for each of ~2500 ~1km PSDs we have attempted to scale all of the measured PSDs from the 14 TRMM, ARM and FIRE1 flights considered onto a single curve by multiplying $\mathrm{d}N/\mathrm{d}D$ by $\eta\Gamma(\theta)D_1^{\theta}/\phi$. We can also obtain the best possible scaled function, given the inherent natural variability, by multiplying $\mathrm{d}N/\mathrm{d}D$ by the measured $\eta\Gamma(\theta)D_1^{\theta}/\phi$, that is $1/\mathrm{N}_0$. Figure 4 is a two dimensional histogram of all the scaled PSDs. The binned frequencies have been normalised for each D/D_1 interval. The dashed line is the overplotted exponential function exp($-D/D_1$) that the scaled PSDs are expected to collapse onto. The contour encompassing the greatest area contains 80% of the data and each subsequent contour contains 20% less. The exponential function fits well for $1 < D/D_1 < 9$. For $D/D_1 < 1$ the small end of the PSD is not dominated by the effects of aggregation and so the scaled function is not expected to apply. So, given $D_1(t)$ and P the PSD can be found. Hence, D_1 and P are required to estimate IWC and radar reflectivity.

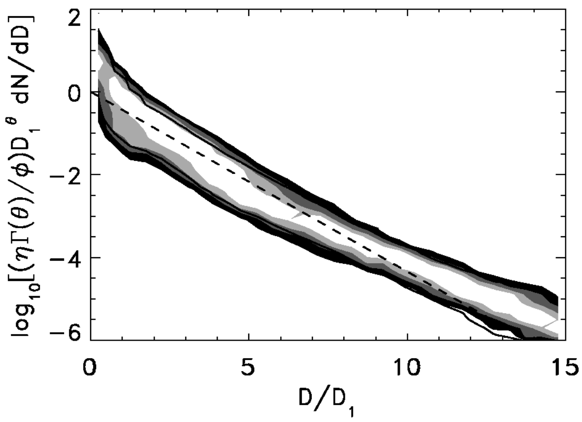

Fig. 4 contour plot of 2500 scaled spectra. Each contour is a quintile of the data. The dashed line is the function exp($-D/D_1$).

4. SUMMARY

Similar to other aggregating particle systems it is seen that PSDs evolved through ice crystal aggregation are scalable when it is assumed that ice mass flux density is conserved during aggregation. This scalability occurs in spite of the fact that the numbers of component crystals in the aggregates are only of order 10 compared to other colloidal systems where aggregates can be composed of many orders of magnitude more component crystals. The form of the scaled PSD is exponential out to at least 9 times the number weighted mean diameter, D_1.

The scaling exponent θ is correlated with precipitation rate and approaches a value of 3 for large precipitation rates. Consideration of theoretical arguments allows the quartet of parameters necessary to state the mass-diamater and fallspeed-diameter power law relations explicitly to be determined. This implies that the mass exponent, β, approaches 2 for large precipitation rates.

The introduction to a GCM of a prognosed mean diameter in tandem with IWC would lead to a more accurate treatment of the ice phase.

5. REFERENCES

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