P2.27 A linear time-delay model for orographic precipitation

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1. Introduction

Even with the development of sophisticated high-resolution mesoscale models, there is still a need for simple models of precipitation in complex terrain. Simple models can help to predict flooding, erosion, avalanche danger, glacier growth and other hydrological phenomenon at very high spatial resolution and little cost. Models of this type were proposed by Collier (1975), Rhea (1978) and Smith (1979) among others, assuming that the condensed cloud water falls immediately to the ground. More recently, Alpert and Shafir (1989) and Sinclair (1994) included the downstream drift of hydrometeors in their simple upslope models. Their methods however used an *ad hoc* wind-dependent Gaussian weighting scheme instead of directly solving an advection equation. In this note, we derive and test a new advection model that can be efficiently applied to small regions with complex terrain. The key element in the model is the idea of characteristic time scales for condensed water conversion and fallout. The concept of a microphysical time constant has been examined by Jiang and Smith (2001).

2. Model derivation

We begin by postulating a pair of linear equations describing the vertically integrated cloud water density ($q_c(x,y)$) and hydrometeor density ($q_s(x,y)$).

$$\frac{Dq_c}{Dt} = S(x,y) - q_c/\tau_c \tag{1}$$

$$\frac{Dq_s}{Dt} = q_c/\tau_c - q_s/\tau_f \tag{2}$$

where τ_c is the time constant for conversion from cloud water to rain or snow and τ_f is the time constant for hydrometeor fallout. In (1), S(x,y) is the rate of cloud water generation by moist adiabatic uplift. The last term in (2) is the precipitation at the ground

$$P(x,y) = q_s/\tau_f \tag{3}$$

In steady state, the material derivatives in (1) and (2) become

$$\frac{Dq}{Dt} = \vec{U}\cdot\nabla q \tag{4}$$

where $\vec{U}$ is the horizontal wind vector with components U and V. To solve (1-4), we Fourier transform each field (S, q_c, q_s or P) according to

$$\hat{S}(k,l) = (1/4\pi^2)\iint S(x,y)e^{-i(kx+ly)}dxdy \tag{5}$$

where k and l are the components of the horizontal wavenumber vector.

Using $\partial/\partial x \to ik, \partial/\partial y \to il$ and

$$\sigma = Uk + Vl \tag{6}$$

in (1, 2, 3, 4), an expression for the Fourier Transform of the precipitation distribution can be obtained.

$$\hat{P}(k,l) = \frac{\hat{S}(k,l)}{[1+i\sigma\tau_c][1+i\sigma\tau_f]} \tag{7}$$

This expression can be inverted to obtain the precipitation distribution (P(x,y)) using

$$P(x,y) = \iint \hat{P}(k,l)e^{i(kx+ly)}dkdl \tag{8}$$

This double Fourier Transform procedure is widely used in 3-D mountain wave theory (e.g. Sawyer, 1962, Smith 1980). The mathematical properties of the double Fourier Transform are given by Sneddon (1951) and elsewhere. The present formulation allows any wind direction to be used, without rotating the coordinate axes (Smith, 2002a). The forward and inverse Fourier transforms (5 and 8) can be done quickly using a Fast Fourier Transform (FFT) algorithm. Note that the two time constants in (7) are mathematically interchangeable.

To illustrate the properties of (7), we consider a point source of cloud water produced by a fixed isolated spot of uplift, so that $S(x,y) = A\delta(x)\delta(y)$, i.e. the product of two

Dirac delta functions. We also assume U>0 and set V=0. Inversion of (7) can be carried out using contour integration of (8). For x>0, the integration path in the complex k-plane closes in the upper half-space, encircling the two simple poles in (7). The residue theorem gives (9);

$$P(x,y) = A\delta(y)[(e^{-x/U\tau_c} - e^{-x/U\tau_f})/U(\tau_c - \tau_f)]$$

For x<0, the integration path closes in the lower half-space, including no poles. Thus P(x,y)=0. If the two tau values are equal, using L'Hospital's rule, (9) becomes

$$P(x,y) = A\delta(y)xe^{-x/U\tau}/(U\tau)^2 \quad (10)$$

while if both tau values are zero (i.e. no time microphysical delay), (7) and (8) or (9) give

$$P(x,y) = S(x,y) = A\delta(x)\delta(y) \quad (11)$$

In the general case (9), the precipitation rises downstream of the source pulse, reaches a maximum and decays exponentially. The area-total precipitation (the integral of 9, 10, or 11) is equal to the source strength A. In the case of equal taus (10), the precipitation peak is located a distance $d=U\tau$ downstream of the input pulse of cloud water. For example, if U=10m/s and τ=500 seconds, =5km.

There is some ambiguity concerning the cloud water source function S(x,y) in (1). According to Smith(1979), for an unsheared saturated moist-neutral atmosphere, the source function is given by positive values of

$$S(x,y) = \rho q_v(z=h)\vec{U}\cdot\nabla h(x,y) \quad (12)$$

In (13), $q_v(z=h)$ is the saturated mixing ratio at the ground, $\vec{U} = Ui + Vj$ is the horizontal wind vector and h(x,y) is the terrain. In real applications, several questions arise. What degree of smoothing should be applied to the terrain? How should q_v be estimated? What if the wind changes with height? The most troubling questions concern regions of descent. Should S be allowed to be negative or should it be set to zero in downslope regions? These issues are discussed in Smith (2002b).

3. An application to orographic precipitation in the Italian Alps

An opportunity to test (7) is given by data from the Mesoscale Alpine Programme (MAP) in 1999 (Bougeault, et al., 2001). An appropriate case of orographic precipitation in northern Italy occurred during the Intensive Observing Period 2b described by Smith et al (2002). On September 20, 1999, a strong southerly stream of moist air lifted over the Alps bringing heavy precipitation.

Data from the Monte Lema radar (46.042N, 8.833E) is available for a small region in northwestern Italy near Lago Maggiore (Joss et al., 1998). The terrain in the test region is complex (Figure 2). On the largest scale, the terrain rises towards the north and then descends beyond the St Gotthard pass into the Rhine Valley. On a somewhat smaller scale, the funnel shaped valley leading to Lago Maggiore is seen. On the smallest scales shown (3 to 5 km) numerous hills and ridges are seen. In spite of the general upslope nature of the southerly flow, parcels rise and fall several times as they move northward.

The predictions from the upslope models were computed in the following way. The q_v field was specified as a function of altitude using a saturated adiabat. The horizontal wind in (12) was given by vertically and time averaged Doppler data from Monte Lema. The same wind values (U=1.7, V=19.1m/s) are used for the advective velocity in (6). The terrain used in (12) is high resolution with a grid spacing of one kilometer.

The correlations in Table 1 indicate that in predicting spatial patterns, the raw upslope model has no skill, while the upslope FFT model has considerable skill. It is nearly as good as the mesoscale models.

The average precipitation values in Table 1 must be viewed with caution. The Monte Lema estimate could be too low due to terrain blockage of the radar beam. The Monte Lema radar fails to register the precipitation falling to the north of the first two ranges of foothills. Raingauge data (not shown) suggests that the COAMPS prediction might be closer to the truth. Even in this case, the upslope model is seen to greatly overestimate the precipitation as it assumes perfect precipitation efficiency. .

4. Discussion

In this paper, we derived and tested an advection model of orographic precipitation. The time-delay algorithm generally advects precipitation downwind and smooths the prediction of the raw upslope model. The smoothing caused by the microphysical time delay is small enough that the effect of cloud-water source regions on windward slopes are still clearly seen. Every hill and ridge still has its own precipitation maximum, although these are typically shifted downstream to the hill peak.

The "upslope-FFT" model (5,7,8,13), with time delays in the range of τ=200 to 1500 seconds, gives reasonable predictions of precipitation patterns in the test case. These delay values are consistent with the cloud water residence times computed from numerical models: 300 to 2000 seconds for the same case (Smith et al. 2002). They also are in the range expected for fallout times using fall speeds of 1m/s and 5m/s for snow and rain. For example, a mean cloud depth of 3km with an average terminal fall speed of 3m/s gives a delay of 1000 seconds.

While the correlation coefficients in Table 1 are good, suggesting some skill in pattern prediction, the amount of precipitation is significantly over estimated by the upslope model. The time-delay algorithm has no influence on this result, as it conserves water. For southerly flow against the Alps, we have the paradox that the overall precipitation efficiency (PE) is close to one (see Smith et al. 2002), while the PE for each hill is rather small, probably in the range of 0.1 to 0.3. Only by repeated lifting events can all the excess water be precipitated.

A more detailed evaluation of the upslope-FFT model is difficult because of uncertainties in the precipitation observations. The Monte Lema radar data are seriously affected by terrain screening and by the bright band at the melting level (3km). The Alpine raingauge network does not even approach the density needed to resolve the actual precipitation patterns.

Given current levels of understanding, the precipitation in complex terrain might best be estimated by using a few raingauges to scale down the upslope-FFT model into a reasonable range Thus, the model is used essentially as an interpolation method. It might also be reasonable to alter the time delays according to the likelihood of rain or snow.

5. Acknowledgements

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Table 1. Monte Lema Area Radar and Model precipitation totals and correlations.

Radar and Models	Parameters (τ_c, τ_f, U, V)	Average precipitation (mm)	Correlation Coefficient
Monte Lema	NA	26	1
COAMPS	NA	60	0.72
MC2	NA	37	0.79
Upslope	0,0, 1.7,19.1	506	0.47
Upslope FFT	500,500, 1.7, 19.1	450	0.71

Figure 1 Accumulated precipitation (mm, shaded) for 20 September, 1999, for a region near Lago Maggiore in northwestern Italy: a) Monte Lema radar, b) COAMPS, c) MC2, d) Upslope model (U=2, V=19, no delay), e) Upslope FFT model (U=2, V=19, τ_c=τ_f=500 seconds). The upslope model values are divided by a factor of 10 to use the same gray scale as the data and mesoscale models. Other lines in the figure show terrain and a section of the Italy-Switzerland border. The width of the scene is about 160km.